



Problems in resumming interjet energy flows with k_t clustering

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Received 16 August 2005; accepted 24 August 2005

Available online 19 September 2005

Editor: N. Glover

Abstract

We consider the energy flow into gaps between hard jets. It was previously believed that the accuracy of resummed predictions for such observables can be improved by employing the k_t clustering procedure to define the gap energy in terms of a sum of energies of soft jets (rather than individual hadrons) in the gap. This significantly reduces the sensitivity to correlated soft large-angle radiation (non-global leading logs), numerically calculable only in the large N_c limit. While this is the case, as we demonstrate here, the use of k_t clustering spoils the straightforward single-gluon Sudakov exponentiation that multiplies the non-global resummation. We carry out an $\mathcal{O}(\alpha_s^2)$ calculation of the leading single-logarithmic terms and identify the piece that is omitted by straightforward exponentiation. We compare our results with the full $\mathcal{O}(\alpha_s^2)$ result from the program EVENT2 to confirm our conclusions. For $e^+e^- \rightarrow 2$ jets and DIS (1 + 1) jets one can numerically resum these additional contributions as we show, but for dijet photoproduction and hadron–hadron processes further studies are needed.

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1. Introduction

Energy flow into gaps between hard jets is a valuable source of information on many aspects of QCD. Since this radiation is typically soft, a perturbative calculation of the corresponding distribution contains large logarithms that need resummation. Comparisons of resummed perturbative estimates with data then en-

able us to test and further our knowledge of soft QCD dynamics.

Moreover the hadronisation corrections are large, making these spectra a useful testing ground for theoretical ideas about power corrections within say a dispersive model for the QCD coupling [1]. Additionally at hadron colliders the activity away from jets has traditionally been used to study the soft underlying event and to refine models thereof [2], which will be an important component of physics at the LHC.

In the present Letter we analyse the first aspect alone, that of perturbative resummation. This re-

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summation is challenging since the leading single-logarithms (there are no collinear enhancements due to the fact that we are away from the hard jets) are generated both by direct angular-ordered emission into the gap by the hard parton (jet) system as well as multiple energy-ordered correlated emission by a complex ensemble of soft emissions outside the gap [3,4], in addition to the hard jets. This latter piece, which cannot yet be computed analytically and more worryingly beyond a large N_c approximation, is known as the non-global component of the result. The non-global term causes a much stronger suppression of the soft energy flows than that obtained by vetoing direct emission off hard partons, into the gap, the Sudakov or bremsstrahlung component of the answer [4]. The final result, for the cross-section with gap energy less than Q_Ω can be expressed as [3–5]

$$\Sigma(Q, Q_\Omega) = \exp[-R(Q/Q_\Omega)]S(Q/Q_\Omega). \quad (1)$$

In the above $\exp[-R]$ is the Sudakov term obtained by exponentiating the single gluon contribution and accounting for gluon branching to reconstruct the scale of the running coupling, while S is the non-global part of the answer obtained by running a large N_c Monte Carlo program that encodes soft evolution of a system of dipoles to single-log (SL) accuracy, for emission into the gap Ω . Equivalently for such away-from-jet energy flows one has to solve numerically a non-linear evolution equation [5] obtained in the large N_c limit.

Given that only large N_c approximations of the non-global component are calculable at present in conjunction with the fact that it dominates the full result at smaller Q_Ω values, it is important to reduce the sensitivity of the measurement to this effect. One method of doing so is to study event-shape–energy-flow correlations [6,7] where measuring an event shape V outside the gap at the same time as the energy flow Q_Ω inside it leads to non-global logarithms in the ratio V/Q_Ω . Thus choosing $V \sim Q_\Omega$ the magnitude of the non-global effects is reduced. However this procedure is more complex to implement in the case of several hard partons (e.g., jet production in hadron–hadron collisions) and is quite restrictive experimentally, amounting essentially to studying a different observable.

