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Determination of Rational Strategies for Players in Two-Target Games

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Abstract—This paper addresses some of the basic issues involved in the determination of rational strategies for players in two-target games. We show that unlike single-target games where the task of role assignment and selection of strategies is conceptually straightforward, in two-target games, many factors like the preference ordering of outcomes by players, the relative configuration of the target sets and secured outcome regions, the uncertainty about the parameters of the game, etc., also influence the rational selection of strategies by players. The importance of these issues is illustrated through appropriate examples.

1. INTRODUCTION

The theory of differential games is an important extension of game theory. It essentially deals with dynamic systems (represented by differential equations) under the control of more than one player or decision-maker. This theory found a ready application in pursuit-evasion games such as combat encounters between two aircraft in which only one of them carries a weapon system, and missile-target engagements [1]. These games are also called single-target games with the weapon envelope of one the vehicles modelled as the only target set. However, in realistic aerial combats between two aircraft, both aircraft have weapon systems of their own. The notion of two-target games, as proposed by Blaquiere, Gerard and Leitmann [2] was found to have some of the essential features to model this kind of aerial combat game. Getz and Leitmann [3] extended this theory of two-target games to demarcate the win regions for the two players in the state space. A similar theory was later used by Getz and Pachter [4,5] to obtain the qualitative solutions of the two-target game versions of the homicidal chauffeur game and the game of two cars. Details of the geometrical construction of the barrier in the two-target game of two cars are given in Pachter and Getz [6]. Further theoretical developments using this approach are reported by Skowronski and Stonier [7,8].

In single-target games, the player having a weapon system always plays offensively and the other always plays defensively. Thus, it is easy to assign the role of pursuer to the player with a target set and the role of evader to the player without a target set. Olsder and Breakwell [9] considered a two-target game in which, depending on all the possible terminations of the game, each player was assigned the role of a pursuer or of an evader in certain regions of the state space, and the game was solved in these regions as a single-target pursuit-evasion game. In [4,6] too, the examples chosen were such that this kind of role assignment was possible.

Merz [10] also addressed the question of role specification and strategy determination for a two-target game between two exactly identical aircraft with identical weapon systems. It was claimed

Typeset by $\mathcal{A}_{\mathcal{M}}\mathcal{S}$-TEX that, under these assumptions, it is possible to unambiguously assign the roles of pursuer and evader to the players from specific initial states in the state space. Time-optimal strategies were also determined for both players in different outcome regions. However, this choice of strategies did not account for certain important issues that arise in two-target games. One such issue was raised by Breakwell [11] in an interesting comment on Merz's paper.

It was pointed out by Ardema, Heymann and Rajan [12,13] that, in a general two-target game, such role assignments may not always be possible. It was shown that players need not always assume the role of a pursuer or an evader. The paper also discussed the possibility of the players adopting different preference orderings of the four possible qualitative outcomes of the game.

Shinar and Davidovitz [14] proposed a set of guidelines to obtain the qualitative outcome regions in a two-target game. In Ghose and Prasad [15], a similar approach was taken with the same objective, but with the additional feature of the possibility of different preference orderings of outcomes for the two players. Based on this notion, the secured outcome regions for the players were demarcated. It was shown that the outcome regions depend significantly on the preference orderings of outcomes by the players. In fact, these papers collectively showed that the simple notion of role assignment, as used in single-target games, must be replaced by a more complex notion based on the preference orderings of outcomes by the players.

The work discussed above was primarily concerned with the qualitative solution of two-target games in which the objective was to demarcate the state space into outcome regions of players. On the other hand, the quantitative solution attempts to obtain optimal strategies for the players. Exactly as in the qualitative solution, it is natural to expect that the preference ordering of outcomes by the players will play an important role here as well. In [16,17], some constrained optimization problems were defined whose solutions not only partially demarcated the state space into secured outcome regions [16], but also yielded some strategies for the players in these regions [17]. In [17], it was also shown that the strategies so obtained were dependent on the preference orderings of outcomes by the players in the following sense. A point in the state space could belong to one particular secured outcome region of a player if he follows one of the preference orderings, while it may belong to another secured outcome region if he follows a different preference ordering. Consequently, the player had the option of choosing two different strategies as specified by the solution to the optimization problems. However, in the present paper, we show that the selection of strategies by the players must depend, at least to some extent, on the preference ordering of players irrespective of any optimization problems posed to solve the quantitative/qualitative game. We also show that even if a point in the state space belongs to the same secured outcome region of a player for two different preference orderings of outcomes, the player may select different strategies for these different preference orderings. Moreover, in this paper we also identify a few other issues like the possibility of non-optimal play by the opponent, the relative configuration of the target sets and secured outcome regions in the state space, uncertainty over the parameters of the game, etc., within the framework proposed here, and discuss their effect on the selection of strategies. All these points will be mainly illustrated through the solution of a modified version of the homicidal chauffeur game.

