Cold dark matter with MOND scaling

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A B S T R A C T

We provide a holographic dual description of Milgrom's scaling associated with galactic rotation curves. Our argument is partly based on the recent entropic reinterpretation of Newton's laws of motion. We propose a duality between cold dark matter and modified Newtonian dynamics (MOND). We introduce the concept of MONDian dark matter, and discuss some of its phenomenological implications. At cluster as well as cosmological scales, the MONDian dark matter would behave as cold dark matter, but at the galactic scale, the MONDian dark matter would act as MOND.

1. Introduction

One of the most outstanding puzzles of contemporary physics is the nature of the "missing mass" or dark matter [1]. That cold dark matter (CDM) should exist is strongly supported by various observations such as the galactic rotation curves, the large scale structure surveys and the cosmic microwave background [1]. On the other hand, there is a remarkable observation due to Milgrom, regarding a very successful scaling observed in the galactic rotation curves that goes by the name of modified Newtonian dynamics, or MOND [2]. Milgrom's approach aims to reinterpret the "missing mass" problem as the "acceleration discrepancy", and thus points to a radical modification of gravity and the laws of motion [3]. Specifically, Milgrom postulates that the acceleration of a test mass m due to the source M is given by

$$a = \begin{cases} a_N, & a \gg a_c, \\ \sqrt{a_N a_c}, & a \ll a_c, \end{cases} \tag{1}$$

where $a_N = GM/r^2$ is the magnitude of the usual Newtonian acceleration. Coincidentally, the critical acceleration a_c is related to the speed of light c and the Hubble scale H: $a_c \sim cH/(2\pi) \sim 1.2 \times 10^{-8}$ cm/s^2. It turns out that MOND fits hundreds of galactic rotation curves very well and the Tully–Fisher relation is automatically satisfied. Since the galactic dynamics is very complex, it is not

surprising that MOND doesn't explain all of the observed galactic rotation curves. But the success of this simple MOND relation seems to suggest that it may hold the key to the problem.

The basic difference between the above two approaches is nicely encapsulated in two possible modifications of Einstein's equations of motion $G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \Lambda g_{\mu\nu} + 8\pi G_N T_{\mu\nu}$. One can either change the source term, $T_{\mu\nu} \to T_{\mu\nu} + T_{\mu\nu}^{DM}$, which is the conventional dark matter approach, or one can attempt to change the Einstein tensor $G_{\mu\nu} \to F(G_{\mu\nu})$ (where F is either a local or possibly non-local operator), and thus modify gravity, which is the approach associated with MOND.

While the CDM paradigm has its attractive features, it cannot easily explain the observed galactic flat rotation curves and the observed Tully–Fisher relation [4] that the MOND scenarios can. On the other hand, there are problems with MOND at the cluster and cosmological scales, where apparently CDM works much better [1]. This inspires us to ask: Could there be some kind of dark matter that can behave like MOND at the galactic scale?

The two ideas, one based on the existence of dark matter and one that denies it, but requires the radical modification of the laws of motion, are apparently contradicting and hence irreconcilable. However, that seemingly incompatible ideas can be incorporated in a new concept is well known in the history of physics, with the wave-particle duality being one of the most astonishing instances. Inspired by such lofty examples, we would like to suggest a reconciliation of the dark matter and MOND approaches by introducing a new concept of "MONDian dark matter". In a nutshell, *we propose a scheme such that the MONDian dark matter looks like CDM at cluster and cosmological scales, but it behaves like MOND at the galactic*

scale. This would perhaps explain the apparent failure of MOND at cluster and cosmological scales.

In fact, a preliminary attempt to mimic Milgrom's scaling from the CDM paradigm has been conducted in [5], although it has completely ignored the low-surface-brightness galaxies in which the acceleration is everywhere smaller than a_c. We think that it is important to have a deeper theoretical understanding about the connection between the nature of dark matter and Milgrom's scaling. *It is our intention to combine the salient successful features of both CDM and MOND into a unified scheme, by introducing the concept of MONDian dark matter.* Hopefully our proposal could also point to quantum gravitational origins of the "missing mass".

