

Electronic Notes in Theoretical Computer Science 46 (2001)
URL: <http://www.elsevier.nl/locate/entcs/volume46.html> 11 pages

An Efficient Algorithm for Helly Property Recognition in a Linear Hypergraph

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Abstract

In this article we characterize bipartite graphs whose associated neighborhood hypergraphs have the Helly property. We examine incidence graphs of both hypergraphs and linear hypergraphs and we give a polynomial algorithm to recognize if a linear hypergraph has the Helly property.

1 Introduction

Computer vision is an attractive field of applications for two main reasons. First, it is directly applicable to many interesting problems, including face recognition, scene understanding, autonomous vehicle navigation, and content-based image and video retrieval. Second, it employs techniques from many mathematical disciplines such as linear algebra, graph theory, combinatorial and continuous optimization, ordinary and partial differential equations, probability and statistics, and pattern recognition, to name a few. This makes it an ideal testing ground for new and adapted concepts and algorithms, whose impact can be immediately felt in such diverse applications as the analysis of medical images, video surveillance, target identification, etc. Furthermore,

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the often imposed real-time performance requirements necessitate the development of efficient and practical algorithms. While computer vision has a long history, its main success stories come from solutions to problems that can be adequately formulated in terms of the mathematical tools it employs. For example, depth estimation from stereo can be cast as a matching problem (combinatorial optimization) followed by depth reconstruction (robust statistics and linear algebra). Image segmentation is a classic problem in computer vision and pattern recognition, instances of which arise in areas as diverse as object recognition, motion and stereo analysis. We have recently introduced a solution to this problem by providing a novel way of deriving an image neighborhood hypergraph based on the hypergraph-theoretic notions of visibility [6,7,8,9]. We have proved that in this new formulation allows to develop efficient algorithms. The framework has also been extended to handle low level vision problems such as noise cancellation, edge extraction. This framework is attractive because it casts computer vision problems as a pure hypergraph theoretic problem, for which a solid theory and powerful algorithms have been developed.

A hypergraph is a generalization of a graph in which binary relations between pairs of vertices are generalized to n -ary relations. Hyperedges are formed by structural or relational patterns between the attributed vertices. The hypergraph knowledge representation is local and stable. That is, its subgraphs can be manipulated locally and small changes induce local variations in attributes and topology. This knowledge representation has been used for the synthesis and recognition of 3D object models [23,5] and for robot path planning and navigation [18,22,19,15].

The Helly property has developed into a very broad field since its introduction by C. BERGE [4]. It is one of the most important concepts in hypergraph theory. It has been the focus of intense study for several years [14,17,20]. It provides a common property in several classes of these set systems, such as: balanced hypergraphs, unimodular hypergraphs, normal hypergraphs, arbo-real hypergraphs [3] ...

Some properties can be derived from the Helly property. It is the case for the very interesting theorem of SANTALO published in 1940. The usefulness of this theorem can be found in [12]

More recently this property has proved its importance in image analysis [7,8]. Helly property is a term that has arisen for the study of geometric properties of digital image. It generalizes the geometric notion of visibility. Hence, in image processing this property can be interpreted as a local uniformity notion. In this article we are going to give some results about this property. We will deduce a polynomial algorithm to recognize if a linear hypergraph has (or not) the Helly property.

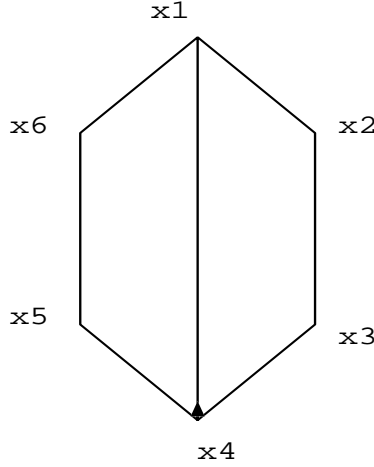


Fig. 1. This figure shows a cycle C_6 with a well chord $\{x_1, x_4\}$.

