Proton decay, supersymmetry breaking and its mediation

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Abstract

We study the breaking of supersymmetry and its transmission to the light states in the context of the minimal SU(5) grand unified theory, with no additional singlets. This simple theory can be taken as a prototype for a program of breaking simultaneously grand unified symmetry and supersymmetry. The main predictions are: (i) $d = 6$ proton decay is completely negligible and $d = 5$ is in accord with experiment, (ii) supersymmetry breaking is mainly mediated by gravity.
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1. Introduction

After more than 30 years of supersymmetry playing a prominent role in particle physics we still know nothing about the source of its breaking or the nature of its mediation to the standard model supermultiplets. The most appealing scenario of spontaneous supersymmetry breaking in the MSSM fails by predicting sfermions lighter than the light fermions [1] and so the desired spontaneous breaking is assumed to happen in the SM gauge invariant sector and then transmitted to our world through either gravity or other interactions.

The most natural messengers of supersymmetry breaking are the Higgs doublets, H and $\overline{H}$, as suggested some 10 years ago by Dvali and Shifman [2]. Unfortunately this gives a negative contribution (proportional to the square of the Yukawa couplings $y^\dagger y$) to the squares of sfermion masses, so that the stop becomes tachyonic [3].

In a sense this is a blow to the whole program. After all, the large y_t plays an important role in supersymmetry for it leads naturally to the tachyonic property of the Higgs [4] and it was also predicted [5] originally in order to achieve unification of couplings in the MSSM [5,6]. New vector-like multiplets can be added in order to mediate supersymmetry breaking but this typically means introducing new Yukawa type couplings [7]. One assumes that they are zero and speaks of pure gauge mediation, but this is true only if the gauge quantum numbers do not allow direct Yukawas, which is rare. Recently it was argued that the job could be done by the Higgs [8] or gauge [9] fields of some grand unified theory. In view of nonvanishing neutrino masses a particular interesting candidate is the Higgs supermultiplet responsible for the type II seesaw [8]. The crucial issue here is to know who dominates the mediation and by how much. This can be only answered in a simple and predictive theory, a kind we describe below.

Whoever the messenger is, an important question remains regarding the source of supersymmetry breaking. The conventional perturbative scenarios which use gauge singlets work kind of trivially due to the absence of constraints on the singlet couplings. Low energy supersymmetry has its principal role in grand unified theories, where it protects the Higgs from the large scale once the

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doublet–triplet (DT) splitting is achieved. Thus the most natural source of supersymmetry breaking is provided by the GUT Higgs supermultiplet, the SM singlet component. It turns out that this was studied very little [10–12]. We come back to the comparison with these works over and over in this Letter.

At the same time supersymmetric grand unification is normally plagued by large threshold effects which impede precise predictions of the proton decay rate. For example, in the minimal supersymmetric SU(5) the dominant $d = 5$ operator depends crucially on the ratio of the colour octet (σ_8) mass m_8 and the weak triplet (σ_3) mass m_3 of the surviving remnants of the adjoint Higgs: varying m_3/m_8 from 1 to 4 increases τ_p by a factor of 10^3 [13,14]. Furthermore, in general even soft supersymmetry breaking may obscure proton decay predictions, if the soft terms in the heavy and light sector are strongly split (for recent work see [15] and references therein).

All of this indicates that by itself none of the above questions can be easily answered. It is strongly suggestive that our best hope is a consistent correlated treatment of all the three questions above (mediation, supersymmetry breaking and unification) in the context of a well defined simple grand unified theory. This is the main scope of our Letter. For the sake of simplicity, clarity and predictivity, we discuss this program in the very minimal supersymmetric SU(5) theory. By this we mean besides the usual generations of quarks and leptons only 24_H and 5_H, $\bar{5}_H$ supermultiplets. Of course the already existing phenomenology requires the inclusion of higher-dimensional terms.

An additional issue to be faced in SU(5) is the neutrino mass. Here there are number of ways which basically do not change anything we do in this Letter. One simple possibility is for example bilinear R-parity breaking [16] (this means tuning away the baryon number violating contribution) which does not require any change. Another simple possibility is to have right-handed neutrinos as SU(5) singlets and utilize the so-called type I seesaw mechanism [17]. As long as these fields have zero vacuum expectation value and zero F-term, everything we say here goes through unchanged. In the opposite case one faces a danger of having potentially uncontrollable R-parity breaking which we prefer to avoid. Yet another simple possibility is to utilize type II seesaw [18] through the introduction of 15_H and $\overline{15}_H$ fields. These fields are potential messengers of supersymmetry breaking and we will comment on their role in Section 5. Finally, one can use the triplet and singlet fermions in 24_H as a combination of type I and type III seesaw [19].

