Pricing and hierarchical logit-based mode choice models in a multimodal corridor with trip-chain costs

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Abstract

This paper proposes four pricing models for a multimodal corridor transportation system with trip-chain costs. The transportation system comprises a subway parallel to a bottleneck-constrained highway between a residential area and a workplace. Commuters can get their destination by either auto mode or transit mode only; besides these, they can first drive to the bottleneck, park there and then take subway to the destination. The solutions of these four models correspond to the hierarchical logit-based stochastic equilibria among travel costs and trip benefits with different optimization objectives. It is shown that when transit and park-and-ride place are operated by government and parking lot at working area belongs to the private enterprise, lower transit fares and higher parking fee in the central area can effectively encourage P&R mode choice, increase transit attraction and maximize the total net benefit of the system. Numerical results support the on-going differentiated parking charge policy in Beijing.

Keywords: systems engineering; pricing; mode choice; park-and-ride; trip chain; hierarchical logit model; bottleneck model

1. Introduction

As the result of urban revitalization, many metropolitan areas have witnessed explosive growth of traffic demand. Due to limited supply, traffic demand management is imperative at present. One of the strategies is to encourage park and ride (P&R) travel, namely guiding auto commuters to park at bottleneck, and then take the high capacity public transport mode to finish rest part of the trip. In Beijing, 26 large sized P&R facilities are expected to be built to connect new transit lines during the Twelfth Five-Year Plan period. Meanwhile, parking fee mechanism will be established according to the parking lots’ areas, positions, time intervals and forms. Since April 1, 2011, a new differentiated parking fee policy has been implemented, keeping lower charges in P&R parking lot while raising fees at center parking lot. However, it should be pointed out that though the economic regulation measures are helpful to improve the public transit attraction and ease the metropolitan transportation pressure, it will certainly intensify the competition among government and parking lot owners. Therefore, the nature of the differentiated pricing policy should be further discussed.

Many studies on trip distribution in a multimodal corridor have been carried out [1-4]. Tabuchi [5] and Huang [6]...
analyzed the choice behavior between private car and subway based on deterministic and multinomial logit stochastic equilibrium respectively. On the basis of [5], Tian et al. [7] made an important extension by adding a P&R option at bottleneck, yet they still assumed commuters’ mode choice behavior follows multinomial logit stochastic equilibrium. Although this model is more close to the reality than deterministic model [8], it assumes all options are independent. In fact, it is not suitable for the case with P&R option which is a combination mode of auto and transit. On the contrary, hierarchical logit model is more appropriate for mode split prediction when correlation exists among the involved transport modes.

Traditional trip distribution models mainly focus on a single trip such as “home to work” or “work to home” trip. It may lead to inappropriate prediction of travel distribution or evaluation of traffic policy because of the separation of round trip [9]. In fact, there exists close connection between ‘to’ and ‘from’ travelers, especially for those by car. Whether the car is parked at work area or P&R station, the driver has to pick it up at the parking lot then back home. Such “home-work-home” round travel is the most simple and common trip chain [10].

Considering the “home-work-home” trip chain costs, this paper investigate fares, parking fees and corresponding modal split under four market schemes in the railway/highway parallel corridor with P&R option. Commuters first decide to travel by car or subway, and then those by auto will select either P&R at bottleneck or continuing driving to work. Their choice behavior is depicted by a hierarchical logit stochastic equilibrium model. These four schemes are: net social benefit maximization, profit maximization of a transit company only, profit maximization of a parking lot management company only and duopoly price competition, respectively. Findings provide insights for parking fee policy and transportation system design.

2. Hierarchical logit-based models with trip chain

2.1. Basic description

As shown in Fig.1, H (a residential area or home) and W (a workplace) are connected by a simplified two-direction corridor with a parallel railway/highway system. A P&R parking lot (P2) and a transfer hub (TS) are located at the highway bottleneck (B). Also, a parking lot (P1) is located in the workplace. Denote distance from H to B and B to W as $L$ and $l$ respectively, and the highway capacities of the two directions are $s$ and $\bar{s}$ respectively. For simplification, directions of home to work and back home are represented by $\rightarrow$ and $\leftarrow$, respectively [11].