2 Graph and Hypergraph Definitions

The general terminology concerning graphs and hypergraphs in this article is similar to the one used in [2,3]. All graphs in this paper are both finite and undirected. One will consider that these graphs are simple; graphs with no loops or multiple edges. All graphs will be considered as connected with no isolated vertex. We denote them $G = (V; E)$. Given a graph G , we denote the *neighborhood* of a vertex x by $\Gamma(x)$, i.e. The set formed by all the vertices adjacent to x is defined by:

$$\Gamma(x) = \{y \in V, \{x, y\} \in E\}.$$

The number of neighbors of x is the *degree* of x (denoted by dx).

A *chain* in a graph is a sequence of distinct edges—one following another—and the number of edges is the *length* of this chain.

A *cycle* is a chain such that the first vertex and the last vertex are the same.

Let C_n be a cycle, a *chord* of a cycle is an edge linking two non-consecutive vertices of this cycle.

Let $G = (V, E)$ be a graph, a cycle C_{2n} , $n > 2$ has a *well chord* if there exist an edge e belonging to G such that e shares C_{2n} in two cycles C_{n+1} . An example is given in figure 1.

$G' = (V'; E')$ is a *subgraph* of G when it is a graph satisfying $V' \subseteq V$ and $E' \subseteq E$. If $V' = V$ then G' is a *spanning subgraph*.

An *induced subgraph* (generated by A) $G(A) = (A; U)$, with $A \subseteq V$ and $U \subseteq E$ is a subgraph such that: for $x, y \in A$, when $\{x; y\} \in E$ implies $\{x; y\} \in U$.

A subgraph which is a cycle without chord and with a length equal to n will be denoted by C_n .

A cycle C_n is *centered* if there exists a vertex of G adjacent to every vertex of C_n . (If this vertex is on the cycle, one will consider that it is adjacent to it.)

A graph $G = (V, E)$ is bipartite if $V = V_1 \cup V_2$ with $V_1 \cap V_2 = \emptyset$ and every edge joins a vertex of V_1 to a vertex V_2 . We will denote a bipartite graph by $G(V_1, V_2)$.

A graph $G = (V, E)$ is *bipartite* if and only if it does not contain any cycle with an odd length.

A *hypergraph* H on a finite set \mathbf{S} is a family $(E_i)_{i \in I}$, $I = \{1, 2, \dots, n\}$ $n \in \mathbb{N}$ of non-empty subsets of \mathbf{S} called hyperedges with:

$$\bigcup_{i \in I} E_i = \mathbf{S}.$$

Let us denote: $H = (\mathbf{S}; (E_i)_{i \in I})$

The *rank* of H is the maximum cardinality of a hyperedge.

An hypergraph is *simple* if $E_i \subseteq E_j$ then $i = j$. In this article any hypergraph will be considered as simple.

A hypergraph is *linear* if $|E_i \cap E_j| \leq 1$ for $i \neq j$.

For $x \in \mathbf{S}$, a *star* of H —with x as a center—is the set of hyperedges which contains x , and is called $H(x)$. The *degree* of x is the cardinality of the star $H(x)$.

A *partial hypergraph* on \mathbf{S} is a subfamily $(E_j)_{j \in J}$ of $(E_i)_{i \in I}$.

A *subhypergraph* of the hypergraph H is the hypergraph $H(Y) = (Y, (E_i \cap Y \neq \emptyset)_{i \in I})$, (with $Y \subseteq \mathbf{S}$).

The *dual* of a hypergraph $H = (E_1, E_2, \dots, E_m)$ on \mathbf{S} is a hypergraph $H^* = (X_1, X_2, \dots, X_n)$ whose vertices e_1, e_2, \dots, e_m correspond to the hyperedges of H , and with hyperedges $X_i = \{e_j, x_j \in E_j\}$.

A family of hyperedges is an *intersecting family* if every pair of hyperedges has a non-empty intersection.

A hypergraph has the *Helly property* if each intersecting family has a non-empty intersection—belonging to a star. An example of this property is given in figure 2.

A hypergraph has the *the strong Helly property* if each subhypergraph has the Helly property.

The *incidence graph* of a hypergraph $H = (S; E)$ is a bipartite graph with a vertex set $V = S \cup E$, where two vertices $x \in X$ and $e \in E$ are adjacent if and only if $x \in e$. We denote it $IG(H)$.

