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# A class of non-supersymmetric open string vacua

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**Abstract**

We analyze non-supersymmetric four-dimensional open string models of type IIB string theory compactified on $T^2 \times$ K3 with Scherk–Schwarz deformation acting on an $S^1$ of the $T^2$ torus. We find that there are always two solutions to the tadpole conditions that are shown to be connected via Wilson lines in an non-trivial way. These models although non-supersymmetric, are free of R–R and NS–NS tadpoles.


## 1. Introduction

One of the outstanding problems in string theory is the construction of realistic non-supersymmetric string vacua. In particular, for open strings, an important program is the cancellation of tadpoles that appear in the massless limit of the transverse (or tree) channel amplitudes associated to Klein-bottle ($\tilde{\mathcal{K}}$), Annulus ($\tilde{\mathcal{A}}$) and Möbius strip ($\tilde{\mathcal{M}}$) world-sheets, corresponding to the exchange of a closed string between two crosscaps, two boundaries and a crosscap and a boundary, respectively. This cancellation is a necessary condition for the consistency and the stability of the vacuum.

In supersymmetric unoriented closed and open string models NS–NS and R–R tadpoles are equal by supersymmetry and consequently if the R–R tadpoles cancel, so do the NS–NS tadpoles. In models where supersymmetry is broken in the open string sector, one can argue that NS–NS tadpole cancellation implies that supersymmetry should also be broken in the closed string sector [1,2]. However, when supersymmetry is broken already at tree level in the closed string sector, the situation is more involved and one has in general non-zero NS–NS tadpoles even if R–R tadpoles cancel. Recently, many non-supersymmetric open string vacua have been constructed without R–R tadpoles [3–7]. Less is known about vacua which, in addition, have zero NS–NS tadpoles.

The massless spectrum of open string models can be computed either by looking directly at the action of the orientifold group on the massless excitations in the closed and open string sectors [8,9] or by performing appropriate modular transformations on $\tilde{\mathcal{K}}$, $\tilde{\mathcal{A}}$ and $\tilde{\mathcal{M}}$ to obtain the corresponding direct (or loop) channel amplitudes $\mathcal{K}$, $\mathcal{A}$, $\mathcal{M}$ and taking their massless limit [10]. In the former approach the action of the $i$th

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element of the orientifold group $g_i$ on a D$p$-brane is encoded in matrices acting on its Chan–Paton factors, which we call $\gamma_{g_i,p}$. In the latter approach, the Torus $\mathcal{T}$ and $\mathcal{K}$ contain the information about the closed string spectrum and $\mathcal{A}$ with $\mathcal{M}$ contain the information about the open string spectrum [10,11].

The Scherk–Schwarz (SS) deformation [12] is so far the most interesting mechanism for supersymmetry breaking in which supersymmetry is broken by twisting the boundary condition of the fermions along some compact direction. In a recent paper [4] the quantum stability of models with SS supersymmetry breaking have been considered. It has been argued that the one loop cosmological constant has a term power-like in the compactification radii proportional to the difference between fermionic and bosonic degrees of freedom and an exponentially suppressed term. In [2] examples of non-supersymmetric but fermion–boson degenerate models has been presented for the case of M-theory breaking. In the class of models we consider in this Letter the massless spectrum we find is non-degenerate which would imply that we will have a radius dependent one-loop cosmological constant. We think that this question deserves more investigation.

In this Letter we present a class of models in which supersymmetry is broken by a Scherk–Schwarz deformation [12] and have zero tadpoles. In Section 2, we discuss a nine-dimensional model, the simplest possible example in which the main points can be illustrated. In Section 3 we present a novel class of five-dimensional models and in Section 4 we state our conclusions.

## 2. Symmetry breaking in nine dimensions

Consider the orientifold of the $S^1/Z_2'$ compactification of type IIB string theory [3], where $Z_2'$ is the freely-acting orbifold generated by an element $h$ acting as a translation of length $\pi R$ along $S^1$, together with $(-1)^F$, where $F$ is the space–time fermion number [11]. This orbifold, known as Scherk–Schwarz deformation [12,13], breaks spontaneously supersymmetry by assigning different boundary conditions to bosons and fermions.

