Flight Loads and Dynamics of Flexible Air Vehicles

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Abstract: Based on the equations of motion of flexible air vehicles including rigid-body modes and elastic structural modes, and applying influence coefficients of linear aerodynamics, a set of equations are derived and a method is presented for analysis of flight loads and dynamic characteristics. The problems in the fields of flight mechanics and aeroelasticity such as static aeroelastic divergence, trim and deformation, aerodynamic loads distribution, flutter and flight dynamics can be solved by the procedure. An airplane with high aspect ratio wings is analyzed, and the results show that the coupling between rigid-body modes and elastic modes is distinct and should not be overlooked.

Key words: flexible air vehicle; aeroelasticity; flight dynamics; flight load

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摘 要: 在综合考虑了刚体运动自由度和弹性自由度的弹性飞行器运动方程的基础上, 采用了线性气动力的影响系数形式, 提出了一套飞行载荷与动态特性分析的方程和方法, 用于解决飞行力学和气动弹性力学中的静气动弹性发散、配平与结构变形、气动载荷分布、颤振与飞行动态特性等问题。算例表明, 该方法有效可行, 且对于大柔性飞行器, 刚体运动模态与弹性模态之间的耦合是显著的, 应予以重视。

关键词: 弹性飞行器; 气动弹性; 飞行动力学; 飞行载荷
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Aeroelasticity and flight dynamics are two important disciplines in the fields of aircraft design. But for a long period of time, there had been a gap between the two subjects because of the sufficient frequency separation between the rigid-body dynamics and the aircraft's structural dynamics. The traditional flight dynamics puts its interests in the rigid-body motions and control of flight vehicles, including pitch, plunge, roll, spire and Dutch-roll modes, with the hypothesis of rigid body, while the traditional aeroelasticity studies mainly the coupling effects between elastic motions of structure and aerodynamics, such as divergence and flutter, in which the mathematic models generally consider only elastic modes.

With the development of aerospace science and technology, the structure flexibility becomes larger, which results in lower structural dynamic frequencies. For some large aspect-ratio aircrafts, the first elastic frequencies are close to the rigid-body frequencies. For example, the first symmetric bending mode of Boeing's preliminary High Speed Civil Transport conceptual design is predicted to be about 1.4 Hz, and the first symmetric wing bending mode of MIT's human-powered airplane is 0.56 Hz. The coupling between the rigid-body dynamics and the structural dynamics is indicated to be significant, and a set of integrated rigid-body modes and elastic modes equations of motion is needed for analyzing flight dynamics and aeroelastic characteristics of elastic flight vehicles correctly.

Rodden established the equations of motion of elastic airplane for quasi-steady flight in Reference [1], considering the steady aeroelastic characteristics of the restrained airplane. Later his work became the theoretical basis of steady aeroelasticity analysis module in MSC/NASTRAN. Waszak and Schmidt used the Lagrangian approach to develop

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the equations of motion for elastic airplanes in mean-axes in Ref. [2], and applied strip theory to gain the additional unsteady aerodynamic forces due to elastic vibration. The results of B-1 airplane demonstrates the significant influence of the elastic modes on flight dynamics.

The contribution of the paper is to apply the work available and more general aerodynamics form, present the equations and approaches of analyzing flight loads and dynamics for flexible air vehicles, and unify flight dynamics and aeroelasticity.

1 Flexible Airplane's Equations of Motion

In modeling of flexible air airplanes, the assumption is adopted that a flat, non-rotating Earth model, and the absence of rotating machinery and fluid flow in oil tank. In developing equations of motion of any unconstrained elastic system, inertial coupling can occur between the rigid-body degrees of freedom and the elastic degrees of freedom unless an appropriate choice for the local body-reference coordinate system is used. The noninertial reference system, which moves with the body but is not fixed to a material point in the body, is a "mean axis" system. The mean axes are defined as such that the relative linear and angular momenta due to elastic deformation are zero at every instant[3].