We start by readdressing the issue of supersymmetry breaking through a single 24_H field in the supergravity potential. We find that this program can lead to a huge suppression of dimension 5 proton decay rate, due to the automatic appearance of intermediate states. In the case of the simplest possible realistic superpotential (quartic in 24_H), this is actually a firm prediction. As usual in supersymmetric grand unification there are also direct dimension 5 operators suppressed by the Planck scale, such as

$$W(d = 5) = \frac{c}{M_{Pl}} 10_F 10_F 10_F \bar{5}_f. \tag{1}$$

Unless $c \lesssim 10^{-6}$, this destabilizes the proton too much. The bottom line of this discussion is that the $d = 5$ proton decay in supersymmetry cannot be predicted, unless one resorts to flavour symmetries on top of grand unification. Since we stick to the minimal and pure SU(5) theory, we have to assume that the above condition is satisfied.

The possible mediators of supersymmetry breaking are: (1) gravity; (2) heavy gauge bosons X and Y; (3) heavy Higgs supermultiplets σ_3 and σ_8 (weak triplet and colour octet from 24_H), (4) T, $\bar{T}$ (the colour triplets from 5_H and $\bar{5}_H$ which mediate proton decay); (5) light Higgs doublets D and $\bar{D}$. Since the masses of these states are constrained by the requirement of unification, one can compare their contribution to the soft light spartner masses. This program is rather predictive: as we show below, the dominant contribution to the soft breaking terms comes actually from gravity in most of the parameter space. The desired gauge mediation is rather suppressed and the question of flavour violation of neutral currents remains still an open question. However, in a rather small region of the parameter space, where $m_{3,8}$ are particularly fine-tuned, $\sigma_{3,8}$ could be the dominant messengers. The interesting characteristic of this case would be a somewhat unusual spectrum of spartners with right-handed sleptons much lighter than the rest.

Needless to say, we do not wish to argue here that this is the final theory, but rather to indicate how a well defined approach of using a simple model makes simultaneously clear predictions on proton decay, the TeV effective theory and the nature of the soft supersymmetry breaking.

In summary, the main predictions for the reader to carry away from this approach are:

(a) the minimal nonrenormalizable SU(5) with $24_H, 5_H, \bar{5}_H$ and three generations of $10_F, \bar{5}_F$ embedded in supergravity is enough to break supersymmetry and get a realistic low-energy physics (MSSM);

(b) $d = 6$ proton decay is completely negligible and $d = 5$ is not in contradiction with the experiment as often claimed; the importance of this result cannot be overstressed;

(c) gravity dominates soft supersymmetry breaking terms in most of the parameter space, and if it were to be subdominant, the right-handed sleptons become the lightest spartners.

2. Breaking supersymmetry by 24_H

As an example that is enough generic and illustrative, but still simple, we will consider the superpotential up to the fifth order[1] in the adjoint 24_H (Σ) and up to an arbitrary constant W_0

$$W - W_0 = +a_0 \frac{v^3}{M_{Pl}^2} \operatorname{Tr} \Sigma^2 + a_1 \frac{v^2}{M_{Pl}^2} \operatorname{Tr} \Sigma^3 + a_2^{(1)} \frac{v}{M_{Pl}^2} \operatorname{Tr} \Sigma^4 + a_2^{(2)} \frac{v}{M_{Pl}^2} \left(\operatorname{Tr} \Sigma^2\right)^2$$
$$+ a_3^{(1)} \frac{1}{M_{Pl}^2} \operatorname{Tr} \Sigma^5 + a_3^{(2)} \frac{1}{M_{Pl}^2} \operatorname{Tr} \Sigma^3 \operatorname{Tr} \Sigma^2, \tag{2}$$

where v ($= M_{\text{GUT}}$) is the grand unified scale and M_{Pl} stands for the Planck scale ($\approx 10^{19}$ GeV). The reader should keep in mind that some of the coefficients a_i (except the last two) could be bigger than 1 without being in contradiction with perturbativity.

One expands the Σ multiplet as

$$\Sigma = +\frac{\sigma}{\sqrt{30}} \operatorname{diag}(2, 2, 2, -3, -3) + \frac{\sigma_8}{\sqrt{2}} \operatorname{diag}(1, -1, 0, 0, 0) + \frac{\sigma_3}{\sqrt{2}} \operatorname{diag}(0, 0, 0, 1, -1) \tag{3}$$

with σ's canonically normalized, so that the Kähler potential is just

$$K = \operatorname{Tr} \Sigma^\dagger \Sigma. \tag{4}$$

We take here the canonical Kähler only for simplicity, although it is not realistic. A more general case would only help to achieve supersymmetry breaking. Of course, we will not take seriously any prediction that the minimal Kähler leads to, such as for example flavour conservation in neutral currents at high energies. Nothing in the discussion below depends on this assumption.

