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Non-extremal charged rotating black holes in seven-dimensional gauged supergravity

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Abstract

We obtain the solution for non-extremal charged rotating black holes in seven-dimensional gauged supergravity, in the case where the three rotation parameters are set equal. There are two independent charges, corresponding to gauge fields in the $U(1) \times U(1)$ Abelian subgroup of the $SO(5)$ gauge group. A new feature in these solutions, not seen previously in lower-dimensional examples, is that the first-order "odd-dimensional self-duality" equation for the 4-form field strength plays a non-trivial role. We also study the BPS limit of our solutions where the black holes become supersymmetric. Our results are of significance for the $\mathrm{AdS}_7/\mathrm{CFT}_6$ correspondence in M-theory.

1. Introduction

Charged black holes in gauged supergravities provide gravitational backgrounds of importance in the study of the AdS/CFT correspondence. Non-extremal black hole solutions are relevant for studying the dual field theory at non-zero temperature. This has been discussed extensively for static AdS black holes in, for example, [1–3]. See also [4–6], for recent related work. For non-extremal charged rotating black holes in gauged supergravities, little has been known until recently. In [7,8] the first examples of non-extremal rotating charged AdS black holes in five-dimensional $\mathcal{N} = 4$ gauged supergravity were obtained, in the special case where the two angular momenta J_i are set equal. These solutions are characterised by their mass, three electromagnetic charges, and the angular

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momentum parameter $J = J_1 = J_2$. By taking appropriate limits, one obtains the various supersymmetric charged rotating $D = 5$ black holes obtained in [9–11]. If instead the charges are set to zero, the solutions reduce to the rotating AdS_5 black hole constructed in [12], with $J_1 = J_2$. In four dimensions, the charged Kerr–Newman–AdS black hole solution of the Einstein–Maxwell system with a cosmological constant has long been known [13,14]. This can be viewed as a solution in gauged $\mathcal{N} = 8$ supergravity, in which the four electromagnetic fields in the $U(1)^4$ Abelian subgroup of the $SO(8)$ gauge group are set equal. Recently, a more general class of non-extremal charged rotating solutions in the four-dimensional gauged theory were constructed, in which the four electric charges are set pairwise equal [15].

Another case of interest from the AdS/CFT perspective is non-extremal charged rotating black holes in seven-dimensional gauged supergravity, and this forms the subject of the present Letter. The maximally-supersymmetric theory has $\mathcal{N} = 4$ supersymmetry, and the gauge group is $SO(5)$ [16]. It was shown in [17,18] that this theory can be obtained as a consistent reduction of eleven-dimensional supergravity on S^4. A convenient presentation of the Lagrangian for the bosonic sector, in the conventions we shall be using, appears in [19]. The theory is capable of supporting black holes carrying two independent electric charges, carried by gauge fields in the $U(1) \times U(1)$ Abelian subgroup of the full $SO(5)$ gauge group. For the purposes of discussing the solutions, it therefore suffices to perform a (consistent) truncation of the full supergravity theory to the relevant sector, in which all except the $U(1) \times U(1)$ subgroup of gauge fields are set to zero. The fields retained in the consistent truncation comprise the metric, two dilatons, the $U(1) \times U(1)$ gauge fields and a 4-form field strength that satisfies an odd-dimensional self-duality equation. The equations of motion can be derived from the Lagrangian

$$\begin{aligned}\mathcal{L}_7 = {} & R * \mathbb{1} - \frac{1}{2} * d\varphi_i \wedge d\varphi_i - \frac{1}{2}\sum_{i=1}^{2} X_i^{-2} * F_{(2)}^i \wedge F_{(2)}^i - \frac{1}{2}(X_1X_2)^2 * F_{(4)} \wedge F_{(4)} \\ & + 2g^2\big[(X_1X_2)^{-4} - 8X_1X_2 - 4X_1^{-1}X_2^{-2} - 4X_1^{-2}X_2^{-1}\big] - gF_{(4)} \wedge A_{(3)} + F_{(2)}^1 \wedge F_{(2)}^2 \wedge A_{(3)},\end{aligned} \tag{1}$$

where

$$\begin{aligned} & F_{(2)}^i = dA_{(1)}^i, \qquad F_{(4)} = dA_{(3)}, \\ & X_1 = \exp\left\{-\frac{1}{\sqrt{2}}\varphi_1 - \frac{1}{\sqrt{10}}\varphi_2\right\}, \qquad X_2 = \exp\left\{\frac{1}{\sqrt{2}}\varphi_1 - \frac{1}{\sqrt{10}}\varphi_2\right\},\end{aligned} \tag{2}$$