The loop channel Klein-bottle amplitude is obtained by projecting the torus amplitude by $\Omega$ (the notation we follow in this section is the one used in Ref. [11]):

$$\mathcal{K} \sim \frac{1}{4}(V_8 - S_8)P_{2m}. \tag{1}$$

Here $V_8$ and $S_8$ are the standard bosonic and fermionic $SO(8)$ characters respectively and $P_{2m}$ is the momentum lattice with even momenta. By a modular transformation one obtains the tree channel Klein-bottle amplitude

$$\tilde{\mathcal{K}} \sim \frac{2^5}{4}R(V_8 - S_8)W_n, \tag{2}$$

where $W_n$ is the winding lattice. The above amplitude contains massless R–R tadpoles and corresponds to an O9-plane with positive tension and charge, i.e., to an O$^+$-plane. To cancel this tadpole, a stack of 32 D9-branes has to be introduced. The most general Annulus amplitude associated with these D9-branes including Wilson lines is[1]

$$\begin{aligned}\mathcal{A} \sim \frac{1}{4}\Big[&\big(N^2 P_{m-2\theta} + \bar{N}^2 P_{m+2\theta} + 2N\bar{N}P_m\big)(V_8 - S_8)\\ &+ \big(N^2 P_{m-2\theta} + \bar{N}^2 P_{m+2\theta} + 2N\bar{N}P_m\big)\\ &\times (-1)^m (V_8 + S_8)\Big].\end{aligned} \tag{3}$$

The transverse channel amplitude is obtained by a modular transformation, yielding

$$\begin{aligned}\tilde{\mathcal{A}} \sim \frac{2^{-5}}{4}R\Big[&\big(Ne^{2\pi in\theta} + \bar{N}e^{-2\pi in\theta}\big)^2(V_8 - S_8)W_n\\ &+ \big(Ne^{2\pi in\theta} + \bar{N}e^{-2\pi in\theta}\big)^2\\ &\times (O_8 - C_8)W_{n+\frac{1}{2}}\Big].\end{aligned} \tag{4}$$

The tree channel Möbius strip amplitude is then obtained as a state by state geometric mean of the Klein-bottle amplitude $\tilde{\mathcal{K}}$ and the tree channel Annulus amplitude $\tilde{\mathcal{A}}$:

$$\begin{aligned}\tilde{\mathcal{M}} \sim -\frac{2}{4}R&\big(Ne^{2\pi in\theta} + \bar{N}e^{-2\pi in\theta}\big)\\ &\times \big(\hat{V}_8 - (-1)^n\hat{S}_8\big)W_n.\end{aligned} \tag{5}$$

The $(-1)^n$ is introduced due to a sign ambiguity in taking the mean value[2]. Performing a modular

[1] This is actually the most general Wilson line that in the T-dual model moves the stack around the T-dual circle as a whole.

[2] Ignoring this sign will generate a supersymmetric Möbius strip amplitude. Note also that it seems to be possible to put the sign

transformation, one finds the direct channel amplitude

$$\mathcal{M} \sim -\frac{1}{2}\big[(NP_{2m-2\theta} + \bar{N}P_{2m+2\theta})\hat{V}_8 - (NP_{2m-2\theta+1} + \bar{N}P_{2m+2\theta+1})\hat{S}_8\big]. \qquad (6)$$

The Wilson line takes values in $[0,1] \bmod \mathbb{Z}$. We will distinguish two cases corresponding to $\theta = 0, \frac{1}{2}$. The first case ($\theta = 0$) gives $SO(N+\bar{N})$ gauge group with $N = \bar{N} = 16$ and no massless fermions. The second case ($\theta = \frac{1}{2}$) gives a $U(16)$ gauge group with fermions in the symmetric representation. This is because the $Z_2'$ projection gives antiperiodic boundary conditions to the fermions but its effect is cancelled by the $\theta = \frac{1}{2}$ Wilson line.[3] Note that in the supersymmetric case $\theta = \frac{1}{2}$ leads to $SO(32)$. This mismatch in the values of $\theta$ is due to the shift, since putting a shift results in an effective rescaling of the radius by a factor of 2. It is easy to see that for both cases tadpoles cancel.