Another version of the measurement was suggested in Refs. [8,9], to reduce the impact of non-global logarithms. There it was shown that defining the energy

Q_Ω as the sum of energies of soft (mini) jets inside the gap region significantly reduced the non-global component of the result. Defining the jets via a k_t clustering procedure [10], which is also common practice experimentally [11,12], had the effect of pulling soft emissions out of the gap region by clustering with harder emissions outside. As was shown in the simple case of $e^+e^- \rightarrow 2$ jets [8], it is still possible for emissions near the centre of the gap to escape clustering. However since these emissions are well separated in rapidity from their nearest neighbours outside the gap (to escape clustering) the magnitude of non-global effects is reduced. This is because the bulk of the non-global piece arises in the region when the emitted soft gluon does not have too large an opening angle relative to those involved in the emitting ensemble [4]. Forcing a relatively large opening-angle/rapidity separation between the softest gluon and the harder emitters, as is required to escape clustering, does reduce the size of the non-global effects [8].

We would like to point out, however, that using k_t clustering impacts the general form Eq. (1) and does not leave the primary emission Sudakov piece $\exp[-R]$ unchanged as has been assumed till now [8,9]. In fact we find the exact calculation of R becomes non-trivial at higher orders since it depends at n th order on the n gluon geometry and the use of the clustering algorithm. The departure of R from its naive one-gluon form starts with two gluons and the resulting piece does not have the properties of non-global logs, neither in the colour structure nor in the dynamical properties. This conclusion is unfortunate especially in the case of dijet photoproduction [9] or hadron–hadron studies where the missing piece we compute at leading order here, will have a highly non-trivial colour structure since it reflects the colour flow of the primary emission piece, computed, e.g., in [9, 13]. This would impact accurate theoretical studies of such observables even though the non-global part is reduced.

The current Letter is organised as follows. In the subsequent section we identify the problem with naive one-gluon exponentiation to obtain the supposed Sudakov part of the answer, with k_t clustering. We compute the piece that will be missed by one-gluon exponentiation, at leading order $\mathcal{O}(\alpha_s^2)$. We then provide the full answer up-to order α_s^2 , for the leading logarithms, and show via comparisons with EVENT2 [14]

that the extra piece we compute is needed to agree with fixed order estimates while the expansion to order α_s^2 of the Sudakov term, as currently computed in the literature, is insufficient. We conclude by pointing out that while analytical control over the Sudakov term is lost, due to clustering, it is possible in the simpler cases $e^+e^- \rightarrow$ two jets and DIS (1 + 1) jets, to numerically compute the additional piece at all orders, with existing programs [8], as we show. For higher jet topologies such as those studied in Ref. [9] it may only be possible to numerically compute this term in the large N_c limit, thereby reducing the accuracy of the theoretical results compared to current expectations.

2. Independent soft gluon emission

We specialise to the process $e^+e^- \rightarrow 2$ jets purely for reasons of simplicity. We perform our calculation for emission into a rapidity slice of width $\Delta\eta$, centred at zero rapidity with respect to the jet (thrust) axis, although similar considerations apply to any geometry one may choose for the interjet region Ω . We first consider the Sudakov exponent generated by assuming exponentiation of single-gluon emission into the gap Ω (cf. Eq. (3.9) of Ref. [8]):

$$\Sigma_{\Omega, P}(t) = \frac{1}{\sigma} \int_0^{Q\Omega} \frac{d\sigma}{d\omega} d\omega = e^{-4C_F t \Delta\eta}, \quad (2)$$

with $t = \frac{1}{2\pi} \int_{Q\Omega}^{Q/2} \frac{dk_t}{k_t} \alpha_s(k_t) = \frac{1}{4\pi\beta_0} \ln \frac{1}{1-2\lambda}$, where we used the one-loop running coupling to obtain up-to SL accuracy and defined $\lambda = \beta_0 \alpha_s(Q) \ln \frac{Q}{2Q\Omega}$, with $\beta_0 = (11C_A - 2n_f)/(12\pi)$.