![Fig. 1 Two-direction corridor with a bottleneck](image)

There are three types of modes providing transportation service: transit mode only, auto mode only and P&R mode. Denote $N_1$, $N_2$ as the numbers of commuters who not use and use the private car, respectively. $N_{21}$ and $N_{22}$ represent the numbers of commuters choosing P&R and auto only, respectively, as in Fig. 2. $N$ is the total number of travelers between H and W. We then have $N_{21} + N_{22} = N_2$, and $N_1 + N_2 = N$.

2.2. Mode choice based on hierarchical logit stochastic equilibrium model

Since P&R has correlation with auto and transit modes, hierarchical logit stochastic equilibrium model is more suitable to describe mode choice behavior than multinomial logit model [8]. The decision process is shown in Fig. 2.
Here we assume that the P&R and auto only modes are relevant options. Similar analysis can also be carried out supposing that the P&R and transit only modes are relevant.

\[
\text{CBD commuters}
\]

\[
\begin{array}{c}
\text{Auto-2} \\
\text{Transit only-1} \\
\text{Auto only-22} \\
\text{P&R-21}
\end{array}
\]

Fig. 2 Structure of a hierarchical logit model

We use the generalized utility function to characterize each mode as follows:

\[
V_i = U_0 - C_i + \xi_i, \quad (1a)
\]
\[
V_{2i} = U_0 - C_{2i} + \xi_{2i}, \quad (1b)
\]
\[
V_{22} = U_0 - C_{22} + \xi_{22}, \quad (1c)
\]

where \( U_0 \) is a constant term representing the utility received through a working trip, it could be related to individual’s daily income; \( C_i \), \( C_{2i} \) and \( C_{22} \) are the travel costs of selecting transit, auto and P&R modes respectively; \( \xi_i \) and \( \xi_{2i} \) represent the perception errors in specifying the mode utilities. Hence, the conditional utilities of selecting P&R and auto modes are:

\[
V_{qP} = -C_{2i} + \xi_{2i}, \quad (2a)
\]
\[
V_{2P} = -C_{22} + \xi_{22}, \quad (2b)
\]

Suppose the random terms \( \xi_i \) and \( \xi_{2i} \) be identically and independently distributed Gumbel variables with mean zero, and variances of them are \( \sigma_0 \) and \( \sigma_{\omega} \), respectively, then at equilibrium the modal split at aggregate demand level is governed by the two logit formula specified below:\[8^\text{th}]

\[
N_{2i} = N \frac{\exp(-\omega C_{2i})}{\sum_{j=1,2} \exp(-\omega C_{2j})}, i = 1, 2, \quad (3)
\]
\[
N_i = N \frac{\exp(-\theta C_i)}{\sum_{j=1,2} \exp(-\theta C_j)}, i = 1, 2, \quad (4)
\]

where \( \omega = \pi / \sqrt{6\sigma_0} \) and \( \theta = \pi / \sqrt{6\sigma_0} \). Obviously, the larger the values of \( \omega \) and \( \theta \), the smaller the perception error to travel utility. This means commuters prefer to select the mode with the minimal measured travel cost. In addition, the modal splits (3) and (4) are not affected by adding an identical constant to the function (1).

Using the hierarchical logit model, the expected travel cost of selecting auto mode \( C_2 \) can be calculated by

\[
C_2 = -\frac{1}{\omega} \ln \left( \sum_{j=1,2} \exp\left(-\omega C_{2j}\right) \right). \quad (5)
\]

Similarly, the expected travel cost of all commuters is:

\[
C = -\frac{1}{\theta} \ln \left( \sum_{j=1,2} \exp\left(-\theta C_j\right) \right), \quad (6)
\]
where $\theta \leq \omega$ must hold in Eqs. (3)-(6) in accordance with the discrete choice theory [8]. Generally, the overall demand $N$ is inversely proportional to the expected travel cost $C$, i.e., $N = D(C)$, satisfying $dN/dC < 0$. Let $B(N) = D^{-1}(N)$ denote the inverse demand function or the marginal trip benefit with the property $dB(N)/dN < 0$. According to the representative traveler utility theory [12], an equilibrium between individual travel costs and trip benefit at expectation level can be formulated as follows (proof is omitted here for saving space):

$$
\begin{align}
(\ln N_1 + 1) / \theta + C_1 &= (\ln N_2 + 1) / \theta + C_2 = (\ln N + 1) / \theta + B(N), \\
(\ln N_{21} + 1) / \omega + C_{21} &= (\ln N_{22} + 1) / \omega + C_{22} = (\ln N_2 + 1) / \omega + C_2,
\end{align}
$$