It is easy to check that $\langle \sigma_{3,8} \rangle = 0$ is an extremum. We will see soon that the supersymmetric mass of the weak triplet and color octet is larger than the supersymmetry breaking ones, so the solution is at least locally stable (up to possible tunneling). By definition, $\langle \sigma \rangle = v$ and we look for nonvanishing F in

$$\sigma = v + \theta\theta F. \tag{5}$$

Now everything reduces to the minimization of the supergravity potential with the superpotential and (canonical) Kähler potential

$$W - W_0 = \sum_{n=0}^{3} b_n \frac{v^{3-n}}{M_{Pl}^2} \sigma^{n+2}, \tag{6}$$

$$K = \sigma^* \sigma. \tag{7}$$

The coefficients b's are expressed as

$$b_0 = a_0, \tag{8}$$

$$b_1 = -\frac{1}{\sqrt{30}} a_1, \tag{9}$$

$$b_2 = \frac{7}{30} a_2^{(1)} + a_2^{(2)}, \tag{10}$$

$$b_3 = -\frac{13}{30\sqrt{30}} a_3^{(1)} - \frac{1}{\sqrt{30}} a_3^{(2)}. \tag{11}$$

From (6) it is easy to calculate the various derivatives; the system obtained is linear in the couplings b's:

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 3 & 4 & 5 \\ 2 & 6 & 12 & 20 \\ 0 & 6 & 24 & 60 \end{pmatrix} \begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{pmatrix} = M_{Pl}^2 \begin{pmatrix} (W - W_0)/v^5 \\ W'/v^4 \\ W''/v^3 \\ W'''/v^2 \end{pmatrix}. \tag{12}$$

This system is easily inverted to get

$$\begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{pmatrix} = M_{Pl}^2 \begin{pmatrix} 10 & -6 & 3/2 & -1/6 \\ -20 & 14 & -4 & 1/2 \\ 15 & -11 & 7/2 & -1/2 \\ -4 & 3 & -1 & 1/6 \end{pmatrix} \begin{pmatrix} (W - W_0)/v^5 \\ W'/v^4 \\ W''/v^3 \\ W'''/v^2 \end{pmatrix}. \tag{13}$$

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[1] We will comment on the more restrictive cubic and quartic superpotential at the end of this section.

The last step is to find out for which W, W', W'' and W''' is the vev v a stable minimum of the supergravity potential

$$V = \exp\left(K/M_*^2\right)\left[\left(\frac{\partial W}{\partial \phi^i} + \frac{\partial K}{\partial \phi^i}\frac{W}{M_*^2}\right)(K^{-1})^i_j\left(\frac{\partial W^*}{\partial \phi_j^*} + \frac{\partial K}{\partial \phi_j^*}\frac{W^*}{M_*^2}\right) - 3\frac{|W|^2}{M_*^2}\right], \tag{14}$$

where $M_* = M_{Pl}/\sqrt{8\pi} \approx 2 \times 10^{18}$ GeV is the so-called reduced Planck mass and $(K^{-1})^i_j$ is the inverse matrix of $\partial^2 K/\partial \phi^i \partial \phi_j^*$. The fine-tuning of the cosmological constant requires $V = 0$ at the minimum $\sigma = v$. Together with the constraint of the minimum $(dV/d\sigma = 0$ and all scalar masses square positive) this after some calculation leads to

$$\frac{W'}{W} = \frac{\sqrt{3}\eta^*}{M_*} - \frac{v^*}{M_*^2}, \tag{15}$$

$$\frac{W''}{W} = \left(\frac{\sqrt{3}\eta^*}{M_*} - \frac{v^*}{M_*^2}\right)^2 - \left(\frac{\eta^*}{M_*}\right)^2, \tag{16}$$

$$\frac{W'''}{W} = 3\left(\frac{\sqrt{3}\eta^*}{M_*} - \frac{v^*}{M_*^2}\right)\left(\left(\frac{\sqrt{3}\eta^*}{M_*} - \frac{v^*}{M_*^2}\right)^2 - \left(\frac{\eta^*}{M_*}\right)^2\right) - 2\left(\frac{\sqrt{3}\eta^*}{M_*} - \frac{v^*}{M_*^2}\right)^3 + \frac{2\eta^{*3}}{\sqrt{3}M_*^3}(1 + \xi), \tag{17}$$

where $|\eta|^2 = 1$ and $|\xi| \leqslant 1$ ($|\xi| = 1$ means one massless scalar).