The ideas presented in this paper, though restricted to the determination of rational strategies for players, form a vital link in the overall solution of two-target games. This method of solution is basically a hierarchical process in which the first step is to obtain the secured outcome regions for the two players. A reasonable set of guidelines to achieve this task was presented in [15]. The present paper discusses the subsequent steps in this solution procedure, the first of which is to obtain certain game-of-kind and game-of-degree strategies for the players in related single-target games from particular initial conditions. The next step involves a selection among these strategies on the boundaries and interior portions of the secured outcome regions, taking into account the preference orderings of outcomes by the players, configuration of the secured outcome regions and target sets, and certain other important factors.

2. FORMULATION OF THE GAME

The dynamics of the game are given by the differential equations,

$$\dot{x} = f(x, u, v), \qquad x(0) = x_0.$$

The control actions of the players P1 and P2 are constrained by $u \in U \subseteq R^p$ and $v \in V \subseteq R^q$, respectively, where U and V are compact and convex sets. The target sets associated with the players are $\mathcal{T}_1$ and $\mathcal{T}_2$. The state vector $x \in R^n$ is assumed to remain bounded. We further assume that the maximum allowable time limit for the game is given by a non-zero finite time T. The game is said to terminate at $t = t_f \leq T$ if either $x(t_f) \in \mathcal{T}_1 \cup \mathcal{T}_2$ or $t_f = T$. The game is said to have terminated with the outcome "P1's win" if $x(t_f) \in \mathcal{T}_1 \setminus \mathcal{T}_2$ and with the outcome "P2's win" if $x(t_f) \in \mathcal{T}_2 \setminus \mathcal{T}_1$. The outcome is a "mutual kill" if $x(t_f) \in \mathcal{T}_1 \cap \mathcal{T}_2 \neq \emptyset$, and a "draw" if $x(t_f) \notin \mathcal{T}_1 \cup \mathcal{T}_2$ (in which case $t_f = T$). The players are assumed to adopt feedback strategies which are functions of the current time and state of the game.

In an earlier paper [15], a sufficiently general set of guidelines was specified to demarcate the state space into secured outcome regions based upon the players' preferences over the outcomes defined above. The approach followed was a purely qualitative one and did not involve the specification of players' strategies. In [16,17], a scheme for obtaining the players' strategies in certain regions of the state space, through the solution of some optimization problems, was proposed. Though the strategies so obtained were reasonable choices in some situations, they were not entirely so in other situations. A closer look reveals that this was due to the fact that the notion of preference ordering of outcomes and its effect on the choice of strategies was not fully utilized in the scheme. Moreover, the possibility of non-optimal play by the opponent or uncertainty about the parameters of the game was also not fully exploited. We shall discuss some of these issues in the next section where we also specify some representative strategies for the players. One of the major considerations in the actual use of these strategies would be that the state does not pass into a less preferred secured outcome region. On the other hand, they should be so used as to be able to exploit the situation of non-optimal play by the opponent and move the state into a better secured outcome region or achieve a better outcome.

3. SOME IMPORTANT CONCEPTS AND DEFINITIONS

3.1. Preference Ordering of Outcomes

It is possible for each player to order the outcomes of the game according to his own individual preference. We elaborate on this further. Obviously, a player prefers his own win over any other outcome and accords the least preference to his opponent's win. The preference attributed to the outcomes of mutual kill (MK) and draw (D) will lie somewhere in between these two outcomes. A player may prefer D over MK, or otherwise, depending upon whether the player attributes more importance to his own survival or to the outcome of opponent's capture. It is possible to classify the outcomes as "capture" and "survival" as follows,

$$\text{Capture} = \{\text{player's win, MK}\},$$
$$\text{Survival} = \{\text{player's win, D}\}.$$

We say that a player prefers capture over survival if he prefers MK over D (denoted by MK/D). Similarly, a player prefers survival over capture if he prefers D over MK (denoted by D/MK). Note that "capture" here refers to the opponent's capture, whereas "survival" refers to the player's own survival.