Let $G = (S; E)$ be a graph; we can associate a hypergraph called *neighborhood hypergraph* to this graph:

$$H_G = (\mathbf{S}, (E_x = \{x\} \cup \Gamma(x))).$$

One will say that the hyperedge E_x is generated by x .

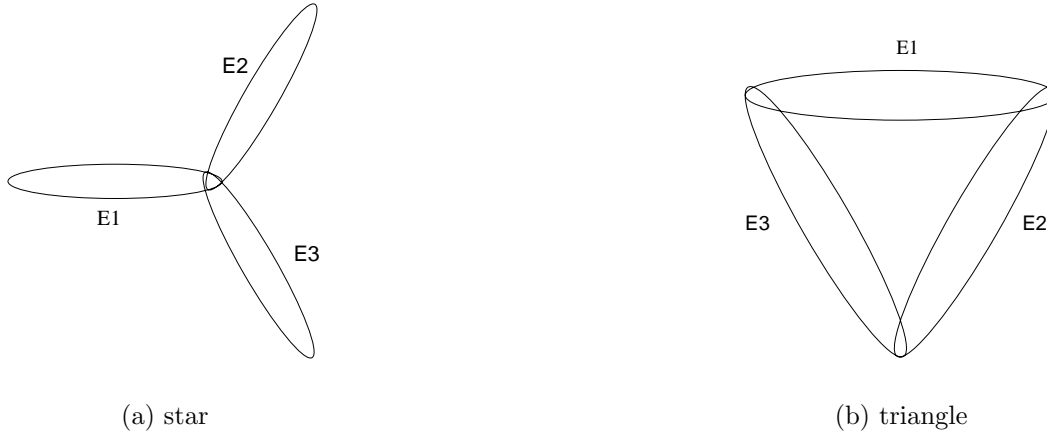


Fig. 2. Assuming that the hypergraphs have only three hyperedges, the hypergraph (b) does not have the Helly property.

3 Hypergraph and Image

Helly type theorem can be written in the following form: “Let \mathbb{F} be a collection of sets and let k be a fixed integer. If every sub-family of k sets in \mathbb{F} has the property P then the collection \mathbb{F} has the property Q .”

One of the most widely known theorem of this type is due to Edward Helly. It is easy to verify that a particular case of the Helly type theorem is the Helly property. Actually this property occurs in numerous mathematical fields. In arithmetics, the Chinese remainder theorem amounts to say that arithmetical progressions have the helly property. Another example of Helly family are families of intervals in a lattice. A computational aspect of this property has been highlighted in discrete geometry and computational geometry [24]. The Helly property is a basic concept in hypergraph theory, because a lot of classes of hypergraph have this property [3].

A distance d' on X defines a *grid* (a graph connected, regular, without both loop and multi-edge).

A *digital image* (on a grid) is a two-dimensional discrete function that has been digitized both in spatial coordinates and in magnitude feature value. So a digital image will be represented by the application:

$$I : X \subseteq \mathbb{Z}^2 \rightarrow \mathcal{C} \subseteq \mathbb{Z}^n \text{ with } n \geq 1$$

where \mathcal{C} identifies the *feature intensity level* and X identifies a set of points called *image points*. The couple $(x, I(x))$ is called a *pixel*. Any image can be represented by a hypergraph in the following way:

Let d be a distance on \mathcal{C} . We have a neighborhood relation on the image defined by:

$$(1) \quad \forall x \in X, \Gamma_{\alpha, \beta}(x) = \{x' \in X, x' \neq x \mid d(I(x), I(x')) < \alpha \text{ and } d'(x, x') \leq \beta\}$$

The neighborhood of x on the grid will be denoted by $\Gamma_{\beta}(x)$.

So, we can associate, to each image a hypergraph called *Image Adaptive Neighborhood Hypergraph* (IANH):

$$\mathcal{H}_{\alpha,\beta} = (X, (\{x\} \cup \Gamma_{\alpha,\beta}(x))_{x \in X}).$$

The attribute α can be calculated in an adaptive way depending on the nature of the neighbour image treated.

As a digital image has geometrical and combinatorial aspects, the Helly property is particular suited to image processing. Let an image be represented by its neighborhood hypergraph, if some connected sub-hypergraphs have the Helly property, the centers of the stars characterize the common neighborhood relations of the stars. Therefore a star center is representative of the whole neighborhood. These centers may be a sufficient representation to extract global information. Consequently it is worth studying the Helly property from a theoretical point of view.