The above equations tell us that effectively the first three derivatives of the superpotential must be highly fine tuned, i.e., the field σ is very close to a flat direction, as physically expected if gravity is to play a substantial role.

Finally, by definition the gravitino mass is

$$m_{3/2} = \frac{|W|}{M_*^2}e^{K/M_*^2}. \tag{18}$$

In principle $m_{3/2}$ can be fine-tuned to be as small as one wants, but in low energy supersymmetry it is expected to lie around TeV.

The parameter W_0 is only constrained to satisfy the upper bound v^5/M_{Pl}^2, which comes from the requirement $b_3 \leqslant 1$. Its value is locally (close to our minimum) completely irrelevant, but plays an important role for the global shape of the potential. For example, besides our local minimum with vanishing energy there is at least one more minimum to worry about, i.e., $\langle\sigma\rangle = 0$, whose energy is given by

$$E\left(\langle\sigma\rangle = 0\right) = -3\frac{|W_0|^2}{M_*^2}. \tag{19}$$

The existence of this supersymmetry preserving minimum is expected on general grounds, due to the absence of an R-symmetry [20].

Of course there could be other local minima, depending on the value of W_0. For example, for $W_0 = 0$ the closest minimum to ours (and lower in energy) lies at approximately $1.4M_{\text{GUT}}$. One is thus faced with an important question of metastability of our local minimum. The tunneling to the ground state turns out to be very slow [21] as expected from the large distance between the minima.

The above discussion is both simple and generic enough to illustrate all the essential points of this program. Still, one may ask, why not a simpler superpotential. The cubic case can be disposed of immediately, since it has only two couplings (a_0 and a_1 in (2)), insufficient to satisfy (15)–(17).

The quartic case on the other hand can suffice, since it adds two new couplings ($a_2^{(1)}$ and $a_2^{(2)}$ as seen from (2)). It leads to interesting predictions

$$W_0 = \mathcal{O}\left(m_{3/2}M_*^2\right), \quad m_3 = 4m_8. \tag{20}$$

These predictions are to be taken with a grain of salt, since they demand ignoring a number of higher-dimensional operators. Still, it is interesting that the latter prediction automatically suppresses sufficiently the $d = 5$ proton decay, as discussed in Section 4. The tunneling is still under control, and formally, although not strongly motivated, this case cannot be ruled out.

3. The particle spectrum and DT splitting

After the SU(5) symmetry breaking, the surviving elements of 24_H, the SU(3) octet σ_8 and the SU(2) triplet σ_3 have the masses

$$m_3 = c_3\frac{M_{\text{GUT}}^3}{M_{Pl}^2}, \qquad m_8 = c_8\frac{M_{\text{GUT}}^3}{M_{Pl}^2}, \tag{21}$$

where

$$c_3 \approx \frac{8}{3}a_2^{(1)} - \frac{28}{3\sqrt{30}}a_3^{(1)}, \qquad c_8 \approx \frac{2}{3}a_2^{(1)} + \frac{1}{\sqrt{30}}a_3^{(1)} \tag{22}$$

after using the symmetry breaking constraints from the previous section. Notice that c_3 and c_8, unlike the coefficients b in (8)–(11), are not constrained by the minimization constraints (13). Thus the colour octet and weak triplet do not need to be taken light as in [10]. The price is the fine-tuning, but the reward is not to have to add ad-hoc new fields, needed in [10] to save gauge coupling unification.

The situation with 5_H and $\bar{5}_H$ supermultiplets require additional fine-tuning as everybody knows. From the additional terms in the superpotential

$$W_5 = m_5 \bar{5}_H 5_H + \sqrt{30}\beta_1 \bar{5}_H \Sigma 5_H + 30\beta_2 \bar{5}_H \frac{\Sigma^2}{M_{Pl}} 5_H \tag{23}$$