We now argue that the above form which exponentiates the single gluon (leading-order) term is not correct starting from two gluon level.¹ Consider two soft emissions k_1 and k_2 that are ordered in energy $\omega_1 \gg \omega_2$, with ω_1 outside the gap and ω_2 inside. The situation is reminiscent of the correlated or non-global configuration at leading order. However that part pertains to the $C_F C_A$ correlated gluon emission piece of the matrix element that is free from collinear singular-

ities. In the present case we shall examine instead the *independent emission* C_F^2 part of the two gluon matrix element along with the corresponding virtual corrections. We have for the independent emission of two real gluons by a dipole ab [15]:

$$\begin{aligned} M^2(k_1, k_2) &= C_F^2 W_{ab}(k_1) W_{ab}(k_2) \\ &= 4C_F^2 \frac{(ab)}{(ak_1)(bk_1)} \frac{(ab)}{(ak_2)(bk_2)}, \end{aligned} \quad (3)$$

where $W_{ab}(k_1)$ represents the emission of k_1 off the hard dipole ab and similarly for k_2 .

Now we examine the region where the two real soft gluons k_1 and k_2 are clustered by the jet algorithm. This happens when

$$(\eta_1 - \eta_2)^2 + (\phi_1 - \phi_2)^2 < R^2, \quad (4)$$

where η and ϕ denote as usual rapidity and azimuth of the partons measured with respect to the axis defined by the back-to-back partons a and b and R is the radius parameter, usually set equal to one in experiment [11, 12].

Since k_1 is outside the gap, k_2 is clustered into k_1 and pulled out of the gap. This configuration then does not contribute to the gap energy distribution $d\sigma/d\omega$ (see Eq. (2)). However now let us take the situation where we have k_1 virtual and k_2 as a real emission. Then k_2 is not clustered away by the algorithm and this configuration contributes with weight [15]:

$$\begin{aligned} M^2(k_{1,\text{virtual}}, k_2) &= -C_F^2 W_{ab}(k_1) W_{ab}(k_2), \\ \omega_1 &\gg \omega_2. \end{aligned} \quad (5)$$

We thus have complete cancellation between the purely real and real-virtual terms, Eqs. (3) and (5) in the region where k_2 is not removed by clustering. However in the angular region mapped by Eq. (4) *only* the one-real-one virtual term will contribute, since k_2 is in the gap.

This contribution can then be computed as below. The four-vectors involved are

$$a = \frac{Q}{2}(1, 0, 0, -1), \quad (6)$$

$$b = \frac{Q}{2}(1, 0, 0, 1), \quad (7)$$

$$k_1 = k_{t1}(\cosh \eta_1, 0, 1, \sinh \eta_1), \quad (8)$$

$$k_2 = k_{t2}(\cosh \eta_2, \sin(\phi), \cos(\phi), \sinh \eta_2), \quad (9)$$

¹ By this we mean that even after accounting for non-global logarithms, the exponentiation of the single-gluon result still does not capture completely the remaining leading logarithms.

where we have exploited the freedom to set $\phi_1 = 0$ and Q is the e^+e^- centre-of-mass energy. We have also neglected the recoil of the hard partons a and b , against the soft emissions k_1 and k_2 , which is valid for our aim of extracting the leading logarithms.

Then the leftover real-virtual two-loop contribution reads (we compute the coefficient of $(\alpha_s/2\pi)^2$)

$$\begin{aligned}
C_2^{\text{primary}} &= 16C_F^2 \int_0^1 \frac{dx_2}{x_2} \int_{x_2}^1 \frac{dx_1}{x_1} \Theta\left(x_2 - \frac{2Q_\Omega}{Q}\right) \\
&\times \int_{k_1 \notin \Omega} d\eta_1 \int_{k_2 \in \Omega} d\eta_2 \int_0^{2\pi} \frac{d\phi}{2\pi} \\
&\times \Theta\left(R^2 - (\eta_2 - \eta_1)^2 - \phi^2\right). \quad (10)
\end{aligned}$$

The above equation requires some explanation. We have introduced the dimensionless scaled transverse momenta $x_{1,2} = 2k_{t1,t2}/Q$ and restricted the region such that virtual emission k_1 is integrated outside the gap region while real emission k_2 inside. We have also inserted a step function that ensures that we are integrating over the region of Eq. (4), where the corresponding double real emissions would be clustered and k_2 would be pulled out of the gap. The additional step function involving x_2 , that constrains the gap energy, is the usual one that corresponds to computing the cross-section for events with energy in the gap greater than Q_Ω . This, by unitarity, is trivially related to that for events with gap energy less than Q_Ω . From the latter quantity the distribution is directly obtained by differentiation with respect to Q_Ω . We have denoted this term C_2^{primary} as it is a second order in α_s piece that has the colour structure and matrix element for independent emission from the primary dipole ab . However it is not derived by expanding the standard Sudakov result to order α_s^2 and is a companion to the non-global correction term S_2 of [8], but with different functional properties and colour structure.