(7a)

(7b)

with $N_{21} + N_{22} = N_2$ and $N_1 + N_2 = N$. $(\ln N + 1) / \theta + B(N)$ is the marginal trip benefit perceived by every commuter, and other items constitute the marginal costs perceived by commuters of each mode. It’s easy to show that the solution of Eq. (7) is a hierarchical logit-based mode split as given by Eqs. (3) and (4). Moreover, Eq. (7) will be reduced to the case of general multinomial logit model when $\omega = \theta$ [7]. Formulation of travel costs of the three modes will be discussed in the next section.

### 2.3. Formulation of trip-chain-based costs

(i) Transit mode only

The daily travel cost of transit mode only consists of travel costs in the morning and evening and the opportunity cost, i.e.,

$$
C_i = \tilde{C}_i + \tilde{C}_i + C_{i0}.
$$

(8)

The cost experienced by a transit commuter should depend on the travel time, the discomfort generated by body congestion in carriage and the transit fare, as below:

$$
\begin{align}
\tilde{C}_1 &= \alpha(L + l) / v_i + \pi_1[\tilde{q}(N_1, L) + \tilde{q}(N_1 + N_2, l)] + f_1, \\
\tilde{C}_i &= \alpha(L + l) / v_i + \pi_1[\tilde{q}(N_1, L) + \tilde{q}(N_1 + N_2, l)] + f_1,
\end{align}
$$

(9a)

(9b)

where $\alpha$ is the unit cost of travel time, $v_i$ is the average velocity of transit, $\tilde{q}(n,x)$ and $\tilde{q}(n,x)$ represent the discomfort experience by a transit commuter in the morning and evening respectively which are increasing functions of the number of commuters selecting this mode and the traveling distance, $f_1$ is the transit fare and $\pi_1$ is the unit cost of discomfort experienced by commuters.

(ii) P&R mode

Similar to (1), daily travel cost of P&R mode consists of four parts, i.e.,

$$
C_{2i} = \tilde{C}_{2i} + \tilde{C}_{2i} + C_{20} + p_i,
$$

(10)

where $C_{20}$ is the opportunity cost and $p_i$ is the parking fee at bottleneck parking lot. The cost experienced by a P&R commuter includes the travel time, the body congestion cost, auto fixed cost and the transit fare, as below:

$$
\begin{align}
\tilde{C}_{2i} &= \alpha(L / v_2 + l / v_i + \bar{T}) + \pi_2 \tilde{q}(N_1 + N_2, l) + F_2 + f_2, \\
\tilde{C}_{2i} &= \alpha(L / v_2 + l / v_i + \bar{T}) + \pi_2 \tilde{q}(N_1 + N_2, l) + F_2 + f_2,
\end{align}
$$

(11a)

(11b)

where $v_2$ is the car velocity, $\bar{T}$ and $\bar{T}$ are the time required for realizing transfer between auto and transit in the
morning and evening, respectively, \( f_2 \) is the transit fare at P&R station, \( F_2 \) is the auto fixed cost and \( \pi_2 \) is the unit cost of discomfort experienced by commuters getting on at the P&R station.

(iii) Auto mode only

If the number of auto users exceeds the bottleneck capacity, a queue develops. In order to avoid or reduce the waiting time in queue, some commuters will leave home earlier or later in the morning which generates schedule delay costs. Let \( \hat{\beta} \) be the unit cost of schedule delay arriving-early, \( \hat{\gamma} \) be the unit cost of schedule delay arriving-late. In accordance with bottleneck theory \cite{13}, at equilibrium \( N_{22} \) auto commuters have the same travel cost in the morning, i.e.,

\[
\tilde{C}_{22} = \alpha \left( L + l \right) / v_2 + \tilde{\delta} N_{22} / \tilde{s} + F_2,
\]

(12)

where \( \tilde{\delta} = \hat{\beta} \hat{\gamma} / (\hat{\beta} + \hat{\gamma}) \) and the second term in (12) is the synthetic cost of queuing time and schedule delay. Similarly, the travel cost in the evening is:

\[
\tilde{C}_{22} = \alpha \left( L + l \right) / v_2 + \tilde{\delta} N_{22} / \tilde{s} + F_2,
\]