one finds for the doublet (D) and triplet (T) mass terms

$$\mu_D = m_5 - 3\beta_1 v + 9\beta_2 \frac{v^2}{M_{Pl}}, \tag{24}$$

$$\mu_T = m_5 + 2\beta_1 v + 4\beta_2 \frac{v^2}{M_{Pl}}. \tag{25}$$

To get the light Higgs mass $\mu_D = \mathcal{O}(m_W)$ one needs to fine-tune the combination of parameters on the right-hand side in (24). This gives for the triplet mass $\mu_T = 5\beta_1 v - 5\beta_2 v^2/M_{Pl}$. Since T and $\bar{T}$ mediate the $d = 5$ proton decay, these masses must be as large as possible and thus β_i cannot be small, i.e., at least $\beta_2 \approx 0.1$–1. This has a dramatic impact on supersymmetry breaking, as seen immediately from the last two terms in (23). Namely, this implies a contribution of order $(-3\beta_1 + 18\beta_2 v/M_{Pl})F$ to the off-diagonal Higgs mass term which, without fine-tuning, requires $F \lesssim$ TeV in order not to destabilize the Higgs masses. Such a small F can work only if the Higgs doublets D and $\bar{D}$ are the dominant mediators of supersymmetry breaking, but as discussed above, it implies a tachyonic stop. The escape from this impasse is to apply a further constraint on the model parameters: $|-3\beta_1 + 18\beta_2 v/M_{Pl}| \ll 1$, so that the dangerous off-diagonal contribution to the Higgs doublet mass is at most $\mathcal{O}(m_W^2)$. To summarize, although at the prize of two fine-tunings, the minimal model survives all the phenomenological constraints.

For those who do not like so many fine-tunings, there is a different option for the doublet–triplet splitting and the hiding of the singlet σ from the light Higgs doublets. This can be accomplished in two different ways. The simplest realization is to add a pair of 50_H and $\overline{50}_H$ multiplets, which contain colour triplets, but no weak doublets. Through the couplings [10]

$$W_{50} = \frac{1}{M_{Pl}}24_H^2\left(\bar{5}_H 50_H + \overline{50}_H 5_H\right) + (M_{50} + \Sigma + \cdots)\overline{50}_H 50_H \tag{26}$$

(the dots stand for possible higher-dimensional operators) one makes the triplets heavy and the doublets remain massless (until supersymmetry gets broken). Clearly, the maximum mass the triplets can have is $\mathcal{O}(M_{GUT}^2/M_{Pl})$.

An alternative is to use 75_H [22] instead of 24_H, since 75_H behave as 24_H^2 in the above example. It has direct renormalizable couplings and in this case $M_T \approx M_{GUT}$.

4. RGE for gauge couplings: Unification and proton decay

We start here with a careful discussion of the minimal supersymmetric SU(5) unification constraints independent of our program. A consistent renormalization group analysis assumes that the three masses m_T, m_3 and m_8 are free. At the renormalizable tree level, $m_3 = m_8$, but minimal supersymmetric SU(5) makes no sense without higher-dimensional terms, since it predicts wrongly fermions masses. Once the higher-dimensional terms are allowed, as in our example, m_3 and m_8 become arbitrary.

At the one loop level, the RGEs for the gauge couplings are (we ignore here for simplicity higher-dimensional terms which split the gauge couplings at the grand unified scale through $\langle \Sigma \rangle \neq 0$; see Section 5)

$$2\pi\left(\alpha_1^{-1}(M_Z) - \alpha_U^{-1}\right) = -\frac{5}{2}\ln\frac{\Lambda_{SUSY}}{M_Z} + \frac{33}{5}\ln\frac{M_{GUT}}{M_Z} + \frac{2}{5}\ln\frac{M_{GUT}}{m_T}, \tag{27}$$

$$2\pi\left(\alpha_2^{-1}(M_Z) - \alpha_U^{-1}\right) = -\frac{25}{6}\ln\frac{\Lambda_{SUSY}}{M_Z} + \ln\frac{M_{GUT}}{M_Z} + 2\ln\frac{M_{GUT}}{m_3}, \tag{28}$$

$$2\pi\left(\alpha_3^{-1}(M_Z) - \alpha_U^{-1}\right) = -4\ln\frac{\Lambda_{SUSY}}{M_Z} - 3\ln\frac{m_8}{M_Z} + \ln\frac{M_{GUT}}{m_T}. \tag{29}$$

We have neglected here the impact of terms $\text{Tr}(\Sigma F_{\mu\nu}F^{\mu\nu})/M_{Pl}$. We demand self-consistently (see below) perturbativity, i.e., $v/M_{Pl} \lesssim 0.1$, which provide about at most a 10% change on gauge coupling unification. We neglect this in our analysis and thus restrain ourselves from the two loop analysis, which would not be appropriate here.