together with a first-order "odd-dimensional self-duality" equation to be imposed after the variation of the Lagrangian. This condition is most conveniently stated by introducing an additional 2-form potential $A_{(2)}$, which can be gauged away in the gauged theory, and defining

$$F_{(3)} = dA_{(2)} - \frac{1}{2}A_{(1)}^1 \wedge dA_{(1)}^2 - \frac{1}{2}A_{(1)}^2 \wedge dA_{(1)}^1. \tag{3}$$

The odd-dimensional self-duality equation then reads[3]

$$(X_1X_2)^2 * F_{(4)} = -2gA_{(3)} - F_{(3)}. \tag{4}$$

This is a first integral of the equation of motion for $A_{(3)}$ that follows directly from (1). (Note that one can alternatively write a Lagrangian that yields the equations of motion directly, with no need for an additional constraint. See, for example, [16,19].)

The fact that the 3-form $A_{(3)}$ satisfies the odd-dimensional self-duality equation presents an interesting new challenge when constructing the charged rotating solutions in the gauged seven-dimensional supergravity. Namely,

[3] Note that if $g \neq 0$, one can absorb A_2 by making a gauge transformation of $A_{(3)}$. If, on the other hand, $g = 0$, then (4) just becomes the defining equation for $F_{(3)}$ as the dual of $F_{(4)}$. When $g = 0$ one can equivalently work either with $A_{(3)}$, or with $A_{(2)}$ in a dual formulation of the theory.

when both charges and rotation are turned on, a non-zero 3-form potential $A_{(3)}$ is induced due to the last term in the Lagrangian (1). Since the self-duality condition (4) contains a term, proportional to $A_{(3)}$, which is linear in g, the Einstein equations now contain a stress energy contribution from $A_{(3)}$ that is also linear in g, and thus the metric contains terms with even *and* odd powers of g.

The trickiest part of finding charged rotating solutions in any of the gauged supergravities is that one has little a priori guidance as to how the dimensionless quantity ag enters the solution, where a is the rotation parameter and g the gauge coupling constant. In the cases that have been constructed previously, in five dimensions [7,8] and in four dimensions [15], the gauge coupling constant appeared always quadratically in the relevant equations of motion, and thus the dimensionless product entered the solutions in the combination a^2g^2. In seven dimensions, by contrast, the gauge coupling constant g appears linearly in the odd-dimensional self-duality equation (4), and so in turn the solution involves linear powers of the product ag. This considerably complicates the task of parameterising possible forms for the solution, in the process of formulating a conjecture and then verifying that it works. It is intriguing that having found the charged rotating black hole, we have the rather uncommon situation of obtaining a solution of seven-dimensional gauged supergravity in which the odd-dimensional self-duality equation (4) is satisfied in a non-trivial way.

In the following sections, we shall construct non-extremal charged rotating black-hole solutions in the seven-dimensional gauged supergravity. In our approach, we begin with the previously-known charged rotating solutions in the ungauged theory. Then, we formulate a conjecture for the generalisation to the gauged theory, and verify it by an explicit checking of all the equations of motion. The charged solutions in the ungauged supergravity were constructed (with the full complement of three independent rotation parameters) in [20]. In order to give a uniform presentation of our results, and also to eliminate some typographical errors that arose in [20], we begin in Section 2 by rederiving the charged rotating black holes in the ungauged seven-dimensional supergravity, in the special case we are addressing in this Letter where the three angular momenta are set equal. Then, in Section 3, we formulate our conjectured generalisation to the gauged supergravity theory, and verify that it does indeed satisfy the equations of motion. As well as obtaining the non-extremal solutions with two independent charges, we also present a somewhat simpler form of the metric in the special case where the two charges are set equal. In Section 4, we discuss the BPS limit, showing how supersymmetric rotating black hole solutions in seven-dimensional gauged supergravity arise for a suitable restriction of the parameters. The Letter ends with conclusions in Section 5.