It is also interesting to note that the notions of capture and survival can be related in some fashion to the notions of pursuit and evasion in single-target games in the sense that the pursuer prefers capture over survival and the evader prefers survival over capture. However, in the case of two-target games, the presence of the second target set makes a significant difference to the choice of strategies by the players and the demarcation of secured outcome regions in the state space.

3.2. Modes of Play

In the literature [2], a differential game between two players has been classified as qualitative (game-of-kind) and quantitative (game-of-degree). In the two-target version of the game-of-kind, a player's only objective is to terminate the game on his own target set while avoiding the opponent's target set. In the game-of-degree, the player has the additional objective of optimizing some performance index, e.g., the time of termination. In this paper, we define some strategies for the players and make some important but reasonable assumptions.

DEFINITION 3.1. GAME-OF-KIND OFFENSIVE STRATEGY (GKOS). *A player is said to play a GKOS when he tries to bring the opponent as close as possible to his own target set.*

DEFINITION 3.2. GAME-OF-KIND DEFENSIVE STRATEGY (GKDS). *A player is said to play a GKDS if he tries to avoid the opponent's target set by as much physical distance as possible.*

DEFINITION 3.3. GAME-OF-DEGREE OFFENSIVE STRATEGY (GDOS). *A player is said to play a GDOS if he tries to terminate the game as quickly as possible, provided that termination on his own target set is possible.*

DEFINITION 3.4. GAME-OF-DEGREE DEFENSIVE STRATEGY (GDDS). *A player is said to play a GDDS if he tries to avoid the opponent's target set for as long as possible.*

The definitions given above are somewhat imprecise and require some additional qualifications. These strategies are basically feedback strategies of a player in a game involving two players, and hence, their optimality also depends on the behavior of the other player. Here, it is assumed that the strategies are obtained by solving a zero-sum game against an antagonistic player. For example, the GKOS of a player is an optimal minimax (security) strategy of a player's target set by as much distance as possible. A similar notion applies to the other strategies too. In fact, in most cases, these strategies can be obtained by solving an appropriate single-target game. However, when the strategies are actually used, the presence of the other player's target set and the resulting demarcation of the state space into secured outcome regions for the player must be accounted for to ensure that the state does not move into a less preferred region in the state space. This also leads to the conclusion that the secured outcome regions of a player must be obtained first before one can discuss an appropriate choice of strategy for the player.

It is also important to note that, although GKOS (or GKDS) has been termed as a game-of-kind strategy to a player in the two-target game, it also involves (like a game-of-degree strategy in a single-target game) the distance between him and his opponent's target set (or the distance between his own target set and the opponent) as the performance index. The reason behind naming it as game-of-kind strategy in the two-target game is that it is this distance that determines whether the game has ended in a capture or not, which is precisely the objective in a game-of-kind solution. Therefore, it should be clearly understood that the phrase "game-of-kind" is used in the above definitions in a somewhat subjective sense. Further, it is the game-of-kind strategies which assume primacy over the game-of-degree strategies as they define the ultimate objective that a player has under a given situation. These assertions will be illustrated later in the examples.

It should also be noted that the notions of offensive and defensive strategies are closely related to the notions of capture and avoidance in the sense that a player playing an offensive strategy tries to use his own target set to capture the opponent whereas a player playing a defensive strategy tries to avoid the opponent's target set.

3.3. Non-Optimal Play by the Opponent

In a realistic aerial combat game (which the two-target game models), a player cannot discount the possibility of non-optimal play by the opponent, and his strategy should be so chosen as to take advantage of the opponent's mistakes without exposing himself to peril. In fact, this factor plays an important role in the selection of strategy by a player based on his preference ordering. For example, consider a point in the win region of the opponent. If the opponent plays non-optimally, he exposes himself either to a mutual kill outcome or to a draw outcome, provided that the player takes advantage of this non-optimal play with a suitable choice of strategy. Obviously, the player will choose a strategy which enhances the possibility of occurance of the more preferred outcome. We shall see some examples of this in the next section.

4. SELECTION OF STRATEGIES

In this section, we shall illustrate, with appropriate examples, that the choice of a rational strategy depends to a great extent on the preference ordering of outcomes by the player. We shall also identify other major factors which contribute to this selection procedure. For this purpose, we use the following modified versions of the two-target homicidal chauffeur game proposed by Davidovitz and Shinar [18].