4 Preliminary Results

We now characterize bipartite graphs such that the associated neighborhood hypergraph has the Helly property.

Theorem 4.1 *Let $G = (V; E)$ be a bipartite graph, and \mathbb{H}_G its associated neighborhood hypergraph. \mathbb{H}_G has the Helly property if and only if G does not contain C_4 and C_6 .*

Proof. The condition is necessary. Suppose that G contains a C_4 . \mathbb{H}_G has the Helly property. Consequently C_4 is centered, so G contains a cycle C_3 . Contradiction. If G contains C_6 , either C_6 is centered and G contains C_3 or there exists a vertex adjacent to three non-consecutive vertices of C_6 , so G contains a cycle C_4 and thus it is a cycle C_3 . Contradiction.

The condition is sufficient. We prove this assertion by induction on the hyperedge number from an intersecting family.

Let $(E_{x_i})_{i \in \{1,2,3\}}$ be an intersecting family of three hyperedges, by hypothesis x_1, x_2, x_3 cannot be on a cycle C_n with $n = 4, 5, \text{ or } 6$. We denote $V = V_1 \cup V_2$. Two cases arise:

- 1 $x_1, x_2, x_3 \in V_1$. We have y_i adjacent to $x_i, x_{i+1} \pmod{3}$. So $y_1 = y_2 = y_3 = y$, otherwise one would have C_4 or C_6 .
- 2 $x_1, x_2 \in V_1$ and $x_3 \in V_2$ necessarily x_3 is adjacent to x_1 and x_2 .

Consequently, there exists a vertex y adjacent to x_1, x_2, x_3 in the first case, and $y = x_3$ is adjacent to x_1 and x_2 in the second case.

Suppose that any intersecting family with $n - 1$ hyperedges belongs to a star, and let $(E_{x_i})_{1 \leq i \leq n}$ be an intersecting family. $(E_{x_i})_{2 \leq i \leq n}$ belongs to a star, so there exists y such that y is adjacent to $(x_i)_{2 \leq i \leq n}$. Suppose that $y \in V_1$ and $(x_i)_{2 \leq i \leq n} \subseteq V_2$.

- $x_1 \in V_1$. Then x_1 is adjacent to x_i $i \in \{2, 3, \dots, n\}$. So $x_1 = y$ otherwise

this would lead to a C_4 .

- $x_1 \in V_2$. Let $u_i, i \in \{2, 3, \dots, n\}$ be the common neighbor of x_1, x_i . Suppose that for all $i \in \{2, 3, \dots, n\}, u_i \neq y$.
 - There exists $i \neq j$ such that $u_i \neq u_j$, consequently this leads to a C_6 .
 - For all $i, j \in \{2, 3, \dots, n\}, u_i = u_j$, consequently this leads to a C_4 .
- So there exists $i \in \{2, 3, \dots, n\}$ such that $u_i = y$ and y is adjacent to x_1 .

We can conclude that \mathbb{H}_G has the Helly property. □

From this theorem we have the following:

Corollary 4.2 *Let $G = (V, E)$ be a bipartite graph and \mathbb{H}_G its associated neighborhood hypergraph. \mathbb{H}_G has the Helly property if and only if it has the strong Helly property.*

Proof. If \mathbb{H}_G has the strong Helly property then obviously it has the Helly property.

Assume now that \mathbb{H}_G has the Helly property. Let $H' = (V', E')$ be a subhypergraph of \mathbb{H}_G . The induced subgraph $G(V')$ contains neither C_4 nor C_6 . Hence H' has the Helly property, so \mathbb{H}_G has the strong Helly property. □

On the strong Helly property we have:

Theorem 4.3 *Let $H = (S, E)$ be a hypergraph. H has the strong Helly property if and only if every C_6 of $IG(H)$ is well chorded.*

Proof. If H has the strong Helly property it is easy to see that any C_6 of $IG(H)$ is well chorded.

Assume now that any C_6 of $IG(H)$ is well chorded. Let H' be a subhypergraph, we are going to prove this assertion by induction on the hyperedge number of H' .