The structural deformation $\boldsymbol{d}$ is assumed sufficiently small, so that it can be linearly superposed by some modes

$$\boldsymbol{d} = \sum_i \Phi_i \xi_i \tag{1}$$

in which, Φ_i is the normal elastic mode of free vibration of the structure, ξ_i is the elastic generalized coordinate. The kinetic and potential energies are deduced in terms of quantities defined in the mean-axes, then the equations of motion of the flexible air vehicles can be gained by using Lagrangian's equation. Considering the symmetry about longitudinal plane, the equations can be expressed as follows[2]

$$\left.\begin{aligned}
&M_i\ddot{\xi}_i + M_i\omega_i^2\xi_i = Q_{\xi_i},\ i = 1, 2, \cdots \\
&m(\dot{u} - rv + qw) = F_x \\
&m(\dot{v} - pw + ru) = F_y \\
&m(\dot{w} - qu + pv) = F_z \\
&I_{xx}\dot{p} - I_{xz}\dot{r} + (I_{zz} - I_{yy})qr - I_{xz}pq = M_x \\
&I_{yy}\dot{q} + (I_{xx} - I_{zz})pr + I_{xz}(p^2 - r^2) = M_y \\
&I_{zz}\dot{r} - I_{xz}\dot{p} + (I_{yy} - I_{xx})pq + I_{xz}qr = M_z
\end{aligned}\right\} \tag{2}$$

where u, v, w and p, q, r are the components in the mean-axes of velocity and angular velocity relative to the inertial system, Q_{ξ_i} is the generalized force corresponding to the ith elastic mode coordinate, F_x, F_y, F_z and M_z, M_y, M_z are the components in the mean-axes of the external forces and moment.

The external forces on the airplane in atmosphere generally include gravity force, jet thrust force and aerodynamic forces. Due to the orthogonality between the elastic modes and rigid-body modes of free vibration, the work of gravity force on the structural deformation equals zero. The Euler angles ϕ, θ, Ψ are used to define the inertial orientations of the mean axes, and the generalized forces can be expressed as the sum of three parts below

$$\begin{bmatrix} Q_{\xi_i} \\ F_x \\ F_y \\ F_z \\ M_x \\ M_y \\ M_z \end{bmatrix} = \begin{bmatrix} A_{\xi_i} \\ A_u \\ A_v \\ A_w \\ A_p \\ A_q \\ A_r \end{bmatrix} + \begin{bmatrix} 0 \\ -mg\sin\theta \\ mg\cos\theta\sin\phi \\ mg\cos\theta\cos\phi \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} T_{\xi_i} \\ T_{Fx} \\ T_{Fy} \\ T_{Fz} \\ T_{Mx} \\ T_{My} \\ T_{Mz} \end{bmatrix} \tag{3}$$

in which, the first term is the generalized aerodynamic forces, the last two terms are the generalized forces due to gravity force and thrust force.

For the convenience of expression, the elastic modes, the rigid-body modes and the control surface deflection are denoted as vectors ξ, η and δ

$$\xi^{\mathrm{T}} = [\xi_1 \ \cdots \ \xi_n]$$

$$\eta^{\mathrm{T}} = [\Delta u \quad v \quad w \quad p \quad q \quad r]$$

$$\delta^{\mathrm{T}} = [\delta_x \quad \delta_y \quad \delta_z]$$

in which $\Delta u = u - V_0$, here V_0 is the flight speed. The generalized forces corresponding the elastic modes and the rigid-body modes are denoted as $\boldsymbol{A}_\xi$ and $\boldsymbol{A}_\eta$

$$\boldsymbol{A}_\xi^{\mathrm{T}} = [A_{\xi 1} \quad \cdots \quad A_{\xi n}]$$

$$\boldsymbol{A}_\eta^{\mathrm{T}} = [A_u \quad A_v \quad A_w \quad A_p \quad A_q \quad A_r]$$

On the assumption of linear perturbation, the aerodynamic forces are linear functions of state vectors ξ, η and control vector δ, that is,

$$\begin{bmatrix} \boldsymbol{A}_\xi \\ \boldsymbol{A}_\eta \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}_{\xi 0} \\ \boldsymbol{A}_{\eta 0} \end{bmatrix} + \frac{1}{2}\rho V_0^2 \begin{bmatrix} \boldsymbol{A}_\xi^\xi & \boldsymbol{A}_\xi^\eta & \boldsymbol{A}_\xi^\delta \\ \boldsymbol{A}_\eta^\xi & \boldsymbol{A}_\eta^\eta & \boldsymbol{A}_\eta^\delta \end{bmatrix} \begin{bmatrix} \xi \\ \eta \\ \delta \end{bmatrix} \quad (4)$$

in which $\rho V_0^2/2$ is the dynamic pressure. The generalized aerodynamic influence coefficient matrices can be calculated by using harmonic oscillating unsteady aerodynamics theories, and they are complex functions which depend on the reduced frequency k. When $k = 0$, the aerodynamicforces becomes steady.