Before ending this section, let us make connection with the Chan–Paton algebra formalism [8]. With vanishing Wilson lines, besides the usual untwisted tadpole condition that fixes the number of D9-branes to $\mathrm{Tr}[\gamma_{1,9}] = 32$, one finds from the Möbius strip amplitude the constraints

$$\gamma^T_{\Omega,9} = \gamma_{\Omega,9},$$
$$\gamma^T_{\Omega h,9} = \pm\gamma_{\Omega h,9}, \qquad (7)$$

which imply that $\gamma^2_{h,9} = \pm 1$ [4], thus giving two possible choices for the $\gamma_{h,9}$ matrix. Note that tadpole cancellation does not impose any constraint on $\mathrm{Tr}[\gamma_{h,9}]$. A solution to Eqs. (7) is

$$\gamma^2_{h,9} = +\mathbf{1}_{32}: \quad \gamma_{h,9} = \mathrm{diag}(-\mathbf{1}_n, \mathbf{1}_{32-n}), \qquad (8)$$

$$\gamma^2_{h,9} = -\mathbf{1}_{32}: \quad \gamma_{h,9} = \mathrm{diag}\big(e^{\frac{i\pi}{2}}\mathbf{1}_{16}, e^{-\frac{i\pi}{2}}\mathbf{1}_{16}\big), \qquad (9)$$

where $\mathbf{1}_n$ the $n \times n$ identity matrix with $n$ an even integer. Solution (8) for $n = 0$ and solution (9) lead to the two distinct gauge groups and spectra we found earlier in our simple model corresponding to integer and half integer $\theta$, respectively.

in front of $V_8$ instead. However, it turns out that this choice is not consistent with the parametrization we have chosen in the Annulus amplitude.

[3] We would like to thank Carlo Angelantonj for very helpful discussion on this point.

For general $n$,[4] the two solutions are just particular realizations of the two possible breaking patterns of an even-dimensional orthogonal group projected out by a $Z_2$ inner automorphism [14]. We could have easily found all these solutions in the simple model as well by choosing an appropriately more general Wilson line in (3). The conclusion therefore is that the seemingly two independent solutions (8) and (9) are in fact related via Wilson lines. Nevertheless, they define two classes of physically inequivalent massless spectra and thus they are both interesting in their own right.

## 3. Non-supersymmetric $T^2$ × K3

Consider the $\mathcal{N} = 4$ orbifold of type IIB string theory in four dimensions, $R^4 \times T^2 \times (T^4/Z_N)$. The $Z_N$ orbifold acts on the complex coordinates $z^1 = x^6 + ix^7$ and $z^2 = x^8 + ix^9$ of the $T^4$ torus as $\theta^k : z^i \to e^{2\pi i k v_i} z^i$, where $v = \frac{1}{N}(1,-1)$ and $k = 1, \ldots, N-1$ labels the different $Z_N$ orbifold sectors. We will concentrate on orbifolds with $N = 2, 3, 4, 6$. In addition, we act with a freely-acting $Z_2'$ orbifold generated by the SS element $h$ acting as a translation of length $\pi R$ along the direction $x^5$ of $S^1$ in the $T^2$ torus together with a $(-1)^F$. We shall consider in the following an orientifold of the type $G + \Omega G$, where $G$ is $Z_N \times Z_2'$ which breaks supersymmetry completely.

Upon projecting this orbifold by the world sheet parity $\Omega$, the massless limit of the tree channel Klein bottle amplitude has non-vanishing R–R tadpoles and thus reveals the presence of orientifold planes in the background. Besides the O9-plane that extends in the non-compact directions, wraps the $T^2 \times T^4$ and it is present for any $N$, for even $N$ the model contains also O5-planes that extend along the non-compact directions, wrap around the $T^2$ and sit at the $\theta^k$-fixed points of the transverse $T^4$. In order to cancel the associated to the orientifold planes massless tadpoles one has to introduce D9- and D5-branes. The contribution of the D-branes to the tadpoles is encoded in the massless limit of the transverse channel Annulus and Möbius strip amplitudes.