From (27)–(29) we obtain

$$2\pi\left(3\alpha_2^{-1} - 2\alpha_3^{-1} - \alpha_1^{-1}\right) = -2\ln\frac{\Lambda_{\text{SUSY}}}{M_Z} + \frac{12}{5}\ln\frac{m_T}{M_Z} + 6\ln\frac{m_8}{m_3}, \tag{30}$$

$$2\pi\left(5\alpha_1^{-1} - 3\alpha_2^{-1} - 2\alpha_3^{-1}\right) = 8\ln\frac{\Lambda_{\text{SUSY}}}{M_Z} + 36\ln\frac{(\sqrt{m_3 m_8}\,M_{\text{GUT}}^2)^{1/3}}{M_Z}. \tag{31}$$

We stick here to low energy supersymmetry, i.e., we take $\Lambda_{\text{SUSY}} \approx M_Z$, as required by one-loop unification. This gives

$$m_T = m_T^0\left(\frac{m_3}{m_8}\right)^{5/2}, \tag{32}$$

$$M_{\text{GUT}} = M_{\text{GUT}}^0\left(\frac{M_{\text{GUT}}^0}{\sqrt{m_3 m_8}}\right)^{1/2}. \tag{33}$$

In the above equations the superscript ^{0} denotes the values in the case $m_3 = m_8 = M_{\text{GUT}}$. Taking $\alpha_1^{-1} = 59$, $\alpha_2^{-1} = 29.57$ and $\alpha_3^{-1} = 8.55$

$$m_{\text{GUT}}^0 \approx 10^{16}\text{ GeV}. \tag{34}$$

If one ignores higher-dimensional terms, one predicts $m_3 = m_8$ and thus $m_T = m_T^0$. It is known that m_T^0 is not large enough to bring $d = 5$ proton decay in accord with experiment (unless one goes through painful gymnastics or arbitrary cancellations [14]). At the same time, M_{GUT} is obviously not predicted and can be as large as 10^{18} GeV, as long as $m_3 \approx m_8 \approx 10^{13}$ GeV (we stick to a perturbative theory and demand $M_{\text{GUT}} \leqslant M_{Pl}/10$). This clearly requires a large amount of fine-tuning, since the mass of the SM singlet σ must be about ten orders of magnitude smaller (recall that $m_\sigma = \mathcal{O}(m_{3/2})$). Notice again that in the minimal model, to which we stick, there is no other option for the masses $m_{3,8}$. This is the same issue as in the doublet–triplet splitting: in minimal theories fine-tuning is unavoidable.

The intermediate values of $m_{3,8}$ on the other hand simply imply that the Yukawa $\text{Tr}(\Sigma^3)$ coupling is small. On the other hand, higher-dimensional terms in the superpotential are the simplest possibility of curing wrong fermion mass relations in the theory; once they are included m_3 and m_8 become arbitrary, as in our case. This means that m_T can be arbitrary large, and in what follows we demand $m_T \geqslant 10^{17}$ GeV in order to stabilize the proton.

The above message cannot be overstressed. We have argued that the theory does not predict either the GUT scale or the mass of the colour triplets, and we will need the experiment to learn their values. Instead of endlessly worrying about the nonexistent predictions of this prototype theory of supersymmetric grand unification, a correct procedure requires to take into account the whole parameter space without ad-hoc unphysical prejudices. The strong indication of large m_T and thus M_{GUT} requires only intermediate states σ_3 and σ_8, completely consistent with theory and experiment. Actually, simply demanding that supersymmetry be broken in the minimal scheme without any hidden sector implies automatically these intermediate states. The bottom line of all of this is that the dimension 6 proton decay operators can be completely ignored: $\tau_p(d=6) \approx 10^{40}$ yrs for $M_{\text{GUT}} \approx 10^{17}$ GeV.

In the minimal theory we considered, the triplet has a mass of order M_{GUT}^2/M_{Pl}. Due to the requirements of safe $d = 5$ proton decay ($m_T \geqslant 10^{17}$ GeV) and $M_{\text{GUT}} \ll M_{Pl}$, the only possibility is to have $M_{\text{GUT}} \approx 10^{18}$ GeV. This determines the masses $m_3 \approx 2m_8 \approx 10^{13}$ GeV as seen from (32) and (33).

We comment here on the alternatives that we mentioned in the previous section. If one wants to employ the missing partner mechanism, and give up the minimal model, there are more options. The simplest situation here is to consider 50_H and $\overline{50}_H$ as complete multiplets at M_{GUT}^2/M_{Pl}. It can be checked that this also guarantees no Landau pole below M_{Pl}. In the case of 75_H, $m_T \approx M_{\text{GUT}}$. As there are more states which contribute to the increase of the gauge couplings, one is forced to have again $M_{\text{GUT}} \approx 10^{18}$ GeV in order to avoid a Landau pole below M_{Pl}.

The common characteristic of all the above cases is a large $M_{\text{GUT}} \approx 10^{18}$ GeV, which completely suppresses $d = 6$ proton decay, and makes it out of reach of even a future generation experiment. Dimension 5 proton decay is clearly in accord with experiment and in the last case above it may not be easily visible.

One may not be happy with such a high value of M_{GUT}, maybe too close to M_{Pl}. A possible way-out is to add another 24_H and stick to the fine-tuned doublet–triplet splitting. Clearly there are no other constraints here except for $M_{\text{GUT}} \geqslant m_T \geqslant 10^{17}$ GeV.