2 Flight Loads and Dynamics

Flight loads and dynamics can be analyzed on the basis of the equations of motion presented above, which integrate the elastic-body and the rigid-body degrees of freedom of flexible ari vehicle. The analysis contents involve the areas of aeroelasticity and flight mechanics, including static aeroelastic divergence, trim and structural deformation, distribution of aerodynamic loads, flutter and flight dynamics.

2.1 Static aeroelastic divergence

Neglecting the work of thrust force on the structural deformation, the equations corresponding to the elastic modes in Eq. (2) on the condition of steady deformation $\dot{\xi} = \ddot{\xi} = \mathbf{0}$ have the expression as follows

$$\boldsymbol{K}\xi = \boldsymbol{A}_{\xi 0} + \frac{1}{2}\rho V_0^2 (\boldsymbol{A}_\xi^\xi \xi + \boldsymbol{A}_\xi^\eta \eta + \boldsymbol{A}_\xi^\delta \delta) \quad (5)$$

where $\boldsymbol{K} = \mathrm{diag}(M_1\omega_1^2 \quad \cdots \quad M_n\omega_n^2)$ is the stiffness matrix corresponding to the elastic modes, $\boldsymbol{A}_\xi^\xi$, $\boldsymbol{A}_\xi^\eta$ and $\boldsymbol{A}_\xi^\delta$ are the steady aerodynamic influence coefficients at $k = 0$.

If the coefficient matrix of ξ is singular, then

$$\det(\boldsymbol{K} - \frac{1}{2}\rho V_0^2 \boldsymbol{A}_\xi^\xi) = 0 \quad (6)$$

If $V_0 > 0$ and the above formula is satisfied, then the static aeroelastic divergence occurs, and the critical V_0 is called divergence speed.

2.2 Trim and structural deformation

Because of the effects of flexibility, the air vehicle would have different trim parameters under some steady flight conditions. In the region of divergence avoidance, the elastic generalized coordinates can be solved from Eq. (5),

$$\xi = (\boldsymbol{K} - \frac{1}{2}\rho V_0^2 \boldsymbol{A}_\xi^\xi)^{-1} \cdot \left[\boldsymbol{A}_{\xi 0} + \frac{1}{2}\rho V_0^2 (\boldsymbol{A}_\xi^\eta \eta + \boldsymbol{A}_\xi^\delta \delta)\right]$$

Substituting the results into Eq. (2), the nonlinear equations of motion of six degrees of freedom are gained. In the equations, there are six independent variables, and the others are non-independent. In some flight conditions, if the non-independent variables are given, then the trim variables η and δ can be determined. And applying mode coordinates transformation, the structural deformation $\boldsymbol{d}$ can be yielded by Eq. (1).

2.3 Distribution of flight loads

Knowing the state vectors ξ, η and control vector δ in some flight conditions, the distribution of the aerodynamic pressures on the aerodynamic surfaces can be obtained as follows

$$\overline{A}_e = \overline{A}_{e0} + \frac{1}{2}\rho V_0^2 (\overline{\boldsymbol{A}}_e^\xi \xi + \overline{\boldsymbol{A}}_e^\eta \eta + \overline{\boldsymbol{A}}_e^\delta \delta) \quad (7)$$

in which, $\overline{\boldsymbol{A}}_e^\xi$, $\overline{\boldsymbol{A}}_e^\eta$ and $\overline{\boldsymbol{A}}_e^\delta$ are the steady pressure influence coefficients calculated by aerodynamic computing approaches.