4.1. Example 1

Consider a planar aerial combat between an aircraft and a helicopter. The aircraft (which is modelled by the dynamics of the chauffeur-driven car) has limited turn capability but is capable of high speed. It carries a weapon system with limited boresight effectiveness. For convenience, we assume that the boresight is no more than the angle corresponding to the usable part of the aircraft's target set, had the target set been assumed circular. The helicopter (which is modelled by the pedestrian's dynamics) is assumed to have lesser speed but is capable of instantaneous turns. We also make an additional assumption that the range of the weapon system used by the two flight vehicles is equal. This model has been used earlier in [15] and [17] to obtain the qualitative (and partial quantitative) solution of the game. For convenience, we call the aircraft P1 and the helicopter P2. Their corresponding target sets are $\mathcal{T}_1$ and $\mathcal{T}_2$.

It should be noted here that because the weapon ranges of the aircraft are equal, the outcome of win for the aircraft does not arise. Thus, there are only three outcomes in this game: helicopter's win, mutual kill (MK, in which both vehicles are destroyed), and draw (D, in which both vehicles survive).

In the previous section, we had defined four kinds of strategies. They are by no means the only strategies that a player can choose, but by and large they represent most of the rational strategies a player may wish to choose in a game such as this. We shall now examine the situations under which a player will have occasion to choose these strategies. The secured outcome regions, obtained by using the method given in [15], are shown in Figure 1. They are classified for the two relevant preference orderings as follows.

	MK/D	D/MK
P1's win region	—	—
P2's win region	A,H	A,F
P1's secured MK region	B,F	B,F
P1's secured D region	D	D
P2's secured MK region	B,F	B,F
P2's secured D region	D	D

In this case, the secured outcome regions for the two players match regardless of the preference ordering of outcomes they choose. But we shall see that the strategies they follow could be different.

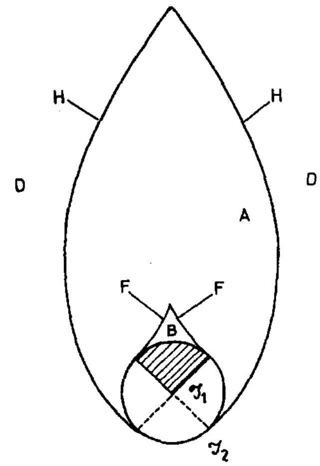

Figure 1. Secured outcome regions for Example 1.

In the interior of the win region of a player, the player may choose either GKDS or GDOS. However, on or very near the barrier separating his win region from the other outcome regions, the player chooses the relevant barrier strategy such that the state does not leave the win region. In the interior, the player chooses GKDS if he wishes to terminate the game on his own target set but as far away from his opponent's target set as possible. This is also the minimum risk capture strategy (MRCS) as defined in [17]. The player chooses a GDOS if he wishes to terminate the game as quickly as possible. This strategy is also the closest approach survival strategy (CASS) with a suitable value of δ as defined in [17]. The player never chooses GDDS. Any win strategy for the player is also a GKOS and therefore, in this case, both GKDS and GDOS are also GKOS. In this particular case there does not appear to be any reason for the player choosing either GKDS or GDOS over the other, based on his preference order of outcomes. However, this choice of strategy may be governed here by the uncertainty about the opponent's target set or the uncertainty about T. This can be easily seen from Figure 1, in which A is the win region for P2. If there is uncertainty about the boundary of $\mathcal{T}_1$, P2 will not like the game to terminate close to $\mathcal{T}_1$ and so uses a GKDS. If P2 has limited operational life represented by T, and there is some uncertainty about T, then P2 tries to terminate the game as quickly as possible and, hence, may use a GDOS. Of course, P2 may use other strategies (all of which are GKOS) too, depending on other considerations.