Performing the integration over ϕ in Eq. (10) we get

$$C_2^{\text{primary}} = \frac{16}{\pi} C_F^2 L^2 \int_0^R du \min(u, \Delta\eta) \sqrt{R^2 - u^2}. \quad (11)$$

Choosing for instance values of the gap size $\Delta\eta \geq R$ we get

$$C_2^{\text{primary}} = \frac{16}{3\pi} C_F^2 L^2 R^3, \quad (12)$$

with $L = \ln \frac{Q}{Q_\Omega}$. Alternatively choosing a smaller gap $\Delta\eta \leq R$ we get

$$\begin{aligned}
C_2^{\text{primary}} &= \frac{16}{\pi} C_F^2 L^2 \left[\frac{1}{3} (R^3 - (R^2 - (\Delta\eta)^2)^{3/2}) \right. \\
&+ \frac{\Delta\eta}{2} \left(\frac{\pi R^2}{2} - \Delta\eta \sqrt{R^2 - (\Delta\eta)^2} \right. \\
&\left. \left. - R^2 \tan^{-1} \left(\frac{\Delta\eta}{\sqrt{R^2 - (\Delta\eta)^2}} \right) \right) \right]. \quad (13)
\end{aligned}$$

It is clear that although this piece has the same colour structure as that for independent two gluon emission, it cannot arise from expanding the single-gluon generated Sudakov equation (2). The expansion of the naive Sudakov would give a term independent of R and which goes as $(\Delta\eta)^2$ at $\mathcal{O}(\alpha_s^2)$.

In the following section we shall show that the expansion of the Sudakov equation (2) needs to be supplemented with the results equations (12), (13) as appropriate, as well as the correlated non-global $C_F C_A \alpha_s^2 L^2$ piece computed in [8], in order to agree with the full $\alpha_s^2 L^2$ result, for $\Sigma(Q, Q/Q_\Omega)$ generated by the program EVENT2.

3. Full $\mathcal{O}(\alpha_s^2)$ result and comparison to EVENT2

First we expand the Sudakov exponent equation (2) to $\mathcal{O}(\alpha_s^2)$. The result is

$$\begin{aligned}
\Sigma_{\Omega, P}(t) &= 1 - \bar{\alpha}_s L (4C_F \Delta\eta) \\
&+ \bar{\alpha}_s^2 L^2 \left(8(\Delta\eta)^2 C_F^2 - \frac{22}{3} \Delta\eta C_F C_A \right. \\
&\left. + \frac{4C_F n_f \Delta\eta}{3} \right), \quad (14)
\end{aligned}$$

where we denoted $\bar{\alpha}_s = \frac{\alpha_s}{2\pi}$.

An additional $C_F C_A \alpha_s^2 L^2$ term is indeed the non-global term computed in Ref. [8]. We compute this piece numerically for different values of the parameters $\Delta\eta$ and R and add it to the result from Eq. (14) for comparison with the fixed order program EVENT2. For example, with $R = 1$ and $\Delta\eta = 1$ one gets $S_2 = -1.249 C_F C_A$ where S_2 is the first coefficient of the non-global log contribution $S = 1 + \sum_{n=2} S_n t^n$, with t defined as before.