(13)

where \( \tilde{\delta} = \hat{\beta} \hat{\gamma} / (\hat{\beta} + \hat{\gamma}) \), \( \hat{\beta} \) is the unit cost of schedule delay leaving-early, \( \hat{\gamma} \) be the unit cost of schedule delay leaving-late. Then the total daily travel cost of auto mode commuters is:

\[
C_{22} = \tilde{C}_{22} + \tilde{C}_{22} + p_0,
\]

(14)

where \( p_0 \) is the parking fee at working area. In accordance with the theoretic conditions for stability \cite{13}, we assume that \( \hat{\gamma} \geq \alpha \geq \hat{\beta} \) and \( \hat{\beta} \geq \alpha \geq \hat{\gamma} \).

3. Strategies of transit fare and parking fee under four market schemes

In this section, it should be pointed out that we assume the fare and parking fee in P&R station are collected as a whole through scanning the public transit IC card, which can effectively contain the parking behavior without transferring and encouraging the use of P&R mode. Then fares and parking fees of the round trip are compared under different schemes.

3.1. Net social benefit maximization

We derive the optimal transit fare and parking fee through maximizing the net social benefit of the system and solve the corresponding modal split. This means all facilities are operated and managed by government. The optimization problem is as follows:

\[
\text{maximize } \text{NSB} = \int_0^N B(w) dw + \left[ N \ln N / \theta - \left( N_1 \ln N_1 + N_2 \ln N_2 \right) / \theta \right]
+ N_2 \ln N_2 / \omega - \left( N_{21} \ln N_{21} + N_{22} \ln N_{22} \right) / \omega\]
- \left[ N_1 (C_1 - 2f_1) + N_{21} (C_{21} - 2f_2 - p_1) + N_{22} (C_{22} - p_0) + 2(N_1 + N_{21})c_1 + 2F_1 \right]

\]

s.t. (7) and

\[
N_1 + N_2 = N, \quad N_{21} + N_{22} = N_2, \quad N_1 \geq 0, \quad N_{21} \geq 0, \quad N_{22} \geq 0.
\]

(16)
where $c_i$ is the variable cost and $F_i$ is the fixed operating cost of transit. In (15), the integration term is the deterministic trip benefit of all commuters from traveling; the first square bracket term represents the difference between the expected values of random parts of trip benefit and cost; the second square bracket term is the total social cost eliminating fares and fees.

**Theorem 1.** In order to maximize the net social benefit, the transit fare (round trip) should be set as 

$$
F = \pi_1 [\bar{q} (N_1, L) + \bar{q} (N_1, L)] + (\pi_1 + \pi_2 N_2) \{q (N_1 + N_2, l) + q (N_1 + N_2, l) \} + 2c_i \cdot ; \text{ commuters selecting P&R mode (round trip) should be charged amounting to } (\pi_1 + \pi_2 N_2) \{q (N_1 + N_2, l) + q (N_1 + N_2, l) \} + 2c_i \cdot ; \text{ commuters who park in the workplace should pay an amount of } (\bar{q} / \bar{s} + \bar{q} / \bar{s}) N_{22} .
$$

Theorem 1 suggests that the single trip transit fare is the sum of the body congestion externality caused by a marginal transit commuter and the variable cost; the fare in the P&R station of a single trip is the sum of the body congestion externality caused by a marginal P&R commuter and the variable cost; parking fee in the workplace is the queue externality generated by an additional auto commuter. If we only consider the morning peak or evening peak travel, then parking fee will be underestimated to the amount of $\delta N_{22} / \bar{s}$ or $\delta N_{22} / \bar{s}$, ignoring the queue externality caused by evening or morning congestion.

### 3.2 Transit company’s profit maximization and parking lot operated by government

If the parking lot in the workplace is still operated by government, then the parking fee is given by $p_p = (\tilde{\delta} / \tilde{s} + \tilde{\delta} / \tilde{s}) N_{22}$. While the transit company wants to optimize the fare for maximizing profit, the problem is formulated as follows:

$$
\text{maximize } F = [2N_1 f_1 + N_2 (2f_2 + p_1)] - 2(N_1 + N_2) c_i - 2F_i \quad (17)
$$

s.t. (7), (16) and

$$
p_p = (\tilde{\delta} / \tilde{s} + \tilde{\delta} / \tilde{s}) N_{22} . \quad (18)
$$