Let $(E_i)_{i \in \{1,2,3\}}$ be an intersecting family. This family generates a cycle $(x_1, e_1, x_2, e_2, x_3, e_3, x_1)$ in the incidence graph of H' . This cycle is well chorded, so there exists a vertex $x_i, i \in \{1, 2, 3\}$ which belongs to $\bigcap_{i \in \{1,2,3\}} E_i$.

Assume this is true for any intersecting family of H with $p - 1$ hyperedges. Let $(E_i)_{i \in \{1,2,3,4,\dots,p\}}$ be an intersecting family with p hyperedges. The following families $(E_i)_{i \in \{2,3,4,\dots,p\}}, (E_i)_{i \in \{1,3,4,\dots,p\}}, (E_i)_{i \in \{1,2,4,\dots,p\}}$ are stars, by induction hypothesis. So one can stand for respectively by $H(u), H(v), H(w)$ these three stars. These three vertices are on a cycle $(u, E_{u,v}, v, E_{v,w}, w, E_{w,u}, u)$ ($E_{a,b}$ being a hyperedge containing the vertices a, b). These cycles being well chorded, u, v, w belong to any hyperedge of $(E_i)_{i \in \{1,2,3,4,\dots,p\}}$. Hence this family is a star. So H' has the Helly property. □

We say that the hypergraph $H = (V; E)$ has the *separation property* (briefly, SP) if for every pair of distinct vertices $x, y \in V$ there exists a hyperedge $E_i \in E$ such that either $x \in E_i$ and $y \notin E_i$ or $x \notin E_i$ and $y \in E_i$.

Corollary 4.4 *A hypergraph H with the SP property has the strong Helly property if and only if its dual H^* has the strong Helly property.*

Proof. It is easy to see that the incidence graph of a hypergraph with the *SP* property is isomorphic to the incidence graph of its dual H^* . Moreover, from theorem above, H has the strong Helly property if and only if every C_6 of $IG(H)$ is well chorded. The corollary is proved. \square

5 Linear Hypergraph and Helly Property

The existence of an efficient polynomial algorithm for checking if a hypergraph has the Helly property follows from classical theory. For example, from the following corollary at page 23 of [2]:

Corollary 5.1 *A hypergraph H has the Helly property if and only if for any three vertices a_1, a_2, a_3 , the family of hyperedges which contains at least two of these vertices has a non-empty intersection.*

This corollary comes directly from the following:

Theorem 5.2 [3] *A hypergraph H is k -Helly if and only if for every set A of vertices with $|A| = k+1$, the intersection of the hyperedges E_j with $|E_j \cap A| \geq k$ is non-empty.*

Remark 5.3 We say that a hypergraph $H = (E_1, E_2, \dots, E_m)$ is k -Helly if, for every $J \subset \{1, 2, \dots, m\}$, the following two conditions are equivalent:

- $I \subset J$ and $|I| < k$ implies $\bigcap_{i \in I} E_i \neq \emptyset$
- $\bigcap_{j \in J} E_j \neq \emptyset$

It is easy to see that a hypergraph is 2-Helly if and only if it satisfies the Helly property. From corollary 4.1 we get easily the following algorithm:

Algorithm Helly:

Data: Hypergraph H

for all pairs of vertices x and y of H do begin

$X_{xy} :=$ all hyperedges containing both x and y ;

for all vertices v of H do begin

if x and y are both neighbors of v then begin

$X_{xv} :=$ all hyperedges containing both x and v ;

$X_{yv} :=$ all hyperedges containing both y and v ;

$X := X_{xy} \cup X_{xv} \cup X_{yv}$;

if the intersection of all elements of X is empty then begin

output(THE HELLY PROPERTY DOES NOT HOLD);

end if

end if

end for

end for

end

This algorithm is relatively simple. However, the time complexity of this algorithm is $O(n^3\Delta^2r^2m)$ which is rather large. We are going to give an algorithm with lower complexity for particular class of hypergraphs.

The following result gives a characterization of linear hypergraphs which have the Helly property.