2. Charged rotating black holes in the ungauged theory

Charged solutions in ungauged supergravity can be obtained from uncharged ones by making use of global symmetries of the theory, employed as solution-generating transformations. In the present case, one way of doing this is to recognise that in the ungauged ($g = 0$) limit, the seven-dimensional theory described by (1) can be obtained as the dimensional reduction of the eight-dimensional "bosonic string" theory described by

$$\mathcal{L}_8 = \hat{R}\,\hat{*}\,\mathbb{1} - \frac{1}{2}\,\hat{*}\,d\varphi \wedge d\varphi - \frac{1}{2}\exp\left\{-\frac{2}{\sqrt{3}}\varphi\right\}\hat{*}\,\hat{F}_{(3)} \wedge \hat{F}_{(3)}. \tag{5}$$

This yields the seven-dimensional theory in a formulation in which the 4-form $F_{(4)}$ has been dualised to the 3-form $F_{(3)}$ (see footnote 3). The strategy for introducing charges is then to begin with an uncharged, Ricci-flat solution in seven dimensions, take its product with a circle, and hence obtain a Ricci-flat solution of the eight-dimensional theory. Next, one performs a Lorentz transformation in the (t, z) plane, with Lorentz boost parameter δ_1, where

$$t \to t\cosh\delta_1 + z\sinh\delta_1, \qquad z \to z\cosh\delta_1 + t\sinh\delta_1, \tag{6}$$

where z is the circle coordinate of the eighth dimension. Upon reduction to $D = 7$ on the Lorentz-transformed circle coordinate z, one obtains a seven-dimensional solution in which the Kaluza–Klein vector carries an electric charge. The next step is to use the discrete Z_2 subgroup of the seven-dimensional global symmetry group that exchanges the Kaluza–Klein and winding vectors. This allows one to repeat the lifting, Lorentz boosting and reduction steps, with a second boost parameter δ_2, thereby ending up with a seven-dimensional solution where each of the Kaluza–Klein and winding vectors carries an electric charge.

In principle, we can apply this charge-generating procedure starting from any Ricci-flat metric in seven dimensions. In our present case, we take as our starting point the generalisation of the rotating Kerr black hole to seven dimensions, obtained by Myers and Perry [21]. The most general such solution has three independent rotation parameters in the three orthogonal 2-planes of its six-dimensional transverse space. For reasons of simplicity, we restrict attention to the case where the three rotation parameters are set equal.

The uncharged seven-dimensional rotating black hole, with the three rotation parameters set equal, can be written as

$$ds_7^2 = -dt^2 + \frac{2m}{\rho^4}(dt - a\sigma)^2 + \frac{\rho^4\,dr^2}{V - 2m} + \rho^2\big(d\Sigma_2^2 + \sigma^2\big), \tag{7}$$

where

$$\rho^2 \equiv r^2 + a^2, \qquad V \equiv \frac{1}{r^2}\big(r^2 + a^2\big)^3, \tag{8}$$

$d\Sigma_2^2$ is the standard Fubini–Study metric on $\mathbb{CP}^2$, and σ is the connection on the $U(1)$ fibre over $\mathbb{CP}^2$ whose total bundle is the unit 5-sphere. Thus we may write [22]

$$d\Sigma_2^2 = d\xi^2 + \frac{1}{4}\sin^2\xi\,\big(\sigma_1^2 + \sigma_2^2\big) + \frac{1}{4}\sin^2\xi\cos^2\xi\,\sigma_3^2, \qquad \sigma = d\tau + \frac{1}{2}\sin^2\xi\,\sigma_3, \tag{9}$$

where σ_i denotes a set of left-invariant 1-forms on $SU(2)$, satisfying $d\sigma_i = -\frac{1}{2}\epsilon_{ijk}\sigma_j \wedge \sigma_k$. Note that we have

$$d\sigma = 2J, \tag{10}$$

where J is the Kähler form on $\mathbb{CP}^2$.

After implementing the sequence of steps described above in order to introduce electric charges, we find that the charged rotating non-extremal seven-dimensional black holes are given by

$$\begin{aligned}
ds_7^2 &= (H_1 H_2)^{1/5}\bigg[-\frac{\rho^4 - 2m}{\rho^4 H_1 H_2}\,dt^2 - \frac{4ma}{\rho^4 H_1 H_2}\,dt\,\sigma + \frac{2ma^2}{\rho^4 H_1 H_2}\bigg(1 - \frac{2ms_1^2 s_2^2}{\rho^4}\bigg)\sigma^2 \\
&\quad + \rho^2\big(d\Sigma_2^2 + \sigma^2\big) + \frac{\rho^4\,dr^2}{V - 2m}\bigg], \\
A_{(1)}^1 &= \frac{2ms_1}{\rho^4 H_1}(c_1\,dt - ac_2\sigma), \qquad A_{(1)}^2 = \frac{2ms_2}{\rho^4 H_2}(c_2\,dt - ac_1\sigma), \\
A_{(2)} &= \frac{mas_1s_2}{\rho^4}\bigg(\frac{1}{H_1} + \frac{1}{H_2}\bigg)dt \wedge \sigma, \qquad X_i = (H_1 H_2)^{2/5} H_i^{-1},
\end{aligned} \tag{11}$$