The first square bracket term in objective function (17) is the total income of the transit corporation, including fares collected from two stations, where the parking fee at the P&R station is covered in the transit fare. The second term is the variable costs varying with the load and the last term is the fixed operating cost. Maximizing $F$ with respect to $N_1$ and $N_{21}$, we obtain:

$$
2f_1 = \pi_1 N_1 [\bar{q} (N_1, L) + \bar{q} (N_1, L)] + (\pi_1 + \pi_2 N_2) \{q (N_1 + N_{21}, l) + q (N_1 + N_{21}, l) \} + 2c_i + 1 / \theta
+ [(N_1 + N_{21}) g_{22} - N_{21} / (\eta N_2)]/(h - g_{22}),
$$

$$
2f_2 + p_1 = (\pi_1 N_1 + \pi_2 N_{21}) [\bar{q} (N_1 + N_{21}, l) + \bar{q} (N_1 + N_{21}, l)] + 2c_i + 1 / \omega
+ [(N_1 + N_{21}) h - N_{21} / (\eta N_2)] [g_{22} - 1 / (\eta N_2)]/(h - g_{22},
$$

where $1 / \eta = 1 / \theta - 1 / \omega$, $\eta > 0$, $h = 1 / (\theta N) + B'(N)$, $g_{22} = 1 / (\omega N_{22}) + 1 / (\eta N_2) + 2(\tilde{\delta} / \tilde{s} + \tilde{\delta} / \tilde{s})$ . $h$ represents the expected marginal profit of traveling.$^7$

**Theorem 2.** The fares for transit and P&R commuters generated by maximizing the profit of transit company are both higher than those by maximizing the net social benefit. The differences are

$$
\Delta_1 = l / \theta + h(N_1 + N_{21}) g_{22} - N_{21} / (\eta N_2)]/(h - g_{22}) .
$$

$$
\Delta_{21} = 1 / \omega + [(N_1 + N_{21}) h - N_{21} / (\eta N_2)] [g_{22} - 1 / (\eta N_2)]/(h - g_{22}) .
$$

Clearly, the above differences become identical when the model is the multinomial logit one. This says, when $\omega = \theta$, we have $\eta \to \infty$ and then $\Delta_1 = \Delta_{21} = 1 / \theta + (N_1 + N_{21}) / (1 / g_{22} - 1 / h)^7$. 

3.3. Parking lot owner’s profit maximization and transit operated by government

If the transit is operated by government, while the parking lot owner wants to maximize the revenue from charging the commuters by auto only. The problem is formulated as follows:

\[
\text{maximize } P = N_{22}p_o
\]  
\text{s.t. (7), (16) and}

\[
2f_1 = \pi_1N_1[q^*(N_1, L) + \hat{q}^*(N_1, L)] + (\pi_1N_1 + \pi_2N_{21})[\hat{q}^*(N_1 + N_{21}, I) + \hat{q}^*(N_1 + N_{21}, L)] + 2c_i, \\
2f_2 + p_i = (\pi_1N_1 + \pi_2N_{21})[q^*(N_1 + N_{21}, I) + \hat{q}^*(N_1 + N_{21}, L)] + 2c_i.
\]

However, the analytical comparisons of \( p_o \) with other mechanisms cannot be carried out because of its complex formulae. Section 4 will demonstrate this by numerical results.

3.4. Pricing game between transit and parking lot owner

In reality, parking lot and transit usually belong to different companies. Both of them expect to maximize own profit via pricing policy. The pricing game will reach the Nash equilibrium state after a long time. The optimization problem is as follows:

To transit company, the objective function is

\[
\text{maximize } F = [2N_1f_1 + N_{21}(2f_2 + p_1)] - 2(N_1 + N_{21})c_i - 2F_1
\]  
\text{s.t. (7) and (16).}