[4] $n \neq 0$ amounts to splitting the stack of D9-branes into two smaller stacks.

For sake of brevity we will skip the details of the calculation and present directly the result for the massless tadpole conditions. The action of the $Z_N \times Z_2'$ orbifold $g_i = (1, \theta^k, h, \theta^k h)$ on the Chan–Paton matrices carried by the D9- and D5-branes is described by $32 \times 32$ matrices $\gamma_{g_i,9}$ and $\gamma_{g_i,5}$. The matrices $\gamma_{1,9}$ and $\gamma_{1,5}$ that correspond to the identity element of $Z_N \times Z_2'$ can be chosen to be the $32 \times 32$ identity matrices, so that $\mathrm{Tr}[\gamma_{1,9}] = \mathrm{Tr}[\gamma_{1,5}] = 32$. This is a constraint on the number of D-branes that originates from tadpole cancellation in the untwisted sector. The twisted tadpole conditions on the other hand in the $\theta^k$ twisted sector, for $N$ even are given by [9]

$$\mathrm{Tr}[\gamma_{\theta^{2k-1},9}] - 4\sin^2\frac{(2k-1)\pi}{N}\,\mathrm{Tr}[\gamma_{\theta^{2k-1},5}] = 0, \quad (10)$$

$$\mathrm{Tr}[\gamma_{\theta^{2k},9}] - 4\sin^2\frac{2\pi k}{N}\,\mathrm{Tr}[\gamma_{\theta^{2k},5}] - 32\cos\frac{2\pi k}{N} = 0, \quad (11)$$

whereas for $N$ odd they read

$$\mathrm{Tr}[\gamma_{\theta^{2k},9}] - 32\cos^2\frac{\pi k}{N} = 0. \quad (12)$$

From the $\theta^k h$ and $h$ twisted sectors we do not get further constraints on $\mathrm{Tr}[\gamma_{\theta^k h,9}]$, $\mathrm{Tr}[\gamma_{\theta^k h,5}]$, $\mathrm{Tr}[\gamma_{h,9}]$ and $\mathrm{Tr}[\gamma_{h,5}]$. Notice that for $N$ even, the tadpole conditions are consistent with T-duality transformations along the $T^4$ torus that exchanges the D9- and D5-branes. On the other hand, for the circle along which the shift is performed, we have a freedom in taking $\gamma_{h,9}^2 = \pm 1$ and also $\gamma_{h,5}^2 = \pm 1$, however T-duality constrains them to have the same sign. In summary, we will obtain two open string spectra for each $N$, related by Wilson lines, as we have explained in the previous section.

Let us describe the massless spectrum starting from the closed string sector. The closed string spectra of the supersymmetric $T^4/Z_N$ orientifolds have been computed in [8,9]. Sectors twisted by $h$ do not contribute to the massless part of the torus and the Klein-bottle since they correspond to half integer winding [11]. Every other massless sector in the torus is the same as in the corresponding supersymmetric model[5] plus an identical sector where the sign of the fermions is reversed. This simply means that $h$ projects out the fermions altogether from the closed string sector. The bosons remain multiplied by a factor of two which is cancelled by the $1/2$ of the $h$-projector $(1+h)/2$ in the trace. The Klein-bottle on the other hand remains the same as in the corresponding supersymmetric model. The extra $1/2$ from the $h$-projector is now cancelled by a factor of two coming from the doubling of the surviving the $\Omega$ projection states, since any sector and its projected by $h$ counterpart give the same contribution to the Klein-bottle. The closed string spectrum therefore for any $N$ is just the bosonic part of the corresponding supersymmetric model compactified on a $T^2$ torus.