where

$$H_i = 1 + \frac{2ms_i^2}{\rho^4}, \tag{12}$$

and we have defined

$$s_i \equiv \sinh\delta_i, \qquad c_i \equiv \cosh\delta_i. \tag{13}$$

(A different solution-generating technique, making use of global symmetries of the three-dimensional theory obtained by dimensional reduction, was used in [20] to construct rotating charged black holes in D-dimensional supergravities for $4 \leqslant D \leqslant 9$, with 2 independent charges and $(D-1)/2$ independent angular momenta. When the three angular momenta in the $D=7$ solution are set equal, the situation considered in [20] reduces to the one we have considered in this Letter.[4])

Note that the 3-form $F_{(3)} = dA_{(2)} - \frac{1}{2}A^1_{(1)} \wedge dA^2_{(1)} - \frac{1}{2}A^2_{(1)} \wedge dA^1_{(1)}$ can be dualised to the 4-form $F_{(4)} = dA_{(3)}$, in which case one has

$$A_{(3)} = \frac{2mas_1s_2}{r^2+a^2}\sigma \wedge J \tag{14}$$

in place of the expression for $A_{(2)}$ in (11).

3. Charged rotating black holes in the gauged theory

In this section, we construct non-extremal charged rotating solutions in the gauged seven-dimensional supergravity theory. Note that the global symmetries that allowed us to generate charged solutions from uncharged ones are broken in the gauged theory, and so there is no longer a procedure available that delivers the charged solutions by mechanical means. Instead, we have constructed the charged solutions by means of "educated guesswork", followed by an explicit verification that all the seven-dimensional equations of motion are indeed satisfied. In order to conjecture the form of the solution, we have made extensive use of previously-known limiting cases, including, especially, the charged solutions of the ungauged theory, which we described in the previous section.

We find that the charged and rotating non-extremal black hole solution of the seven-dimensional gauged supergravity is given by[5]

$$\begin{aligned}
ds_7^2 &= (H_1H_2)^{1/5}\left[-\frac{Y\,dt^2}{f_1\Xi_-^2} + \frac{\rho^4\,d\rho^2}{Y} + \frac{f_1}{\rho^4H_1H_2\Xi^2}\left(\sigma - \frac{2f_2}{f_1}\,dt\right)^2 + \frac{\rho^2}{\Xi}\,d\Sigma_2^2\right],\\
A^i_{(1)} &= \frac{2ms_i}{\rho^4\Xi H_i}(\alpha_i\,dt + \beta_i\sigma),\\
A_{(2)} &= \frac{mas_1s_2}{\rho^4\Xi_-^2}\left(\frac{1}{H_1}+\frac{1}{H_2}\right)dt\wedge\sigma, \qquad A_{(3)} = \frac{2mas_1s_2}{\rho^2\Xi\,\Xi_-}\sigma\wedge J,\\
X_i &= (H_1H_2)^{2/5}H_i^{-1}, \qquad H_i = 1 + \frac{2ms_i^2}{\rho^4}, \qquad \rho^2 = r^2+a^2,\\
\alpha_1 &= c_1 - \frac{1}{2}\big(1-\Xi_+^2\big)(c_1-c_2), \qquad \alpha_2 = c_2 + \frac{1}{2}\big(1-\Xi_+^2\big)(c_1-c_2),\\
\beta_1 &= -a\alpha_2, \qquad \beta_2 = -a\alpha_1, \qquad \Xi_\pm = 1\pm ag, \qquad \Xi = 1-a^2g^2 = \Xi_-\Xi_+,
\end{aligned} \tag{15}$$

where the functions f_1, f_2 and Y are given by

$$\begin{aligned}
f_1 &= \Xi\rho^6H_1H_2 - \frac{4\Xi_+^2m^2a^2s_1^2s_2^2}{\rho^4} + \frac{1}{2}ma^2\big[4\Xi_+^2 + 2c_1c_2\big(1-\Xi_+^4\big) + \big(1-\Xi_+^2\big)^2\big(c_1^2+c_2^2\big)\big],\\
f_2 &= -\frac{1}{2}g\Xi_+\rho^6H_1H_2 + \frac{1}{4}ma\big[2\big(1+\Xi_+^4\big)c_1c_2 + \big(1-\Xi_+^4\big)\big(c_1^2+c_2^2\big)\big],
\end{aligned}$$