5. Transmitting supersymmetry breaking

As discussed in the introduction, there are a number of possible mediators of supersymmetry breaking: (1) gravity; (2) X and Y heavy vector supermultiplets; (3) heavy Higgs supermultiplets σ_3 and σ_8; (4) heavy colour triplets T, $\overline{T}$ from 5_H, $\overline{5}_H$ (and possibly 50_H and $\overline{50}_H$); (5) light Higgs doublets D and $\overline{D}$.

All the supersymmetry breaking terms are necessarily proportional to F, which is the auxiliary field of the singlet supermultiplet $\sigma = v + \theta\theta F$. Due to the requirement of zero cosmological constant, it is connected to the gravitino mass

$$F \approx m_{3/2} M_*. \tag{35}$$

We now carefully study each of these contributions. The end result will turn out to be the domination of gravity. For this reason we only present the estimates of the single contributions, i.e., the order of magnitude values for the soft terms.

5.1. Gravity

Gravity is an automatic messenger in any theory, and its contribution to the sfermion masses and A-terms is

$$m_{\tilde{f}} \approx A \approx m_{3/2}. \tag{36}$$

The situation with gaugino masses depends on the following higher-dimensional operator

$$\int d^2\theta \, \frac{f}{M_{Pl}} \, \mathrm{Tr}(\Sigma W^\alpha W_\alpha), \tag{37}$$

where W^α is the supersymmetric generalization of the Yang–Mills field strength. One gets generically for the gaugino masses

$$m_\lambda \approx f m_{3/2}. \tag{38}$$

If f is of order 1, the unification constraints must of course be reanalyzed. For smaller f one expects lighter gauginos, a fact that helps further suppressing the $d = 5$ proton decay. This encouraged us to focus on the case $f \ll 1$ in the above renormalization group study.

5.2. Heavy gauge bosons X and Y

In this case one gets for the soft terms the intuitively expected result [9]

$$m_{\tilde{f}} \approx m_\lambda \approx A \approx \frac{\alpha}{\pi} \frac{F}{M_{\mathrm{GUT}}}. \tag{39}$$

Since in this theory M_{GUT} is expected to be near M_*, barring accidental cancellations involving complicated Kähler potentials, this contribution is negligible compared to gravity mediation.

5.3. Physical states in 24_H: σ_3 and σ_8

The contribution to the masses is given at two-loops, and is of the order (a similar contribution is also for the A terms at one-loop)

$$m_{\tilde{f}} \approx A \approx \frac{\alpha}{\pi} \frac{F_i}{m_i}, \tag{40}$$

where $i = 3$ and/or 8 and

$$F_i = F \left.\frac{\partial m_i}{\partial \sigma}\right|_{\sigma=v}. \tag{41}$$

Typically $F_i/m_i = \mathcal{O}(F/M_{\mathrm{GUT}})$, which would make this contribution subdominant with respect to gravity, precisely because of the loop suppression. To overcome it one needs to fine-tune m_i without suppressing at the same time F_i. In the model discussed here this reduces to fine-tune c_3 and c_8 in (21)–(22) without the coefficients $a_2^{(1)}$, $a_3^{(1)}$ being much less than 1. Since m_3 and m_8 must be of the same order of magnitude in order to prevent m_T being much bigger than M_{GUT} (see Eq. (32)), this is clearly impossible. Here σ_3 and σ_8 contribute no more than X and Y.

The above is not a rigorous result, though. After all, one can include even higher-dimensional terms in the superpotential in order to have the necessary freedom to fine-tune m_3 and m_8 to be small. Since at the same time one should keep F_3 and F_8 as large as possible, the ideal case is to stop at Σ^6/M_{Pl}^3. At first glance one could enhance arbitrarily the mediation of σ_3 and σ_8, but recall that

$$m_3 \approx m_8 > 10^{12-13} \text{ GeV} \tag{42}$$

in order to keep M_{GUT} below M_{Pl}. It is a simple exercise to check that, although this contribution can be made bigger than the one of X and Y, it is at most of order $m_{3/2}$. In short, even after a fine-tuning, gravity still tends to dominate.

5.4. Heavy colour triplets T and $\overline{T}$

Here the situation is very simple. Since these states must be rather heavy in order to stabilize the proton, their contribution, as in the case of X and Y is much smaller than the gravitational one. Similarly the possible contribution of 50_H and $\overline{50}_H$ states is also negligible since they lie at the GUT scale for the sake of unification and perturbativity up to M_{Pl}.