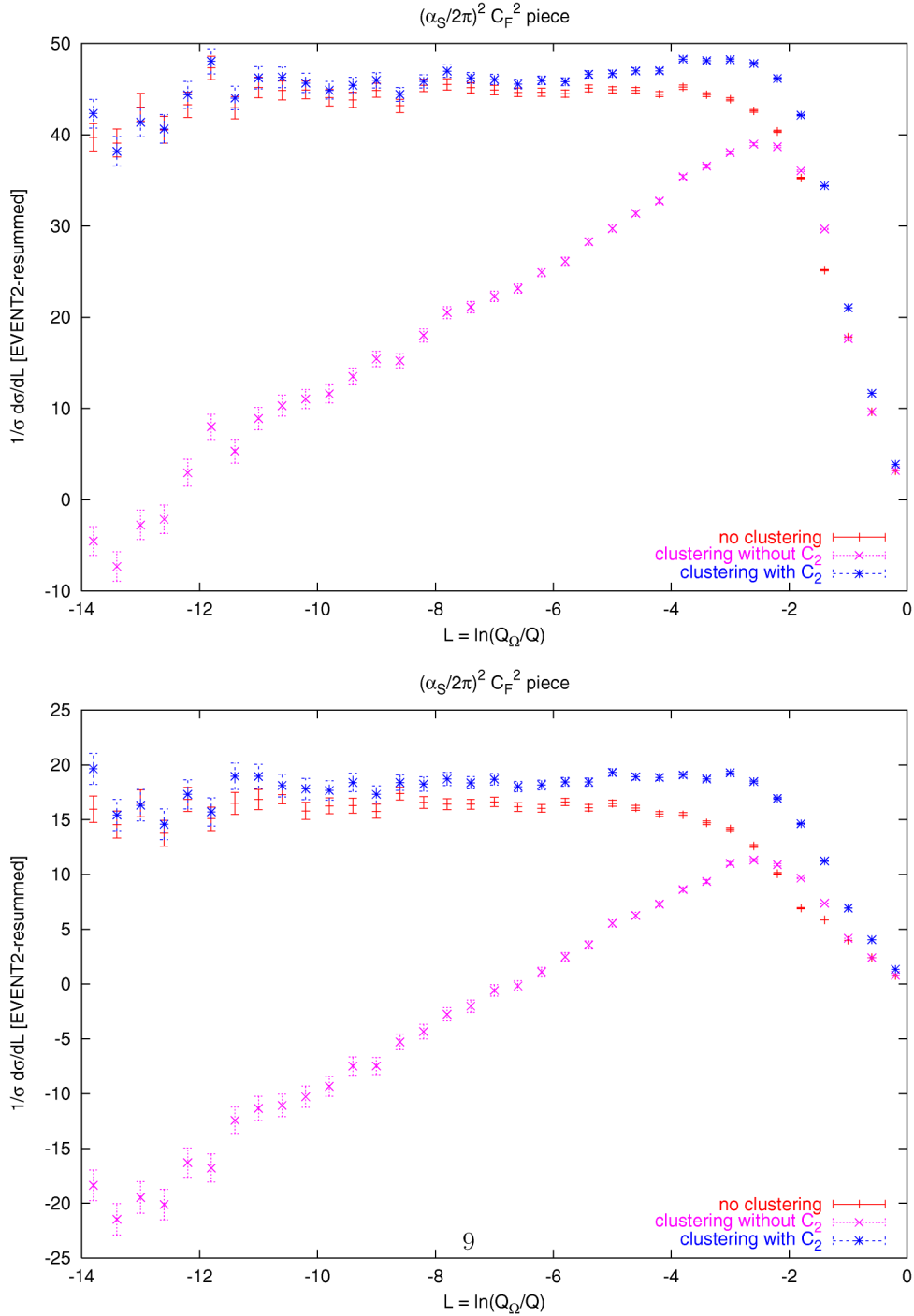


Fig. 1. Comparison of the $C_F^2 \alpha_s^2 L$ term produced by EVENT2 and the analytical calculation (referred to as resummed since it is derived by expanding the naive Sudakov resummation to NLO) with and without C_2^{primary} . The figures are for $R = 1$ and $\Delta\eta = 1.0$ (above) and $\Delta\eta = 0.5$ (below). The agreement for the unclustered case is also shown for comparison.