[4] The general solution in [20] (Eq. (12) of [20]) has a few typographical errors: a term $2N\ell_i^2\mu_i^2$ in the metric coefficient for $d\phi_i^2$ should be $2N\Delta\ell_i^2\mu_i^2$, the 2-form potential components $B_{\phi_i\phi_j}$ should be set to zero, and the quantity mr in $B_{t\phi_i}$ should be N.

[5] It should be emphasised that in the solution (15), $A_{(3)}$ is the potential for the fundamental field $F_{(4)} = dA_{(3)}$ in the gauged supergravity, while, as discussed in footnote 3, $A_{(2)}$ is a term that could, if one wished, be viewed as being absorbed into $A_{(3)}$ via a gauge transformation of $A_{(3)}$. It happens to be convenient to present it in the form we have done; we are *not* saying that $A_{(2)}$ is an independent fundamental field.

$$
\begin{aligned}
Y &= g^2\rho^8 H_1 H_2 + \Xi\rho^6 + \frac{1}{2}ma^2\big[4\Xi_+^2 + 2\big(1-\Xi_+^4\big)c_1c_2 + \big(1-\Xi_+^2\big)^2\big(c_1^2+c_2^2\big)\big] \\
&\quad - \frac{1}{2}m\rho^2\big[4\Xi + 2a^2g^2\big(6+8ag+3a^2g^2\big)c_1c_2 - a^2g^2(2+ag)(2+3ag)\big(c_1^2+c_2^2\big)\big].
\end{aligned}
\tag{16}
$$

It is a purely mechanical exercise, which we performed with the aid of MATHEMATICA, to verify that this configuration indeed satisfies the equations of motion of seven-dimensional gauged supergravity, following from (1) together with the odd-dimensional self-duality equation (4). Note that as mentioned in the introduction, unlike the charged rotating AdS black holes in $D=5$ and $D=4$, the metric in $D=7$ depends on odd powers of g as well as even powers, in consequence of the odd-dimensional self-duality equation.

If one specialises to the case where the two charges are set equal, the solution may be written in a somewhat simpler form, as

$$
\begin{aligned}
ds_7^2 &= H^{2/5}\left[-\frac{V-2m}{\rho^4H^2\Xi^2}(dt-a\sigma)^2 + \frac{1}{r^2H^2\Xi^2}(h_1\,dt-h_2\sigma)^2 + \frac{\rho^4\,dr^2}{V-2m} + \frac{\rho^2}{\Xi}\,d\Sigma_2^2\right], \\
A_{(1)}^1 &= A_{(1)}^2 = \frac{2msc}{\rho^4H\Xi}(dt-a\sigma), \qquad A_{(2)} = \frac{2ms^2a}{\rho^4H\Xi_-^2}\,dt\wedge\sigma, \qquad A_{(3)} = \frac{2mas^2}{\Xi\,\Xi_-(r^2+a^2)}\sigma\wedge J,
\end{aligned}
\tag{17}
$$

where

$$
\begin{aligned}
V &= \frac{1}{r^2}\Big(\big(r^2+a^2\big)^3\big(1+g^2r^2\big) + 2gm\big(2gr^4+3a^2gr^2-2a^3\big)s^2 + 4g^2m^2s^4\Big), \\
h_1 &= -a + gr^2H + \frac{2a^2gms^2}{\rho^4}, \qquad h_2 = -a^2 - r^2H + \frac{2a^3gms^2}{\rho^4}, \\
H &= 1 + \frac{2ms^2}{\rho^4}, \qquad \rho^2 = r^2+a^2, \qquad s \equiv \sinh\delta, \qquad c \equiv \cosh\delta.
\end{aligned}
\tag{18}
$$

4. The supersymmetric limit

The charged rotating black hole solutions in seven-dimensional gauged supergravity that we have derived in this Letter are in general non-extremal, with the mass and the electric charges freely specifiable. It is of interest also to study the extremal limit, in which one obtains supersymmetric BPS black hole solutions. For simplicity, we shall just present the results for the case where the two electric charges are set equal here.