5.5. Light Higgs

As we discussed repeatedly, the light Higgs is never allowed to dominate, since it makes the stop tachyonic. Actually, in the cases when one splits the doublet and the triplet using the missing partner mechanism, light Higgses are completely decoupled from the source of supersymmetry breaking. In the opposite case, when one fine-tunes this coupling (the way one does for the μ term), the light Higgs contribution cannot be predicted, since it depends on the amount of fine-tuning. All one can say here is that the light Higgs cannot dominate.

A few words are needed regarding the issue of neutrino mass. As we said in the introduction, one possibility are the bilinear R-parity violating terms, which do not affect anything of the above. The same is true of the type I seesaw. The situation with the type II seesaw requires some discussion. The 15_H and $\overline{15}_H$ fields have been argued recently to be interesting messengers of supersymmetry breaking [8]. These fields are taken as complete multiplets at some intermediate scale in order not to affect the unification constraints. They couple to the adjoint and thus clearly transmit the supersymmetry breaking. In principle, with some fine-tuning they could be made to dominate the mediation of supersymmetry breaking. We prefer not to incorporate this case here seriously, for otherwise it requires an in-depth study of its impact on unification constraints and perturbativity. In any case the possibility of these fields dominating supersymmetry mediation has been carefully studied in [8].

As claimed, it is clear that in general no contribution except for gravity can be the dominant one. In any case, it is only σ_3 and σ_8 that can compete with gravity, which does not complicate things much, since gravity mediation makes no clear statements regarding the flavour structure of soft terms. It is worth emphasizing that the so-called mSUGRA with universal soft terms does not emerge in supergravity since it is based on a completely unphysical assumption of canonical Kähler. In our study the assumption of a canonical Kähler was used only for simplicity and transparency and no prediction is based on it. We wanted to emphasize that generically supersymmetry can get broken by the 24_H once higher-dimensional terms are allowed; non-canonical Kähler makes the task only easier.

In the extreme case of σ_3 and σ_8 being maximally fine-tuned and giving a somewhat bigger contribution than gravity, the singlet sleptons would be somewhat lighter (σ_3 and σ_8 carry no hypercharge). This is based on incomplete and rough estimates and it would have to be quantified in order to be taken very seriously. This task is beyond the scope of this Letter, although it could be a useful exercise for the future.

6. Conclusions

The scenario of low energy supersymmetry is plagued by our complete ignorance of the source and the nature of supersymmetry breaking and its transmission to the spartners of the SM model particles. Perturbative approaches typically use gauge singlet fields to break supersymmetry which renders them prediction free. As we discussed in the introduction, there were important attempts, though, to use the GUT Higgs (the adjoint of SU(5)) to do the job, but with the price of introducing ad-hoc new fields [10–12].

On the other hand, the mediation of the breaking, when not argued to be dominated by gravity, is typically attributed to new vector-like states, introduced ad-hoc for this purpose. On top of that, one often ignores their possible Yukawa couplings and speaks of gauge mediation. Notable exceptions are the attempts to use the GUT gauge multiplets [9] and the SM model triplet responsible for the type II seesaw [8].

In this Letter we have studied supersymmetry breaking and its transmission to the light states in a simple grand unified theory such as SU(5) without any ad-hoc singlets. The adjoint Higgs 24_H breaks the GUT symmetry and supersymmetry at the same time. While the SM gauge singlet direction must be quite flat, the color octet and the weak triplet end up at the intermediate scale; their impact on the running is to increase in general the GUT scale and possibly the masses of the color triplets states which mediate $d = 5$ proton decay. We wish to emphasize again that this requires a large amount of fine-tuning, since the singlet σ is at the TeV scale, while $\sigma_{3,8}$ are at intermediate scale of about 10^{13} GeV. The alternative would be adding more fields just to fix the unification constraints [10]. The bottom line: (1) the minimal theory 24_H, 5_H and $\bar{5}_H$ suffices; (2) $d = 6$ proton decay gets out of reach; (3) $d = 5$ is slowed enough to be in accord with the experimental limits.

The gauge structure of the theory (i.e., the absence of gauge singlets) makes it quite predictive even when it comes to the transmission of supersymmetry breaking to the MSSM particles. It turns out that gravity dominates in most of the parameter space, while, at the price of fine-tuning, the octet and the triplet of 24_H could compete with gravity. In the extreme and improbable situation of their domination, the signal would be the lightness of singlet sleptons. In short, this simple theory is an example of a predictive program of using grand unification to be responsible for breaking supersymmetry and for the subsequent mediation without any

new ad-hoc singlets whose existence makes the program both trivially achievable and prediction free. The generic prediction in this program is the existence of intermediate scale particles that push up the unification scale and keep the proton safe.

Acknowledgements

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