Lemma 5.4 *Let $H = (S; E)$ be a hypergraph. We have the three following properties:*

- (i) *If H is linear then H has the Helly property if and only if $\mathbb{H}_{IG(H)}$ has the Helly property.*
- (ii) *H is linear if and only if $IG(H)$ does not contain C_4 .*
- (iii) *If $IG(H)$ does not contain C_6 then H has the Helly property.*

Proof. (i) Suppose that H is linear and that $\mathbb{H}_{IG(H)}$ has the Helly property. Let $I = (E_i)_{i \in \{1, \dots, p\}}$ be an intersecting family of hyperedges of H . In $\mathbb{H}_{IG(H)}$ I is a set of vertices such that $(\{e_i\} \cup \Gamma(e_i))_{i \in \{1, \dots, p\}}$ is an intersecting family. $\mathbb{H}_{IG(H)}$ having the Helly property $(E_i)_{i \in \{1, \dots, p\}}$ has a non-empty intersection.

Suppose that H has the Helly property and suppose that $\mathbb{H}_{IG(H)}$ does not satisfy the Helly property. From theorem1 $IG(H)$ contains C_4 or C_6 .

If $IG(H)$ contains C_4 , there exist two vertices of C_4 — e_1, e_2 representing two hyperedges of H —and two vertices x_1, x_2 of S belonging to C_4 . So x_1, x_2 belong to E_1 and E_2 . Consequently $|E_1 \cap E_2| > 1 - |E_1 \cap E_2|$ is the cardinality of $E_1 \cap E_2$ —and H is not linear. Contradiction, $IG(H)$ does not contain C_4 .

If $IG(H)$ contains C_6 : $x_1, e_1, x_2, e_3, x_3, e_2, x_1$ There exists three vertices of C_6 — e_1, e_2 and e_3 representing three hyperedges of H —and three vertices x_1, x_2, x_3 of S belonging to C_6 . But H has the Helly property, so there exists $y \in S$ such that $y \in E_1 \cap E_2 \cap E_3$. Either $y = x_i, i = 1, 2$ or 3 . For example $y = x_1$, so x_1, e_1, x_2, e_3, x_1 is a C_4 contradiction, or $y \neq x_i, i = 1, 2$ or 3 , but for example $|E_1 \cap E_2| > 1$ and H is not linear, contradiction. Consequently $\mathbb{H}_{IG(H)}$ does not contain C_6 . We can conclude that $IG(H)$ has the Helly property.

(ii) is obvious from the proof of (i).

(iii) Suppose that $IG(H)$ does not contain C_6 and that H does not has the Helly property. There exists an intersecting family of hyperedges with no common vertex. Let $F = (E_i)_{i \in \{1, \dots, p\}}$ be a minimal intersecting family of hyperedges not satisfying the Helly property— F does not satisfy the Helly property but any subfamily satisfy this property. One has $|F| > 2$. So $x_1 \in \bigcap_{i \in \{1, 2, 4, \dots, p\}} E_i, x_2 \in \bigcap_{i \in \{2, 3, 4, \dots, p\}} E_i$ and $x_3 \in \bigcap_{i \in \{1, 3, 4, \dots, p\}} E_i$. Hence $IG(H)$ contains a cycle C_6 ; for instance we have the cycle $\{x_1, e_1, x_3, e_3, x_2, e_2, x_1\}$. Contradiction, so H has the Helly property. \square

We now give an important result.

6 Algorithm

The theorem below shows that there exists a polynomial algorithm to recognize if a linear hypergraph has (or not) the Helly property.

Theorem 6.1 *There exists an algorithm with a complexity in $O((pm)^{\frac{5}{3}})$ to recognize if a linear hypergraph has (or not) the Helly property.*

Proof. The recognition algorithm for a Helly-hypergraph—hypergraph having the Helly property—is directly given from theorem1, lemma1 and [1].

In [1], it has been shown that deciding if a graph $G = (V; E)$ contains (or not) a simple cycle C_{2k} or C_{2k-1} can be done in $O(m^{2-\frac{1}{k}})$ time (m is the edge number of G).

The edge number of $IG(H)$ is $\sum_{i=1}^m |E_i| - |E_i|$ is the cardinality of the hyperedge E_i . One has $\sum_{i=1}^m |E_i| \leq \sum_{i=1}^m p = mp$ (p being the rank of H). So the computational time to decide if G contains (or not) C_4 or C_6 is in $O((pm)^{\frac{3}{2}})$ and $O((pm)^{\frac{5}{3}})$ respectively. \square

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