The criterion for supersymmetry is that there should exist supersymmetry parameters ϵ such that the supersymmetry variations of the spin-$\frac{3}{2}$ and spin-$\frac{1}{2}$ fields ψ_μ and λ_i in the gauged supergravity theory should vanish. This is most easily checked by looking at the integrability condition for the spin-$\frac{3}{2}$ field, and by looking directly at the transformation rule for the spin-$\frac{1}{2}$ field. This latter, in the case where the electric charges are set equal, takes the form

$$
\delta\lambda_i = -\frac{1}{4}\Gamma^\mu\epsilon X^{-1}\partial_\mu X + \frac{i}{40}X^{-1}F_{\mu\nu}\Gamma^{\mu\nu}\epsilon - \frac{1}{480}X^2F_{\mu\nu\rho\sigma}\Gamma^{\mu\nu\rho\sigma}\epsilon + \frac{1}{5}g\big(X-X^{-4}\big)\epsilon.
\tag{19}
$$

By studying the eigenvalues of the matrix that acts on ϵ, we find that there can exist Killing spinors if the parameter δ satisfies

$$
\tanh\delta = \frac{\pm 1}{1+ag},
\tag{20}
$$

which implies that

$$
s \equiv \sinh\delta = \frac{\pm 1}{\sqrt{\Xi_+^2-1}}, \qquad c \equiv \cosh\delta = \frac{\Xi_+}{\sqrt{\Xi_+^2-1}}.
\tag{21}
$$

Specifically, we find that the 8×8 matrix has two zero eigenvalues if Eq. (20) holds.

Our results for supersymmetric black holes reduce to previously-known cases if we specialise to $g = 0$ or $a = 0$. In either of these cases, we find that the number of zero eigenvalues increases to four, with the BPS condition (20) reducing now to the familiar one that the "extremality parameter" is given by $\delta \to \pm\infty$ and $m \to 0$ with $m \sinh^2 \delta$ fixed in the BPS limit. Thus we see that, as is the case also in four dimensions, BPS rotating black holes in gauged supergravity have only one half of the supersymmetry that occurs if either the rotation or the gauge coupling is set to zero.

It is not uncommon, for certain ranges of the parameters, for a rotating black hole to have naked closed timelike curves (CTCs). In our solution, with the two charges set equal, it is easy to see that

$$H^{-2/5} g_{00} = \left(\frac{4 f_2^2}{\rho^4 H^2 \Xi^2} - \frac{Y}{\Xi_-^2} \right) \frac{1}{f_1} = \frac{2m(1 - (\Xi_+^2 - 1)s^2) - \Xi_+^2 \rho^4}{\Xi^2 H^2 \rho^4}. \tag{22}$$

The horizon is located at the outer root of $Y = 0$. The absence of CTCs requires that $f_1 > 0$, and so a necessary condition for no naked CTCs is that on the horizon, the expression on the second line be non-negative. This can be satisfied if $s^2 < s_0^2 \equiv 1/(\Xi_+^2 - 1)$, provided that m is sufficiently large. However, in the BPS limit, where $s = s_0$, the metric will necessarily have naked CTCs. (In fact recently an alternative supersymmetric limit of our seven-dimensional non-extremal black hole solution has been found, which does include a regular black hole with no CTCs or singularities on or outside the event horizon [23].)

5. Conclusions

In this Letter, we have constructed non-extremal charged rotating black hole solutions in seven-dimensional gauged supergravity. The solutions carry two independent charges, associated with gauge fields in the $U(1) \times U(1)$ Abelian subgroup of the $SO(5)$ gauge group. In order to simplify the problem we set the three angular momenta of a generic rotating black hole equal. An interesting new feature that arises in seven dimensions is that the 4-form field $F_{(4)}$, which is also non-zero when the two electric charges are both non-vanishing, satisfies a first-order "odd-dimensional self-duality" equation. This implies that the structure of the solutions is considerably more complicated than in previous examples that were studied in four and in five dimensions. As well as obtaining the non-extremal black hole solutions, we also considered their BPS limits, showing that one can obtain supersymmetric rotating black hole solutions of seven-dimensional gauged supergravity.

The results presented in this Letter are of significance for the $\mathrm{AdS}_7/\mathrm{CFT}_6$ correspondence in M-theory.

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