The full open string spectrum will be presented in Tables 1 and 2 for each value of $N$ considered here. As we mentioned before we have two inequivalent spectra for each $N$ corresponding to $\gamma_h^2 = \pm 1$. The effect of the SS deformation on the open strings in a given supersymmetric model is to break the gauge group for $\gamma_h^2 = +1$ as

$$U(N) \to U(n) \times U(N-n),$$
$$SO(N) \to SO(n) \times SO(N-n), \quad (13)$$

whereas for $\gamma_h^2 = -1$ as

$$U(N) \to U(n) \times U(N-n),$$
$$SO(2N) \to U(N). \quad (14)$$

For example, for $N = 2$ and $\gamma_h^2 = +1$ the 99 and 55 sectors contain gauge bosons and scalars (corresponding to the $T^2$ torus) in the adjoint of $U(a) \times U(b)$ with $a + b = 16$ and the remaining scalars (corresponding to the $T^4$ torus) in the $(\boxminus, 1)$ and $(1, \boxminus)$ where $\boxminus$ is the antisymmetric representation of the corresponding gauge group, together with their complex conjugates. The fermions are in the bifundamental representation $(a, b)$ and $2 \times (a, \bar{b})$ plus their complex conjugates. The 95 sector contains bosons in $(a, 1; \bar{a}, 1)$ and $(1, b; 1, \bar{b})$ and fermions in $(a, 1; 1, \bar{b})$ and $(1, b; \bar{a}, 1)$ plus their complex conjugates. On the other hand, for $\gamma_h^2 = -1$ the gauge group is again $U(a) \times U(b)$ with $a + b = 16$. All the scalars are in the $(a, b)$ and the fermions are in the $(\boxminus, 1)$, $(1, \boxminus)$ and $2 \times (a, \bar{b})$ representations plus their complex conjugates. The 95 sector is identical to the previous case. It is easy to check that the above spectrum as well as the spectra for $N = 3, 4, 6$ do not suffer from irreducible gauge anomalies. This is due to the fact that all fermions are in vector

[5] By corresponding supersymmetric model we simply mean the model obtained by eliminating the SS part, which is supersymmetric for all values of $N$ discussed here.

Table 1
The $h$ action on the Chan–Paton charges breaks the gauge group of the six-dimensional supersymmetric orientifolds compactified on K3. For $Z_3$ and $Z_4$ $a+b=c+d=8$