Similar to the method used before, we obtain:

\[
2f_1 = \pi_1N_1[q^*(N_1, L) + \hat{q}^*(N_1, L)] + (\pi_1N_1 + \pi_2N_{21})[q^*(N_1 + N_{21}, I) + \hat{q}^*(N_1 + N_{21}, L)] + 2c_i + \theta h[N_1 + N_{21}]/(\eta N_2 - h_{21}],
\]

\[
2f_2 + p_i = (\pi_1N_1 + \pi_2N_{21})[q^*(N_1 + N_{21}, I) + \hat{q}^*(N_1 + N_{21}, L)] + 2c_i + 1/\omega
\]

\[
p_o = N_{22}(\delta / \bar{s} + \tilde{\delta} / \bar{s}) + 1/\omega
\]

\[
+ N_{22} [h_{21} - 1/(\eta N_2)][(h - h_1)/(\eta N_2) + hh_2] - \pi_1\pi_2[q^*(N_1 + N_{21}, I) + \hat{q}^*(N_1 + N_{21}, L)]/[h - \pi_1\pi_2[q^*(N_1 + N_{21}, I) + \hat{q}^*(N_1 + N_{21}, L)] - (h - h_1)(h - h_2)]
\]

where

\[
h_1 = 1/(\theta N_1) + \pi_1[q^*(N_1, L) + \hat{q}^*(N_1, L) + \hat{q}^*(N_1 + N_{21}, I) + \hat{q}^*(N_1 + N_{21}, L)],
\]

\[
h_{21} = 1/(\omega N_{21}) + 1/(\eta N_2) + \pi_2[q^*(N_1 + N_{21}, I) + \hat{q}^*(N_1 + N_{21}, L)]
\]

\[h_{22} = 1/(\omega N_{22}) + 1/(\eta N_2) + (\delta / \bar{s} + \tilde{\delta} / \bar{s}).\]

They are the expected travel costs of transit, P&R and auto, respectively.
Theorem 3. Fares for transit and P&R commuters in competitive situation are higher than the results for the net social benefit maximization and the differences are \( \Delta_1 = 1/\theta + h[(N_i + N_{21})h_{22} - N_{21}/(\eta N_2)](h - h_{22}) \), 
\[ \Delta_{21} = 1/\omega + [(N_i + N_{21})h_{22} - N_{21}/(\eta N_2)][h_{22} - 1/(\eta N_2)](h - h_{22}) \], respectively.

Clearly, the two differences are equal when the model degenerates into the multinomial logit case. That is, when \( \omega = \theta \), \( \eta \to \infty \), we have \( \Delta_1 = \Delta_{21} = 1/\theta + (N_i + N_{21})/(1/h_{22} - 1/h) \).

Theorem 4. Fares for transit and P&R commuters in competitive situation are lower than the results for transit company profit maximization.

Clearly, the differences are equal when the model degenerates into the multinomial logit case, i.e., when \( \omega = \theta \), \( \eta \to \infty \), the difference is \( h^2(N_i + N_{21})(h_{22} - g_{22})/[h(h_{22} - g_{22})] \).

Theorem 5. Parking fee for auto commuters in competitive situation are higher than the result for the net social benefit maximization, the difference is
\[ \Delta_{22} = 1/\omega + N_{22} \left[ h_{22} - 1/(\eta N_2) \frac{(h - h_i)}{(\eta N_2) + hh_i} - \pi_1\pi_2 \left[ \frac{\tilde{q}(N_i + N_{21}, l) + \tilde{q}(N_i + N_{21}, l)}{\tilde{q}(N_i + N_{21}, l) + \tilde{q}(N_i + N_{21}, l)} \right] \right] \frac{[h - 1/(\eta N_2)]}{[h - \pi_1[\tilde{q}(N_i + N_{21}, l) + \tilde{q}(N_i + N_{21}, l)]\{h - \pi_2[\tilde{q}(N_i + N_{21}, l) + \tilde{q}(N_i + N_{21}, l)]\} - (h - h_i)(h - h_{21})} \].

Clearly, when the model degenerates into the multinomial logit case, i.e., when \( \omega = \theta \), \( \eta \to \infty \), the difference is
\[ 1/\theta + N_{22} \frac{h_{22}(h_{22} - h_i) - \pi_1\pi_2 \left[ \frac{\tilde{q}(N_i + N_{21}, l) + \tilde{q}(N_i + N_{21}, l)}{\tilde{q}(N_i + N_{21}, l) + \tilde{q}(N_i + N_{21}, l)} \right]}{\{h - \pi_1[\tilde{q}(N_i + N_{21}, l) + \tilde{q}(N_i + N_{21}, l)]\{h - \pi_2[\tilde{q}(N_i + N_{21}, l) + \tilde{q}(N_i + N_{21}, l)]\} - (h - h_i)(h - h_{21})} \] .