As a remark, it is not clear how MOND follows from a relativistic modification of Einstein's gravity. One possibility is to modify the effective metric [3] by introducing other degrees of freedom $g_{\mu\nu} \to \tilde{g}_{\mu\nu}(g_{\mu\nu}, A_\mu, \varphi, \ldots)$, where A_μ and φ correspond to the vector and scalar degrees of freedom respectively. Then, a theory is sought in terms of these degrees of freedom, which reproduces MOND and is consistent with some general symmetry principles (such as relativistic invariance, causality, etc.). However, we are not committed to a concrete model for such a modification of gravity. We simply assume that the bulk space–time gravitational theory could be deformed by these new degrees of freedom so as to imply MOND at the galactic scale (GS): $\int d^4x \sqrt{-g}[R_g + L_{SM}(\phi_g)] \to \int_{GS} d^4x \sqrt{-\tilde{g}}[R_{\tilde{g}} + L_{SM}(\phi_{\tilde{g}})]$, where L_{SM} is the Standard Model Lagrangian.

2. Entropic reinterpretations

We start with the recent work of E. Verlinde [6] in which the canonical Newton's laws are derived from the point of view of holography. Using the first law of thermodynamics, Verlinde [6] proposes the concept of entropic force $F_{entropic} = T\frac{\Delta S}{\Delta x}$, where Δx denotes an infinitesimal spatial displacement of a particle with mass m from the heat bath with temperature T. He then invokes Bekenstein's original arguments concerning the entropy S of black holes [7] by imposing $\Delta S = 2\pi k_B \frac{mc}{\hbar}\Delta x$. Using the famous formula for the Unruh temperature, $k_B T = \frac{\hbar a}{2\pi c}$, associated with a uniformly accelerating (Rindler) observer [8], he obtains Newton's second law $\vec{F} = m\vec{a}$, with the vectorial form being dictated by the gradient of the entropy.

Next, Verlinde considers an imaginary quasi-local (spherical) holographic screen of area $A = 4\pi r^2$ with temperature T. Then, he assumes the equipartition of energy $E = \frac{1}{2}Nk_B T$ with N being the total number of degrees of freedom (bits) on the screen given by $N = Ac^3/(G\hbar)$. Using the Unruh temperature formula and the fact that $E = Mc^2$, he recovers exactly the non-relativistic Newton's law of gravity, namely $a = GM/r^2$. This is precisely the fundamental relation that Milgrom is proposing to modify so as to fit the galactic rotation curves. Therefore, in view of Verlinde's proposal for the entropic [7], and thus holographic [9–11] reinterpretation of Newton's law, it is natural to ask: What entropic or holographic interpretation lies behind Milgrom's modification of Newton's second law?

First, a comment on the entropic approach. While it has not yet offered any radically new physics, the entropic approach has consistently brought together a few crucial notions in physics and has provided an alternative unifying point of view. Below we will show that this approach can be used to shed new light on dark matter.

Now to proceed, we first have to recognize that we live in an accelerating universe. This suggests that we will need a generalization of Verlinde's proposal to de Sitter (dS) space. For convenience, we set $\hbar = c = 1$ henceforth. In particular, the Unruh–Hawking

temperature, as measured by an inertial observer in de Sitter space with a positive cosmological constant Λ, is given by $T_{dS} = \frac{1}{2\pi k_B}a_0$ where $a_0 = \sqrt{\frac{\Lambda}{3}}$ [9]. Notice that Λ is related to the Hubble scale H through $\Lambda = 3H^2$. The corresponding Unruh temperature as measured by a non-inertial observer with acceleration a will be [12]

$$T_{dS+a} = \frac{1}{2\pi k_B}\sqrt{a^2 + a_0^2}. \tag{2}$$

This formula can be derived by straightforward but lengthy calculation. Instead, it can also be heuristically derived by noticing that dS^4 can be embedded into a five-dimensional Minkowski space–time M^5. World lines with proper acceleration a in dS^4 (parametrized by $\Lambda = 3a_0^2$) can be viewed as world lines with proper acceleration $\sqrt{a^2 + a_0^2}$ in M^5. Consequently, we can define the *net* temperature as measured by the non-inertial observer (due to some matter sources that cause the acceleration a) to be

$$\tilde{T} \equiv T_{dS+a} - T_{dS} = \frac{1}{2\pi k_B}\left[\sqrt{a^2 + a_0^2} - a_0\right]. \tag{3}$$

As a remark, this formula can be formally applied to anti-de Sitter (AdS) space as well by taking $\Lambda \to -\Lambda$.