| | | |
|---|---|---|
| | $\mathbf{Z}_3$ | |
| $\gamma_h^2=-1$<br>$U(a)\times U(b)\times U(8)$ | (99)/(55) matter | |
| Scalars | adjoint $+(a,b,1)+(\bar a,1,8)+(1,b,\bar 8)+$ c.c. | |
| Fermions | $2\big((a,\bar b,1)+(1,1,\boxminus)\big)+(\boxminus,1,1)$<br>$+(1,\boxminus,1)+(\bar a,1,\bar 8)+(1,\bar b,8)+$ c.c. | |
| $\gamma_h^2=+1$<br>$U(a)\times U(b)\times SO(c)\times SO(d)$ | (99) matter | |
| Scalars | adjoint $+(\boxminus,1,1,1)+(\bar a,1,c,1)$<br>$+(1,\boxminus,1,1)+(1,\bar b,1,d)+$ c.c. | |
| Fermions | $2\big((a,\bar b,1,1)+(1,1,c,d)\big)+(\bar a,\bar b,1,1)$<br>$+(a,1,1,d)+(1,b,c,1)+$ c.c. | |
| | $\mathbf{Z}_4$ | |
| $\gamma_h^2=-1$<br>$\{U(a)\times U(b)\times U(c)\times U(d)\}_{9,5}$ | (99)/(55) matter | (59) matter |
| Scalars | adjoint $+(\bar a,\bar b,1,1)+(a,1,\bar c,1)$<br>$+(1,b,1,\bar d)+(1,1,c,d)+$ c.c. | $(a,1_3;\bar a,1_3)+(1,b,1_2;1,\bar b,1_2)$<br>$+(1_2,c,1;1_2,\bar c,1)+(1_3,d;1_3,\bar d)+$ c.c. |
| Fermions | $2\times\big((a,\bar b,1,1)+(1,1,c,\bar d)\big)$<br>$+(\boxminus,1,1,1)+(\bar a,1,1,\bar d)+(1,\boxminus,1,1)$<br>$+(1,\bar b,c,1)+(1,1,\boxminus,1)+(1,1,1,\boxminus)+$ c.c. | $(a,1_3;1,\bar b,1_2)+(1,b,1_2;\bar a,1_3)$<br>$+(1_2,c,1;1_3,\bar d)+(1_3,d;1_2,\bar c,1)+$ c.c. |
| $\gamma_h^2=+1$<br>$\{U(a)\times U(b)\times U(c)\times U(d)\}_{9,5}$ | (99)/(55) matter | (59) matter |
| Scalars | adjoint $+(\boxminus,1_3)+(\bar a,1,c,1)+(1,\boxminus,1_2)$<br>$+(1,\bar b,1,d)+(1_2,\bar\boxminus,1)+(1_3,\bar\boxminus)+$ c.c. | $(a,1_3;\bar a,1_3)+(1,b,1_2;1,\bar b,1_2)$<br>$+(1_2,c,1;1_2,\bar c,1)+(1_3,d;1_3,\bar d)+$ c.c. |
| Fermions | $2\big((a,\bar b,1,1)+(1,1,c,\bar d)\big)+(\bar a,\bar b,1,1)$<br>$+(a,1,1,\bar d)+(1,b,\bar c,1)+(1,1,c,d)+$ c.c. | $(a,1_3;1,\bar b,1_2)+(1,b,1_2;\bar a,1_3)$<br>$+(1_2,c,1;1_3,\bar d)+(1_3,d;1_2,\bar c,1)+$ c.c. |

like representations. Alternatively, the models we have considered are effectively five-dimensional and therefore do not have anomalies.

## 4. Conclusion

We have presented a class of non-supersymmetric open string vacua without tadpoles. In particular, satisfying conditions (10)–(12) implies the vanishing of the twisted R–R and NS–NS tadpoles, even though supersymmetry is broken both in the closed and the open string sectors. This should not come as a surprise. In the closed string sector the SS deformation just lifts the fermions and therefore it does not affect the R–R or NS–NS states which are the ones that contribute to the tadpoles. In the open string sector there are no $\bar{\mathrm{D}}$-branes necessary to cancel the orientifold plane charge which means that the tree channel Annulus amplitude does not contain sectors projected by $h$. These sectors contain massless states and if they were present, could alter the supersymmetric tadpole cancellation conditions. On the other hand, the tree channel Annulus amplitude does have sectors twisted by $h$, which however do not contain massless states and so do not contribute to tadpoles. In fact, the SS deformation does not seem to alter the tadpole cancellation conditions for any model in which the SS acts along a direction orthogonal to the space where $Z_N$ acts.