4. Numerical experiment

Up to now, we have formulated four pricing models based on hierarchical logit-based equilibrium concept in a multimodal transportation system with trip-chain costs. Note that the groups of nonlinear equations formulated for each modeling must be numerically solved so it is difficult to check the properties of the solutions analytically. In this section, we present a numerical example to demonstrate the results generated by the four pricing schemes.

The parameters of our numerical example are: \((\bar{\beta}_2, \bar{\gamma}_2) = (30, 15)\text{($/hour)}\), \((\bar{\beta}_2, \bar{\gamma}_2) = (15, 30)\text{($/hour)}\), \(\alpha = 20\text{($/hour)}\), \((\bar{\beta}_2, \bar{\gamma}_2) = (4000, 3000)\text{(vehicle/hour)}\), \(v_1 = 20\text{(km/hour)}\), \(v_2 = 30\text{(km/hour)}\), \(L = 30\text{km}\), \(l = 5\text{km}\), \((\bar{T}, \bar{T}) = (0.2, 0.15)\text{(hour)}\), \(F_1 = 0\), \(F_2 = 10\text{($)}\), \(c_1 = 0.5\text{($/person)}\), \(\pi_1 = 0.85e^{-5}\text{($/discomfort)}\), \(\pi_2 = 1.2e^{-5}\text{($/discomfort)}\), \(C_{10} = 0\), \(C_{20} = 0\), \(\theta = 0.1\), \(\omega = 2\). We adopt the following inverse demand function: \(B(N) = -G \ln(N / N_{max})\), where \(N_{max} = 10000\). This function implies that the demand is less sensitive to the marginal trip benefit with a larger value of \( G \) and thus the final realized demand will be higher. The function describing body congestion discomfort takes the form \( \tilde{q}(n, x) = \bar{q}(n, x) = (x / v_1)(Dn^2 + En) \), where \( D = 0.05 \) and \( E = 0.25 \).
Fig. 3 Relative value of fares (to parking fee in the workplace) under different schemes

In order to show the results clearly, we use the relative values of fares to reduce the number of curves in Fig. 3 which gives the fares for transit and P&R commuters by the four schemes. Relative values of fares for transit and P&R commuters are defined by the ratio of fares to parking fee in the workplace. Comparing the results by the four schemes, it can be seen that the two types of fares decreases in the order of schemes II (transit company profit maximization), IV (pricing game), I (net social benefit maximization) and III (parking lot owner’s profit maximization). Coinciding with the pricing levels in Fig. 3, the proportions of commuters by transit and P&R are increasing by the order of II, IV, I and III as shown in Fig. 4.

Fig. 4 Proportion of commuters selecting transit and P&R under different schemes
Comparing the results by schemes II and IV in Fig 4, it can be seen that the scheme IV makes the proportion of P&R commuters increase sharply and transit commuters slightly although the differences between their fares are relatively large (Fig. 3). On the other hand, scheme III increases the proportion of P&R commuters a lot due to the parking fee rising. It is shown that lower transit fares and higher parking fees in working area can effectively encourage P&R mode choice. This means that more auto commuters change to transit mode at bottleneck to ease the central traffic pressures. These numerical results also theoretically support the current differentiated parking fee policy in Beijing.

Fig. 5 Total demand and transit commuters (including commuters by transit and P&R)

Fig. 6 Net benefits of the system under different schemes

Fig. 5 shows the total demand and the number of transit users (including transit and P&R commuters) generated by the four pricing policies. The first policy (scheme I) generates the most number of commuters, while the third
policy (scheme III) gives the second most number of commuters. But number of transit users by the latter is much higher than the former. However, the total net social benefit of the third policy has no significant decrease as shown in Fig. 6. This suggests that the current differentiated parking fee policy in Beijing is efficient.

5. Conclusions

In this paper we propose four pricing models for a multimodal corridor transportation system with trip-chain costs. Commuters can go to work by three modes: transit only, P&R and auto only. The solutions of the four models correspond to the hierarchical logit-based stochastic equilibria among travel costs and trip benefits with different market schemes. It is shown that when transit and park-and-ride parking lot are operated by government and parking lot at working area belongs to private owner, lower transit fares and higher parking fees in central area can effectively encourage parking interchanging, increase public transit attraction and maximize the system’s total net benefit. Numerical results support the current differentiated parking charge policy in Beijing.

The research can be extended by incorporating redistribution of the charge revenue from parking \([14]\), which is an interesting direction.

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