2.4 Flutter and dynamic characteristics

The analysis of flutter and dynamic characteristics is based on the linearized equations at the equilibrium condition. On the assumption of small perturbation, linearizing Eq. (2) at the equilibrium and neglecting 2-order small quantities, the linear small perturbation equations of motion of flexible air vehicle are gained as follows

$$\begin{bmatrix} \boldsymbol{M}\ddot{\xi} + \boldsymbol{K}\xi \\ m\Delta\dot{u} \\ m(\dot{v} + V_0 r) \\ m(\dot{w} - V_0 q) \\ I_{xx}\dot{p} - I_{xz}\dot{r} \\ I_{yy}\dot{q} \\ I_{zz}\dot{r} - I_{xz}\dot{p} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ -mg\Delta\theta\cos\theta_0 \\ mg\phi\cos\theta_0 \\ -mg\Delta\theta\sin\theta_0 \\ 0 \\ 0 \\ 0 \end{bmatrix} +$$

$$\frac{1}{2}\rho V_0^2\begin{bmatrix} \boldsymbol{A}_{\xi}^{\xi} & \boldsymbol{A}_{\xi}^{\eta} & \boldsymbol{A}_{\xi}^{\delta} \\ \boldsymbol{A}_{\eta}^{\xi} & \boldsymbol{A}_{\eta}^{\eta} & \boldsymbol{A}_{\eta}^{\delta} \end{bmatrix}\begin{bmatrix} \xi \\ \eta \\ \delta \end{bmatrix}$$

in which $\boldsymbol{M}=\mathrm{diag}(M_1 \quad \cdots \quad M_n)$ is the generalized mass matrix, and the aerodynamic force term is denoted as the simplified form.

The generalized aerodynamic influence coefficient matrices are complex functions of the reduced frequencies on the assumption of harmonic oscillation. For the convenience of modeling and analysis in the state space, a rational function approximation is required to fix the unsteady aerodynamic matrices in frequency domain to the transfer function matrices in Laplace domain. Usually three approaches are applied, including the least square (LS) method, the modified Pade method and the minimum states (MS) method[4].

Substituting the rational function approximation of the unsteady aerodynamic force, Eq. (1) can be rearranged into the state-space form

$$\dot{\boldsymbol{x}}_s = \boldsymbol{A}_s\boldsymbol{X}_s + \boldsymbol{B}_s\boldsymbol{u}_s \tag{8}$$

in which the state vectors and control vectors are

$$\boldsymbol{x}_s^{\mathrm{T}} = [\xi^{\mathrm{T}} \quad \xi^{\mathrm{T}} \quad \Delta u \quad v \quad w \quad p \quad q \quad r \quad \phi \quad \Delta\theta \quad \boldsymbol{x}_a^{\mathrm{T}}]$$

$$\boldsymbol{u}_s^{\mathrm{T}} = [\delta^{\mathrm{T}} \quad \dot{\delta}^{\mathrm{T}} \quad \ddot{\delta}^{\mathrm{T}}]$$

here $\boldsymbol{x}_a$ is the aerodynamic augmented states.

In the flight control loops of air vehicles, there usually locate sensors, such as accelerometers and angular rate gyros, to perceive the kinetic signals and feed them back to the loops. Generally the signals perceived by the sensors could be effected by the elastic vibration and the positions of the sensors. For instance, the pitch rate perceived by gyro in the longitudinal loop can be expressed as

$$\omega_y = q - \frac{\partial\Phi}{\partial x}\dot{\xi} \tag{9}$$

in which $\partial\Phi/\partial x$ is the slope rate of mode at the position of the pitch rate gyro. Then the output equation of the system can be denoted as follows

$$\boldsymbol{y}_s = \boldsymbol{C}_s\boldsymbol{x}_s + \boldsymbol{D}_s\boldsymbol{u}_s \tag{10}$$

Combining Eqs. (8) and (10), the state space equations of the airframe subsystem are yielded.

According to the theories about linear system, the system is stable if and only if the eigenvalues of the state matrix $\boldsymbol{A}_s$ are located in the left-half complex plane. Here, the state matrix of the airframe subsystem relates to the flight speed, and the eigenvalues change their locus with the speed. When the locus of a eigenvalue crosses the imaginary axis from the left-half complex plane, the system is critically stable. Then the speed is called flutter speed, and the imaginary part of the eigenvalue is flutter frequency.