In the secured draw region, a player may choose GKOS or GKDS but not GDOS. Choice of GKOS or GKDS also implies GDDS if the game actually ends in a draw. Let us elaborate on this further. Choosing GKOS implies that the player tries to bring the opponent as close to the player's target set as possible. The idea is that if the opponent makes a mistake in his choice of strategy, then the player may obtain a win or mutual kill. On the other hand, GKDS implies that the player tries to keep as far away from the opponent's target set as possible. Thus, under certain circumstances, GKOS is chosen if the player prefers mutual kill over draw, whereas GKDS is chosen if the player prefers draw over mutual kill. Of course, all this also depends on the relative configuration of the target sets and the secured outcome regions. For example, consider Figure 1 in which P1 has a wider boresight capability so that it covers the whole of the usable part of $\mathcal{T}_2$. The boundary of the modified $\mathcal{T}_1$ is shown by the broken line. In this case, A is no longer the win region of P2. Now consider a point in D. From this point, P2 will try to bring the state as close as possible to $\mathcal{T}_2$ by using GKOS if he prefers a mutual kill over draw, as this choice of strategy also implies proximity to $\mathcal{T}_1 \cap \mathcal{T}_2$. On the other hand, if P2 prefers draw over mutual kill, then he tries to keep as far away from $\mathcal{T}_1$ as possible by using a GKDS.

A player never uses GDOS since minimization of the time of termination is meaningless in the secured draw region. But there could be many strategies which are GDDS as any strategy which secures a draw also produces the maximum allowable time T as the payoff and hence is a GDDS.

In the secured mutual kill region of a player, it makes sense for the player to choose GKOS under the preference ordering in which the player prefers mutual kill over draw. For, in this case, the player has a GKOS which will fetch him a mutual kill or a win in spite of what the opponent does. In fact, the player may use a GKOS, which is also a GDOS, if he wishes to terminate quickly. The player may also choose a GKDS to enhance his own chances of win. Suppose the player prefers draw over mutual kill, then he may choose a GKOS which is also a GDDS since he would like to delay the termination for as long as possible. The player may also use a GKDS in the interior of the secured mutual kill region.

Region B is the secured mutual kill region for both the players. Suppose P1 prefers mutual kill over draw. Then P1 prefers to use a GKOS which could also be a GDOS. Whereas, if P1 prefers draw over mutual kill, then he uses a GKOS which is also a GDDS. This is so because the best outcome that P1 can get is by termination on $\mathcal{T}_1 \cap \mathcal{T}_2$ and any deviation from this objective might cause P2 to win. However, since he prefers to survive rather than capture the opponent it is rational to suppose that the player chooses a GDDS which would delay mutual kill for as long as possible. Now, consider the case of P2 here. Regardless of whether he prefers mutual kill or draw, he chooses a GKDS (which could also be a GDDS) since this enhances his own chances of win if P1 makes a mistake. This example, in fact, shows that it is not only the preference ordering of outcomes by the players, but also the relative configuration of the target sets and the secure outcome regions which exert an influence on the choice of strategies by the players.

In the opponent's win region, it is logical to assume that a player will use a GKDS if he prefers draw over mutual kill and a GKOS if he prefers mutual kill over draw. Consider the same example (Figure 1), in which A is the opponent's (P2's) win region. If P1 prefers draw over mutual kill, then he tries to avoid the opponent's target set by as much distance as possible. Whereas, if P1 prefers mutual kill over draw (i.e., capture over survival), then he tries to bring the state as close as possible to $\mathcal{T}_1$. However, it should be noted that though the above analysis holds in this example, one can formulate problems in which the target sets and outcome regions are so configured that the choice of strategy for a player may not be as straightforward as given here.

In the next example, we choose a slightly more complex problem and specify the strategies for each player in all the regions of the state space and for both preference orderings of outcomes.

4.2. Example 2

Consider the same example, but with the radius of the target sets larger. In this case, the barriers originating from the usable part of P1's target set $\mathcal{T}_1$ do not intersect. This is shown in Figure 2, in which all the regions and their boundaries are marked. In the following table, the secured outcome regions for the two players are identified, based on the qualitative solution method proposed in [15], for both preference orderings of outcomes.

	MK/D	D/MK
P1's win region	—	—
P2's win region	A,H	A,H
P1's secured MK region	B,F,E,G,C,I	B,F,E
P1's secured D region	D	C,G,I,D
P2's secured MK region	B,F,E	B,F,E,C,G,I
P2's secured D region	C,G,I,D	D

Now, let us identify P1's strategies in terms of the definitions given in Section 3. In P2's win region (which is the region in which P1 loses), P1's strategies are as follows.