Interestingly, Milgrom has suggested in [13] that the difference between the Unruh temperatures as measured by non-inertial and inertial observers in de Sitter space, namely $2\pi k_B \Delta T = \sqrt{a^2 + a_0^2} - a_0$, could give the correct behaviors of the interpolating function between the usual Newtonian acceleration and his suggested MONDian deformation for very small accelerations. Even though $\sqrt{a^2 + a_0^2} - a_0$ could somehow mimic the correct behaviors of his MOND theory, Milgrom was not able to justify why the force is related to the difference between the Unruh temperatures as measured by non-inertial and inertial observers in de Sitter space. Or, in his own words: "it is not really clear why ΔT should be a measure of inertia". Thus, without a reasonable justification, his suggestion remains to be an *ad hoc* mathematical function that can reproduce the behaviors of the MOND theory. As we will see in the next section, adopting Verlinde's entropic force point of view allows us to justify Milgrom's suggestion naturally.

3. CDM–MOND duality

Following Verlinde's approach, the entropic force, acting on the test mass m with acceleration a in de Sitter space, is given by

$$F_{entropic} = \tilde{T}\nabla_x S = m\left[\sqrt{a^2 + a_0^2} - a_0\right]. \tag{4}$$

For $a \ll a_0$, we have $F_{entropic} \approx m\frac{a^2}{2a_0}$. In order to fit the galactic rotation curves as Milgrom did, we require

$$F_{entropic} \approx m\frac{a^2}{2a_0} = F_{Milgrom} \approx m\sqrt{a_N a_c}, \tag{5}$$

$$\Leftrightarrow \quad a = \left(4a_N a_0^2 a_c\right)^{\frac{1}{4}} = \left(2a_N a_0^3/\pi\right)^{\frac{1}{4}}. \tag{6}$$

Numerically, it turns out that $2\pi a_c \approx a_0$, and so we set $a_c = a_0/(2\pi)$ for simplicity. To reproduce the flat rotation curves, we first need to realize that $F_{centripetal} = m\frac{a^2}{2a_0}$ for $a \ll a_0$. Thus, the terminal velocity v should be determined from $m\frac{a^2}{2a_0} = \frac{mv^2}{r}$, with a given by Eq. (6). Obviously, this leads to a constant v (independent of r) and hence the flat rotation curves.

On the other hand, similar to Verlinde's holographic approach which invokes the imaginary holographic screen of radius r, we can write

$$2\pi k_B \tilde{T} = 2\pi k_B \left(\frac{2\tilde{E}}{N k_B} \right) = 4\pi \left(\frac{\tilde{M}}{A/G} \right) = \frac{G\tilde{M}}{r^2}, \tag{7}$$

where $\tilde{M}$ represents the *total* mass enclosed within the volume $V = 4\pi r^3/3$. What is $\tilde{M}$? Suppose we set $\tilde{M} = M$, which means that there is only ordinary matter enclosed by Verlinde's imaginary holographic screen. In that case, we will have $F_{entropic} = ma_N$ even for $a \ll a_0$. But this implies that there is neither dark matter nor consistency with modified gravity given by Eq. (5) and Eq. (6), and as such is obviously incompatible with observations. The only way to be consistent with the observational data is to have $\tilde{M} = M + M'$ where M' is some unknown mass — that is, dark matter. Thus, we need the concept of dark matter for consistency.

In what follows, we *propose* that

$$M' = \frac{1}{\pi} \left(\frac{a_0}{a} \right)^2 M. \tag{8}$$

Note that the above formula can be generalized to AdS space, in which case the missing matter makes a *negative* contribution. With Eq. (8), we can write

$$F_{entropic} = m\left[\sqrt{a^2 + a_0^2} - a_0 \right] = ma_N \left[1 + \frac{1}{\pi} \left(\frac{a_0}{a} \right)^2 \right]. \tag{9}$$

For $a \gg a_0$, we have $F_{entropic} \approx ma \approx ma_N$, and hence $a = a_N$. But, for $a \ll a_0$, we have $F_{entropic} \approx m\frac{a^2}{2a_0} \approx ma_N(1/\pi)(a_0/a)^2$. Solving for a, we get $a = (2a_N a_0^3/\pi)^{\frac{1}{4}}$, which is exactly the same expression as required for the explanation of the galactic rotation curves. *In conclusion, using the proposal as given by Eq. (8), we can actually derive MOND.* We also observe that M' is greater for smaller a, which is consistent with the observations that there is more dark matter in the galactic halos than in the regions closer to the galactic centers.