Based on the state-space equations of the flexible air vehicle, the transfer function matrix from $\boldsymbol{u}_s$ to $\boldsymbol{y}_s$ is obtained as

$$\boldsymbol{G}(s) = \boldsymbol{C}_s(s\boldsymbol{I} - \boldsymbol{A}_s)^{-1}\boldsymbol{B}_s + \boldsymbol{D}_s \tag{11}$$

Then the dynamic characteristics of the system can be analyzed in frequency domain by applying the frequency response functions.

3 Example and Results

3.1 Model and vibration characteristics

A large aspect-ratio unmanned airplane is chosen as an example to demonstrate the theories and the approaches described above. And the flight loads and dynamic characteristics of longitudinal direction in the condition of steady symmetry flight is analyzed.

Firstly, a finite element model (FEM) of the airplane is constructed and its free vibration characteristics are calculated by using MSC/NASTRAN program. The first seven modes and frequencies are listed in Table 1, including the rigid-body modes and the major elastic modes. The seven modes approximately reflect the characteristics of aeroelasticity and flight dynamics.

Table 1 Modes and frequencies of free vibration

Order	Mode	Frequency/Hz
1	plunge	0.0
2	pitch	0.0
3	1st wing bending	2.66
4	1st fuselage bending	6.89
5	2nd wing bending	9.15
6	3rd wing bending	20.97
7	1st wing torsion	24.71

The steady and unsteady aerodynamic forces are calculated by using subsonic doublet-lattice method (DLM), which involves the rigid-body modes, the elastic vibration modes of the airframe, and the deflection of the elevators.

3.2 Flight loads

According to the theory of static aeroelastic divergence described in subsection 2.1, the general eigenvalue problem about the elastic stiffness matrix and the elastic aerodynamic force matrix is solved. The results reveal that the divergence dynamic pressure is 16027Pa, that is, the divergence speed V_D= 162.0m/s at sea level.

The typical flight condition of the airplane is given as follows: sea level H = 0km, cruising speed V= 60m/s, steady level flight in longitudinal direction. In the condition, the undetermined independent trim variables are angle of attack α, deflection of the elevators δ, and elastic generalized coordinates ξ. Table 2 lists the trim variables in the conditions of rigid airplane and flexible airplane.

Table 2 Trim variables in level flight condition

Trim variables	Rigid Airplane	Flexible Airplane
α	2.08°	1.94°
δ	- 1.77°	- 2.22°
ξ_1	0.0	1.87750
ξ_2	0.0	- 0.04236
ξ_3	0.0	- 0.08057
ξ_4	0.0	0.00845
ξ_5	0.0	0.02218

Substituting the elastic coordinates into Eq. (1), the elastic deformation of the structure is gained. Fig. 1 shows the bending deformation of

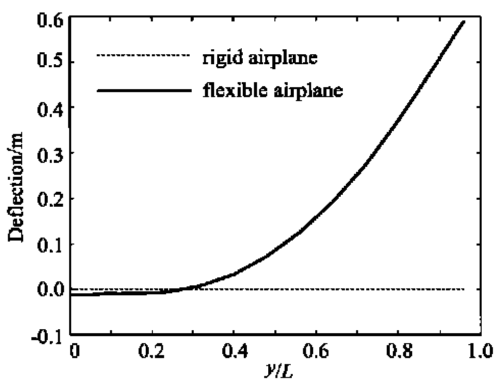

Fig. 1 Bending deformation of the wing

the fore beam in the wing of the airplane, in which the horizontal axis denotes the dimensionless span wise position percentage. The figure demonstrates that the deflection of the wing tip relative to the fuselage axis is about 0.6m, and the angle of rotation at wing tip is abont 0.168 rad. In the condition of large deformation, the geometry nonlinear effects may occur, but they are not discussed here.

Aerodynamic force distribution on the lift surfaces is obtained by applying steady aerodynamic force influence coefficients and trim variables. Fig. 2 shows the lift distribution along the wing span in the conditions of rigid airplane and flexible airplane. The comparison indicates that the lift on the wing tip of the flexible airplane is larger due to flexibility, and it is adverse for strength of the wing root.