The comparison to EVENT2 for the distribution $\sigma^{-1} d\sigma/dL$ is shown in Fig. 1 for the $C_F^2 \alpha_s^2 L$ term, with $L = \ln Q_\Omega/Q$ for $R = 1$ and $\Delta\eta = 1$, as well as $\Delta\eta = 0.5$. If all leading (single) logarithms in the integrated quantity $\Sigma(Q, Q_\Omega)$ are correctly accounted for by the resummed result equation (2), we would expect the difference between the EVENT2 results and the expansion to NLO of the resummation, to be a constant at small Q_Ω corresponding to large (negative) L . As we see this is only the case when C_2^{primary} is included by adding it to the expansion of the Sudakov equation (2). We have considered different values of R and $\Delta\eta$ as mentioned, for example, in the caption for Fig. 1. The comparison for other colour channels $C_F C_A$ and $C_F T_{RNf}$ shows agreement with EVENT2 (see Fig. 2) which reflects the fact that only the C_F^2 channel, corresponding to independent emission, is incorrectly described by Eq. (2).

4. All orders and conclusions

We conclude by pointing out that in the simple cases of $e^+e^- \rightarrow$ two jets and DIS $(1+1)$ jets, the additional terms we describe here can be accounted for numerically, at all orders. This is done by using the Monte Carlo program for large N_c dipole evolution developed in [3] and implemented with the k_t clustering in [8]. By demanding multiple emissions from the primary dipole alone and restoring the colour factor for independent emission by changing $C_A/2 \rightarrow C_F$ one obtains the result for primary emissions only, in the full theory.

Using this procedure we see that the primary emission result, with k_t clustering, differs from the Sudakov result generated by single gluon exponentiation, which is valid in the unclustered case. In Fig. 3, we show the results we obtain for primary emission with clustering and the Sudakov (unclustered) result. The discrepancy grows with the single-log evolution variable $t = \frac{1}{4\pi\beta_0} \ln \frac{1}{1-2\lambda}$ where $\lambda = \beta_0 \alpha_s(Q) \ln \frac{Q}{2Q_\Omega}$ and for $t = 0.25$ we note an increase of around 30% on inclusion of the terms we describe, that start with C_2^{primary} computed analytically here.

We wish to clarify that the full result in the large N_c approximation, including the effect we point out here, is readily obtained by the method described in Ref. [8] and in fact computed there. However its separation

into primary and non-global components (and restoring the proper colour factors where possible) needs to be done with care, keeping in mind our findings.

The procedure to generate the most accurate theoretical results for the $e^+e^- \rightarrow$ two jets and DIS $(1+1)$ jets is the following. We take the results as generated by the code used for Ref. [8], for a given gap geometry. This is the full result in the large N_c approximation and we divide it by the result obtained using the same code for *primary emissions alone* (rather than dividing by the naive Sudakov result), which takes as the only source for emissions the original hard dipole, e.g., the outgoing $q\bar{q}$ pair in e^+e^- annihilation. The result of this division is the non-global piece in the large N_c limit. We can then make use of the fact that one can easily compute the exact $\mathcal{O}(\alpha_s^2)$ non-global term with proper $C_F C_A$ colour factor and parameterise the non-global Monte Carlo result, as a function of t , in a form that retains the correct colour structure for the leading $\alpha_s^2 \ln^2 Q/Q_\Omega$ non-global term (see, e.g., Ref. [3]). This is the non-global result, with the large N_c approximation starting only from $\mathcal{O}(\alpha_s^3 \ln^3 Q/Q_\Omega)$ terms. The overall result is obtained by multiplying the resultant parameterised form with the primary emission result, as computed here with full colour factors. The large N_c approximation is then confined to the non-global term and starting from the next-to-leading such piece (S_3 in the notation of [8]). It is thus still an important finding that the non-global logarithms are reduced considerably by k_t clustering as demonstrated in Ref. [8], since this potentially reduces the impact of unknown non-global terms beyond the large N_c approximation. However the correct procedure for identifying the primary and non-global pieces, pointed out here, must be accounted for while comparing to experimental data to enable accurate phenomenological studies.