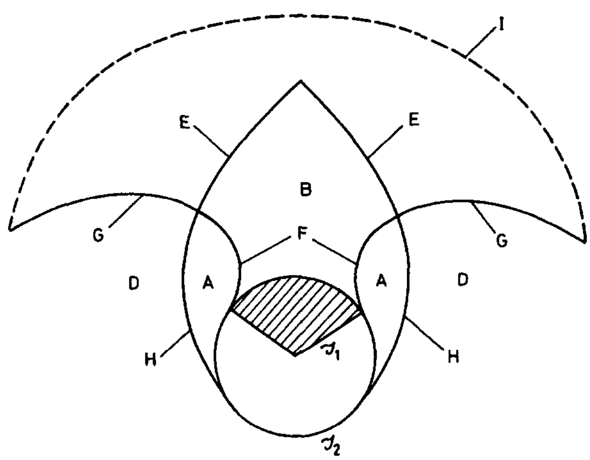

Figure 2. Secured outcome region for Example 2.

	MK/D	D/MK
In A, but close to F	GKOS	GKOS
In A, but close to H and on H	GKDS	GKDS
Elsewhere in the interior of A	GKOS	GKDS

If the initial state lies close to the barrier F (but in A), then P1 uses GKOS (a hard right turn, in the right half region) regardless of his own preference ordering of outcomes. This is a rational choice of strategy as P1 here tries to take advantage of any error in the choice of strategy by P2 and move the state into the region B or on the barrier F since this would secure him a better outcome (mutual kill) than the one of P2's win. This strategy can be regarded as a kind of "Kamikaze" maneuver, i.e., knowing his own destruction to be inevitable, P1 tries to destroy P2, too, in a last desperate attempt.

A similar rationale holds in the choice of GKDS (a hard left turn, again in the right half region) on the barrier H or near H, but inside A. Here, P1 also tries to take advantage of the opponent's error in choice of strategy to secure a better outcome (draw) for himself.

When we refer to a point which is close to a barrier, we don't really specify how close this point should be. This judgement is purely subjective and may depend on the uncertainty of information the player has about the opponent's capability, among other factors.

In P1's secured mutual kill region, the strategies chosen by P1 are as follows.

	MK/D	MK/D
In B, but close to E	GKOS⟶GDOS	GKDS
Elsewhere in B	GKOS⟶GDOS	GKOS⟶GDDS
F	GKOS⟶GDOS	GKOS⟶GDDS
E	GKOS⟶GDOS	GKDS
G,C,I	GKOS⟶GDOS	

The notation GKOS⟶GDOS implies that the strategy chosen by the player is GKOS and there could be many such strategies. Among these strategies, he chooses one which is GDOS. Thus, there is a hierarchy in the choice of strategies.

Note that on the barriers F and G, player P1 may have only one GKOS which keeps the state on the barrier if the opponent plays optimally, and thus, he may not have the choice to select a GDOS or GDDS.

On the barrier E, P1 follows a GKDS when his preference is draw over mutual kill because wrong play by the opponent will then drive the state into the secured draw region C.

	MK/D	D/MK
In D, but close to G	GKOS	GKDS
D	GKDS	GKDS
G,I,C	—	GKDS

In P1's secured draw region, the strategies chosen by P1 are as follows.

Here, too, the phrase "close to G" should be interpreted in a subjective sense only. When the initial state is close to G, by playing an offensive strategy, the player has a chance of moving the state into the mutual kill region, provided P1 prefers MK over D. Whereas in the rest of D, P1 plays a defensive strategy, as otherwise there is a possibility that the state may move into P2's win region. This is irrespective of the preference ordering of outcomes that P1 follows.

Now, let us identify player P2's strategies in a similar way. In P2's secured MK region, the strategies for P2 are as follows.

	MK/D	D/MK
In B, but close to F and on F	GKDS	GKDS
In B, but close to E and on E	GKOS	GKDS
Elsewhere in the interior of B	GKDS	GKDS
C,G,I	—	GKDS

When the state is close to F, P2 plays defensively irrespective of his preference ordering of outcomes, as this enhances the possibility of the state moving into P2's win region if P1 plays wrongly. On the other hand, when the state is close to E and P2 prefers MK over D, he must play offensively, as otherwise the state may move into the secured draw region for a suitable choice of strategy by P1. Note that here the strategies are chosen not to take advantage of non-optimal play by the opponent, but to prevent the opponent from taking advantage of inappropriate play by the player.

In P2's secured draw region, the strategies for P2 are stated above.