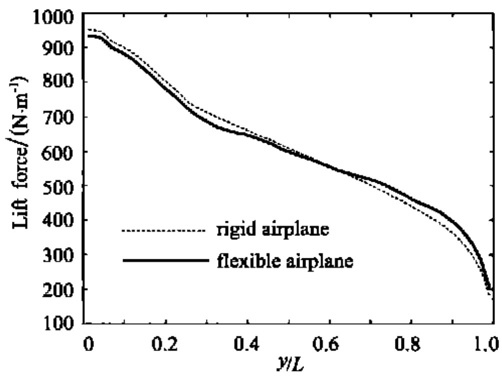

Fig. 2 Lift distribution along the wing span

3.3 Dynamic characteristics

The state space model of the airframe subsystem is gained by linearization at the equilibrium condition. Fig. 3 shows the locus of the eigenvalues of the state matrix with the flight speed increasing. When V = 89.0m/s, the locus of the 5th elastic mode crosses the imaginary axis, and the flutter frequency is 84.7rad/s.

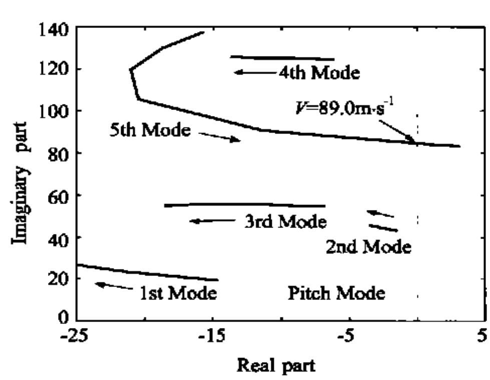

Fig. 3 Eigenvalue locus of the state matrix

In the typical flight condition, the comparison between the rigid airplane and the flexible airplane and the flexible airplane demonstrates that the eigenvalues vary with the flexibility. Under the influence of aeroelasticity, the damping of the short period mode decreases from 0.80 to 0.50. The effects would degrade the flight quality, and should be noticed in the design of flight control system.

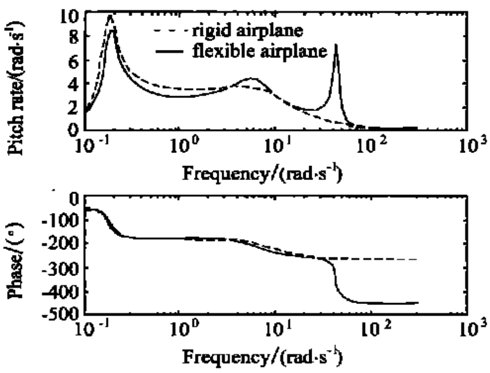

Fig. 4 Frequency response of pitch rate

Fig. 4 shows the frequency response of pitch rate to the deflection of the elevator in the typical condition. From the figure it is shown that the response curves of the rigid airplane and the flexible airplane are basely match together at low frequencies, but have large difference at elastic vibration frequencies. The curves of the flexible airplane show a peak in gain and 180° decrease in phase. Traditional flight control system is designed without considering the influence of elastic modes, which may lead to aeroservoelastic unstability due to adverse coupling between structural flexibility and control dynamics.

4 Concluding Remarks

An integrated flight and structural modes model is needed for analysis and synthesis in aeroelasticity and flight mechanics for large flexible air vehicles. Based on the equations of motion of elastic flight vehicle, a set of equations are derived and a method is presented for analysis of flight loads and dynamic characteristics. The problems in the fields of flight mechanics and aeroelasticity such as static aeroelastic divergence, trim and deformation, distribution of aerodynamic loads, flutter and flight dynamics can be solved by the procedure.

An unmanned airplane with high aspect ratio wing is analyzed as an example. The results reveal that the approach presented in the paper is effective, and on the another hand, it shows that the coupling between rigid-body modes and elastic modes is obvious. It should be noticed in the conceptual and preliminary design of aircraft.

In addition, the state space equations of flexible air vehicles established in the paper can be applied in robust stability analysis and active control design in the further studies.

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