We can now realize the idea of CDM-MOND duality. On one hand, we can interpret Eq. (9) to mean that there is *no* dark matter, but that the law of gravity is modified. On the other hand, we can rewrite it as

$$F_{entropic} = m\frac{G(M + M')}{r^2}, \tag{10}$$

where M' denotes the total mass of dark matter enclosed in the volume $V = 4\pi r^3/3$, which, by construction, is compatible with MOND. We are thus led to the very intriguing dark matter profile $M' = \frac{1}{\pi}(\frac{a_0}{a})^2 M$. Dark matter of this kind can behave *as if* there is no dark matter but MOND. Therefore, we call it "MONDian dark matter". As a remark, to obtain M' as a function of r, one can solve the cubic equation (see Eq. (9)) $\sqrt{a^2 + a_0^2} - a_0 = a_N[1 + \frac{1}{\pi}(\frac{a_0}{a})^2]$ for a general solution of $(\frac{a_0}{a})^2$ and substitute it into the expression for M'.

4. Friedmann's equations

One important issue regarding our theory is to ensure that it is completely compatible with cosmology. Thus, we would like to derive the corresponding Friedmann's equations within our framework. Our derivation follows the procedure of [14]. The FRW metric is given by $ds^2 = -dt^2 + R(t)(dr^2 + r^2 d\Omega^2)$, where $R(t)$ is the scale factor. Assume that the matter sources in the universe form a perfect fluid. Then, in the rest frame of this fluid, the energy–momentum tensor is given by $T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu}$, where $u_\mu = (1, \vec{0})$ is the four velocity of the fluid. Now, consider Verlinde's imaginary holographic screen of comoving radius r. The physical radius would be $\tilde{r} = rR(t)$. In de Sitter space, the net temperature observed by an accelerating observer (with acceleration a)

is $\tilde{T}$, which leads to the entropic force discussed above. As a result, the *effective* acceleration a_{eff} of the observer is $a_{eff} = \sqrt{a^2 + a_0^2} - a_0$, which is also given by $a_{eff} = -\frac{d^2(rR(t))}{dt^2} = -\ddot{R}r$. Using $2\pi k_B \tilde{T} = \frac{G\tilde{M}}{r^2 R^2}$, we get $\ddot{R} = -\frac{G\tilde{M}}{r^3 R^2}$. Following [14], in a fully relativistic situation, we replace $\tilde{M}$ by the active gravitational (Tolman–Komar) mass $\mathcal{M} = \frac{1}{4\pi G} \int dV R_{\mu\nu} u^\mu u^\nu$. By Einstein's field equation, we obtain $\mathcal{M} = 2 \int dV (T_{\mu\nu} - \frac{1}{2}Tg_{\mu\nu} + \frac{\Lambda}{8\pi G}g_{\mu\nu})u^\mu u^\nu = (\frac{4}{3}\pi r^3 R^3)[(\rho + 3p) - \frac{\Lambda}{4\pi G}]$. Finally, it follows that

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}, \tag{11}$$

which, with the continuity equation $\dot{\rho} + 3H(\rho + p) = 0$, can be used to obtain the other Friedmann's equation, viz.

$$H^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3}. \tag{12}$$

We thus conclude that the corresponding Friedmann's equations derived from our framework are exactly the same as the usual ones. The following remark is in order: The entropic approach has *not* replaced general relativity as sometimes misconstrued. To do cosmology, we still need Einstein's general relativity. Indeed we have just shown that our theory would be completely consistent with the ΛCDM model, if the quantity ρ is interpreted as the total energy density of ordinary matter and MONDian dark matter. Furthermore, in our approach, we have assumed a certain symmetry, but one that is consistent with the FRW metric. As usual, one can categorize metric perturbations based on a homogeneous and isotropic background. Of course, ultimately it is important to do cosmological perturbation theory so as to compare with observations such as CMB.

The above computation suggests that one can in principle have Einstein's gravity together with a MONDian dark matter source. The departure from MOND happens when we replace $\tilde{M}$ with $\mathcal{M}$, i.e. when a non-relativistic source is replaced by a fully relativistic source. In that case, Eq. (9) is replaced by $\sqrt{a^2 + a_0^2} - a_0 = \frac{G\mathcal{M}}{\tilde{r}^2}$, where $\tilde{r} = rR(t)$ is the physical radius, i.e.,

$$\sqrt{a^2 + a_0^2} - a_0 = \frac{G(M(t) + M'(t))}{\tilde{r}^2} + 4\pi Gp\tilde{r} - \frac{\Lambda}{3}\tilde{r}. \tag{13}$$

If $M' = \frac{1}{\pi}(\frac{a_0}{a})^2 M$ really gives the correct profile for dark matter, then Eq. (13) works well at the cluster scale without any modification of gravity. The above expression indicates that if we naively use MOND at the cluster scale, we would be missing $4\pi Gp\tilde{r} - \frac{\Lambda}{3}\tilde{r}$ which could be significant. This may explain why MOND doesn't work well at the cluster scale, despite the CDM-MOND duality realized at the galactic scale.