Table 2
For $Z_6$ $2a + 2b = c + d = 2e + 2f = 8$

| | $\mathbf{Z}_6$ | |
|---|---|---|
| $\gamma_h^2 = -1$<br>$\{U(a) \times U(b) \times U(c) \times U(d)$<br>$\times U(e) \times U(f)\}_{9,5}$ | (99)/(55) matter | (59) matter |
| Scalars | adjoint $+ (\bar{a}, \bar{b}, 1_4) + (a, 1, \bar{c}, 1_3)$<br>$+ (1, b, 1, \bar{d}, 1_2) + (1_2, c, 1, \bar{e}, 1)$<br>$+ (1_3, d, 1, \bar{f}) + (1_4, e, f) +$ c.c. | $(a, 1_5; \bar{a}, 1_5) + (1, b, 1_4; 1, \bar{b}, 1_4)$<br>$+ (1_2, c, 1_3; 1_2, \bar{c}, 1_3) + (1_4, e, 1; 1_4, \bar{e}, 1)$<br>$+ (1_3, d, 1_2; 1_3, \bar{d}, 1_2) + (1_5, f; 1_5, \bar{f}) +$ c.c. |
| Fermions | $2\big((a, \bar{b}, 1_4) + (1_2, c, \bar{d}, 1_2) + (1_4, e, \bar{f})\big)$<br>$+ (\bar{a}, 1_2, \bar{d}, 1_2) + (1, \bar{b}, c, 1_3) + (1_2, \bar{c}, 1_2, f)$<br>$+ (1, b, 1_4; \bar{a}, 1_5) + (1_3, \bar{d}, e, 1) + (\boxminus, 1_5)$<br>$+ (1, \boxminus, 1_4) + (1_4, \overline{\boxminus}, 1) + (1_5, \overline{\boxminus}) +$ c.c. | $(a, 1_5; 1, \bar{b}, 1_4) + (1, b, 1_4; \bar{a}, 1_5)$<br>$+ (1_2, c, 1_3; 1_3, \bar{d}, 1_2) + (1_4, e, 1; 1_5, \bar{f})$<br>$+ (1_3, d, 1_2; 1_2, \bar{c}, 1_3) + (1_5, f; 1_4, \bar{e}, 1)$<br>$+$ c.c. |
| $\gamma_h^2 = +1$<br>$\{U(a) \times U(b)$<br>$\times U(c) \times U(d)$<br>$\times U(e) \times U(f)\}_{9,5}$ | (99)/(55) matter | (59) matter |
| Scalars | adjoint $+ (\bar{a}, 1, \bar{c}, 1_3) + (1, \bar{b}, 1, d, 1_2)$<br>$+ (1_2, \bar{c}, 1, e, 1) + (1_3, \bar{d}, 1, f) + (\boxminus, 1_5)$<br>$+ (1, \boxminus, 1_4) + (1_4, \overline{\boxminus}, 1) + (1_5, \overline{\boxminus})$ | $(a, 1_5; \bar{a}, 1_5) + (1, b, 1_4; 1, \bar{b}, 1_4)$<br>$+ (1_2, c, 1_3; 1_2, \bar{c}, 1_3) + (1_4, e, 1; 1_4, \bar{e}, 1)$<br>$+ (1_3, d, 1_2; 1_3, \bar{d}, 1_2) + (1_5, f; 1_5, \bar{f})$ |
| Fermions | $2 \times \big((a, \bar{b}, 1_4), (1_2, c, \bar{d}, 1_2), (1_4, e, \bar{f})\big)$<br>$+ (\bar{a}, \bar{b}, 1_4) + (a, 1_2, \bar{d}, 1_2) + (1, b, \bar{c}, 1_3)$<br>$+ (1_2, c, 1_2, \bar{f}) + (1_3, d, \bar{e}, 1) + (1_4, e, f)$ | $(a, 1_5; 1, \bar{b}, 1_4) + (1, b, 1_4; \bar{a}, 1_5)$<br>$+ (1_2, c, 1_3; 1_3, \bar{d}, 1_2) + (1_2, c, 1; 1_3, \bar{d})$<br>$+ (1_3, d, 1_2; 1_2, \bar{c}, 1_3) + (1_3, d; 1_2, \bar{c}, 1)$ |

We showed that the spectrum for each $N$ splits into two inequivalent branches. The existence of the two branches was understood to have a group theoretic origin associated to the different ways one can embed a $Z_2$ inner automorphism into the $SO(2n)$ and $U(2n)$ Lie algebras and it was shown that the associated vacua are related by Wilson lines.

It would be interesting to extend this analysis to $T^6/Z_N$ and $T^6/Z_N \times Z_M$. In these cases the SS deformation will act in the same direction as the orbifold group. The allowed orbifolds are the ones that commute with the SS deformation [15]. Models where the SS deformation acts along a $Z_2$ direction have been constructed in [3,5].