	MK/D	D/MK
C,G,I,D	GKOS	—
In D but close to G	—	GKDS
Elsewhere in the interior of D	—	GKOS

Hence, when P2 prefers MK over D, he has nothing to lose by playing offensively. On the other hand, he has much to gain if P1 plays non-optimally. This is the reason for his choosing GKOS in the whole of the secured draw region.

But when P2 prefers D over MK, he plays defensively at states close to the barrier G which separates the secured draw region from the secured MK region. In other parts, he plays offensively.

In P2's win region, the strategies for P2 are as follows.

	MK/D	D/MK
In A but close to F	GKDS	GKDS
Elsewhere in the interior of A and on H	GKOS⟶GDOS	GKOS⟶GDOS

Here, close to the barrier F, P2 must play defensively, as otherwise the state may move into secured mutual kill region B, if P1 plays appropriately. Elsewhere, P2 can follow a GKOS which could also be a GDOS. This is irrespective of the preference ordering of outcomes by P2.

An analysis of the strategies followed by the players in this game reveals a number of very significant features.

The first of these is that from the same point in a given secured outcome region, a player may choose different strategies depending on his own preference ordering of outcomes. The opponent's preference ordering of outcomes does not play any role here.

The second significant feature is that from different parts of the same secured outcome region, a player may select different strategies. This may happen in two situations. The first occurs when the player wishes to take advantage of the opponent's non-optimal play and move the state into a more preferred secured outcome region. The other situation occurs when the player tries to prevent the opponent from moving the state into a less-preferred secured outcome region. Obviously, these situations occur close to barrier sections or surfaces separating two different outcome regions.

Another noteworthy feature is the pairs of strategies which the players actually choose in a given region. This is shown in the table which follows. The entries in this table are in the form of pairs of strategies, the first of which is P1's strategy and the second is P2's strategy.

P1's preference⟶ P2's preference⟶	MK/D MK/D	MK/D D/MK	D/MK MK/D	D/MK D/MK
In A, but close to F	GKOS GKDS	GKOS GKDS	GKOS GKDS	GKOS GKDS
In A, but close to H and on H	GKDS GKOS→GDOS	GKDS GKOS→GDOS	GKDS GKOS→GDOS	GKDS GKOS→GDOS
Elsewhere in the interior of A	GKOS GKOS→GDOS	GKOS GKOS→GDOS	GKDS GKOS→GDOS	GKDS GKOS→GDOS
In B, but close to E	GKOS→GDOS GKOS	GKOS→GDOS GKDS	GKDS GKOS	GKDS GKDS
Elsewhere in B and F	GKOS→GDOS GKDS	GKOS→GDOS GKDS	GKOS→GDOS GKDS	GKOS→GDOS GKDS
G,I,C, E	GKOS→GDOS GKOS	GKOS→GDOS GKDS	GKDS GKOS	GKDS GKDS
In D, but close to G	GKOS GKOS	GKOS GKDS	GKDS GKOS	GKDS GKDS
Elsewhere in D	GKDS GKOS	GKDS GKOS	GKDS GKOS	GKDS GKOS

It can be observed that when the state lies close to a section of the barrier which satisfies the condition that it separates two different outcome regions for both the players, regardless of the preference ordering of outcomes by the players, the players invariably choose strategies which are strictly antagonistic (and form a saddle point pair, as in this example) in a game-of-kind sense. In such a case, if one player chooses GKOS, the other chooses GDOS. The fact that this strictly antagonistic choice of strategies does not occur at all barrier sections can be observed from the choice of strategies on E. It is seen that when both players have the same preference orderings of outcomes, E separates two different secured outcome regions for only one of the players, but not for both.

Also, this strictly antagonistic choice of strategies may not happen for states lying in the interior of a secured outcome region, even when both players have opposite preference orderings of outcomes. This can be seen in the choice of strategies in the interior of A.

5. CONCLUSIONS

In this paper, we have attempted to show that, unlike the single-target pursuit-evasion games, in two-target games, the determination of rational strategies by players depends very significantly on the preference ordering of outcomes by players, the relative configuration of the target sets and secured outcome regions, the uncertainty over the parameters of the game, and perhaps other factors, too. This analysis also indicates that the problem of formulating suitable guidelines for selection of player's strategies, even for simple two-target games, may be quite a formidable exercise. However, this paper discusses many of the basic issues which must be addressed in formulating any such universal set of guidelines. Of course, there could be other important issues, too, which have not been discussed here. One such issue is the possibility of limited cooperation between players, as pointed out by Breakwell [11].

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