5. dS/CFT correspondence

So far, our arguments have been thermodynamic but not microscopic, and thus the precise nature of MONDian dark matter is still obscure. Apparently, at the galactic scale, the MONDian dark matter quanta should be massless to realize the MOND-like behavior. However, at the cluster and cosmological scales, they should become massive and hence CDM-like. In what follows, we will argue that the holographic dual picture might shed some light on this issue.

We can at least use the general concept of holography in de Sitter space [15] to understand a possible CDM-MOND crossover. First of all, any holographic formulation of MONDian bulk gravity in de Sitter space should be able to define a dictionary between the modified bulk gravity theory consistent with MOND, and some

non-gravitational degrees of freedom associated with the appropriate "boundary" holographic screen. Currently, a precise holographic dictionary exists only in asymptotic AdS space [11]. But there have been some proposals for such a holographic dictionary in the dS$_4$ space [15]. One idea is to relate the dS fields ϕ to their AdS counterparts ψ through a non-local transformation [15]: $\psi(Y) = \int dX \, K(X, Y)\phi(X)$, where dX is the invariant measure on dS$_4$ and $K(X, Y)$ is a non-local kernel that commutes with the isometries of dS$_4$. As in the AdS/CFT correspondence, one computes the on-shell bulk action S_{bulk} and relates it to the appropriate boundary correlators. Therefore, given a modified bulk (dS$_4$) theory of gravity consistent with MOND, one would expect (in the semiclassical limit) the standard holographic formula: $\langle \exp(-\int JO) \rangle = \exp[-S_{bulk}(\tilde{g}, \phi_{\tilde{g}}, \ldots)]$.

In fact, the holographic dual of a uniformly accelerating observer in de Sitter space has been examined in [16]. It was shown that the Unruh formula in de Sitter space is holographically mapped to a constant one-point function, namely $\langle O \rangle \sim$ constant, in a suitable coordinate system on the boundary [16]. Thus, an interpolation between the Newtonian acceleration and MOND would amount to modifying the one-point function in the boundary theory. Once some non-gravitational degrees of freedom, which are holographically dual to the bulk modified theory of gravity consistent with MOND, are turned on, the relevant one-point function should get "dressed", and in principle, could have different values along the renormalization group flow. The question is: Where is the crossover between the Newtonian and MOND regimes in the bulk? It is reasonable to conjecture that one can separate the global de Sitter metric from the perturbations at smaller bulk scales, while this would not be possible at larger bulk scales. Presumably, this crossover should happen around the galactic scale. Of course, only a complete microscopic theory could answer such a detailed question.

6. Phenomenological implications

The usual way of determining the effective mass of CDM particles is to first assume some couplings to the Standard Model particles and impose some "parity-like" quantum numbers that insure stable dark matter. Then, one computes the relevant cross-sections and plugs them into Boltzmann's kinetic equation. Finally, one compares the relic abundance with cosmological constraints, such as the WMAP data [1]. This logic should be repeatable in our case.

The MONDian dark matter could lead to some *distinctive phenomenological implications*: (1) The nature of MONDian dark matter quanta is constrained by the holographic non-gravitational degrees of freedom dual to the bulk modified theory of gravity. Thus, not any dark matter quantum numbers would be allowed. For instance, vector and scalar degrees of freedom may be preferred, as suggested by [3]. (2) The couplings of MONDian dark matter to the Standard Model particles could be non-standard, and perhaps even of a spin–orbit type, as implied by remarkable particle physics realizations of the Unruh effect [17]. (3) In our proposal, the total mass of MONDian dark matter is related to the cosmological constant as well as the total mass of ordinary matter. This seems to suggest that the microscopic MONDian dark matter degrees of freedom would know about the cosmological constant. Such "non-locality" is obviously a unique feature not shared by any other dark matter candidates. (4) Our scheme (see Eq. (6)) may also hint at a fixed energy density ratio between the different cosmological components of the Universe, thus helping to alleviate the coincidence problem.

7. Conclusions

In this Letter, we have provided a holographic dual description of Milgrom's scaling associated with galactic rotation curves. We have proposed a duality between cold dark matter and modified Newtonian dynamics (MOND), encapsulated in the new concept of MONDian dark matter. Work on a phenomenological model of MONDian dark matter is in progress.

Acknowledgements

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