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## References

[1] E. Kiritsis, seminars at CERN, ENS, 2000, unpublished;
E. Kiritsis, hep-th/0310001.
[2] C. Angelantonj, I. Antoniadis, hep-th/0307254.
[3] I. Antoniadis, E. Dudas, A. Sagnotti, Nucl. Phys. B 544 (1999) 469, hep-th/9807011;
I. Antoniadis, G. D'Appollonio, E. Dudas, A. Sagnotti, Nucl. Phys. B 553 (1999) 133, hep-th/9812118;
I. Antoniadis, G. D'Appollonio, E. Dudas, A. Sagnotti, Nucl. Phys. B 565 (2000) 123, hep-th/9907184;
I. Antoniadis, E. Dudas, A. Sagnotti, Phys. Lett. B 464 (1999) 38, hep-th/9908023;
G. Aldazabal, A.M. Uranga, JHEP 9910 (1999) 024, hep-th/9908072;
A.L. Cotrone, Mod. Phys. Lett. A 14 (1999) 2487, hep-th/9909116;
C. Angelantonj, I. Antoniadis, G. D'Appollonio, E. Dudas, A. Sagnotti, Nucl. Phys. B 572 (2000) 36, hep-th/9911081;

I. Antoniadis, A. Sagnotti, Class. Quantum Grav. 17 (2000) 939, hep-th/9911205;
A. Sagnotti, Nucl. Phys. (Proc. Suppl.) 88 (2000) 160, hep-th/0001077;
C. Angelantonj, I. Antoniadis, E. Dudas, A. Sagnotti, Phys. Lett. B 489 (2000) 223, hep-th/0007090;
C. Angelantonj, A. Sagnotti, hep-th/0010279;
I. Antoniadis, K. Benakli, A. Laugier, Nucl. Phys. B 631 (2002) 3, hep-th/0111209.

[4] M. Borunda, M. Serone, M. Trapletti, Nucl. Phys. B 653 (2003) 85, hep-th/0210075.

[5] C.A. Scrucca, M. Serone, JHEP 0110 (2001) 017, hep-th/0107159;
C.A. Scrucca, M. Serone, M. Trapletti, Nucl. Phys. B 635 (2002) 33, hep-th/0203190;
M. Trapletti, hep-th/0210290.

[6] G. Aldazabal, S. Franco, L.E. Ibanez, R. Rabadan, A. Uranga, J. Math. Phys. 42 (2001) 3103, hep-th/0011073;
L.E. Ibanez, F. Marchesano, R. Rabadan, JHEP 0111 (2001) 002, hep-th/0105155.

[7] R. Blumenhagen, L. Goerlich, B. Kors, D. Lust, JHEP 0010 (2000) 006, hep-th/0007024;
R. Blumenhagen, L. Goerlich, B. Kors, D. Lust, JHEP 0102 (2001) 030, hep-th/0012156.

[8] E.G. Gimon, J. Polchinski, Phys. Rev. D 54 (1996) 1667, hep-th/9601038.

[9] E.G. Gimon, C.V. Johnson, Nucl. Phys. B 477 (1996) 715, hep-th/9604129;
A. Dabholkar, J. Park, Nucl. Phys. B 472 (1996) 207, hep-th/9602030;
G. Aldazabal, A. Font, L.E. Ibanez, G. Violero, Nucl. Phys. B 536 (1998) 29, hep-th/9804026.

[10] G. Pradisi, A. Sagnotti, Phys. Lett. B 216 (1989) 59;
M. Bianchi, A. Sagnotti, Nucl. Phys. B 361 (1991) 519.

[11] C. Angelantonj, A. Sagnotti, Phys. Rep. 371 (2002) 1, hep-th/0204089;
C. Angelantonj, A. Sagnotti, Phys. Rep. 376 (2003) 339, Erratum.

[12] J. Scherk, J.H. Schwarz, Nucl. Phys. B 153 (1979) 61.

[13] E. Kiritsis, C. Kounnas, Nucl. Phys. B 503 (1997) 117, hep-th/9703059.

[14] R. Slansky, Phys. Rep. 79 (1981) 1.

[15] K. Benakli, Phys. Lett. B 386 (1996) 106, hep-th/9509115.