In the case of dijet photoproduction, studied, e.g., in [9], and gaps between jets in hadron–hadron processes, it is less straightforward to account for the missing independent emission terms we point out. They will have a complex colour structure and existing large N_c numerical programs cannot be employed to generate the full answer beyond the large N_c limit. This would mean that the accuracy of the resummed result is limited not just by the unknown beyond-large- N_c non-global logs but similarly in the primary emission terms which are not reduced by the use of cluster-

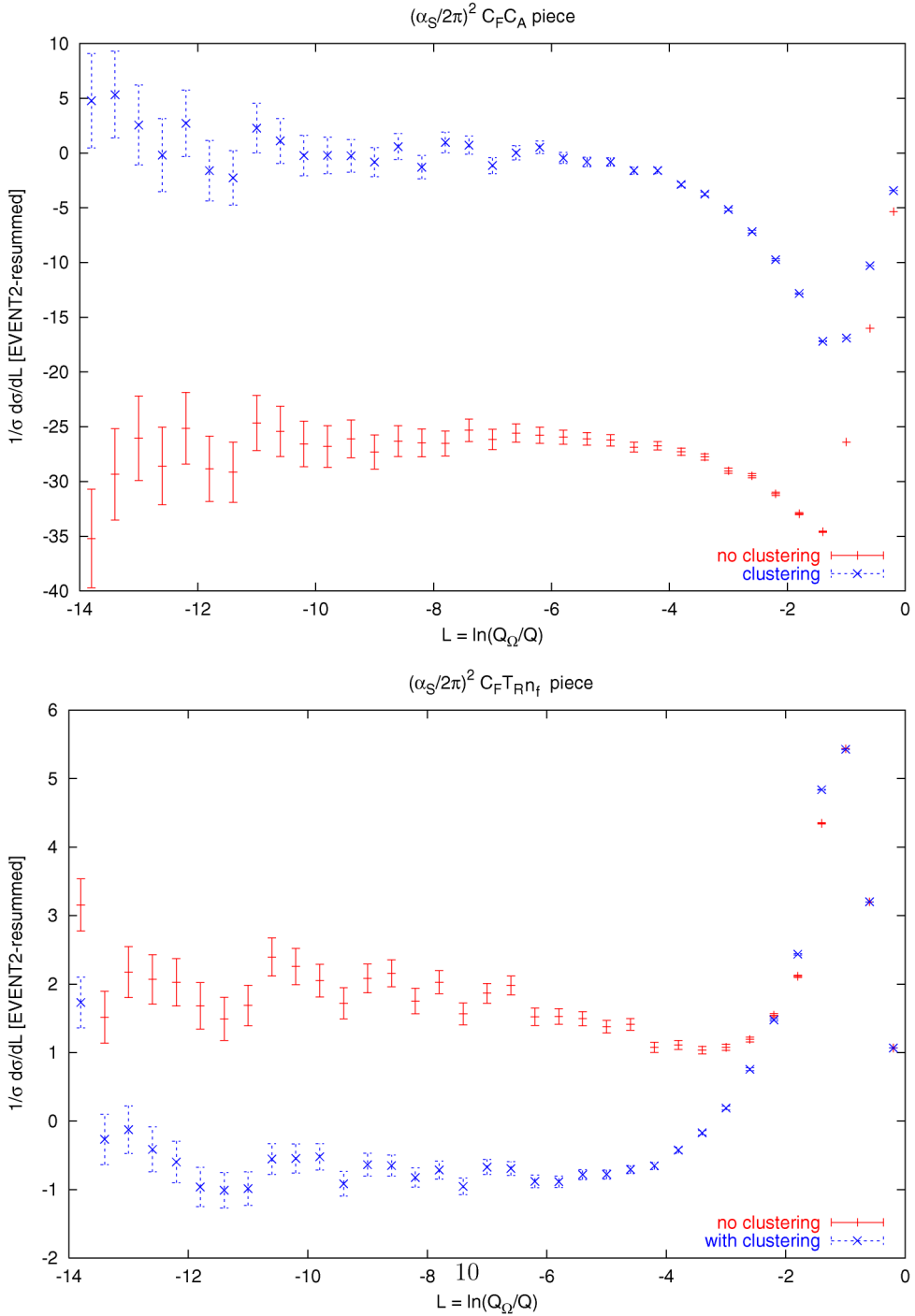


Fig. 2. Comparison of the C_{FC_A} (above) and $C_{F n_f} \alpha_S^2 L$ term (below) produced by EVENT2 and the expanded Sudakov result, supplemented with non-global logs for the C_{FC_A} term. The figures are for $R = 1$ and $\Delta\eta = 1.0$ and as we expect the difference between the exact and resummed result expanded to NLO is a constant at large L .

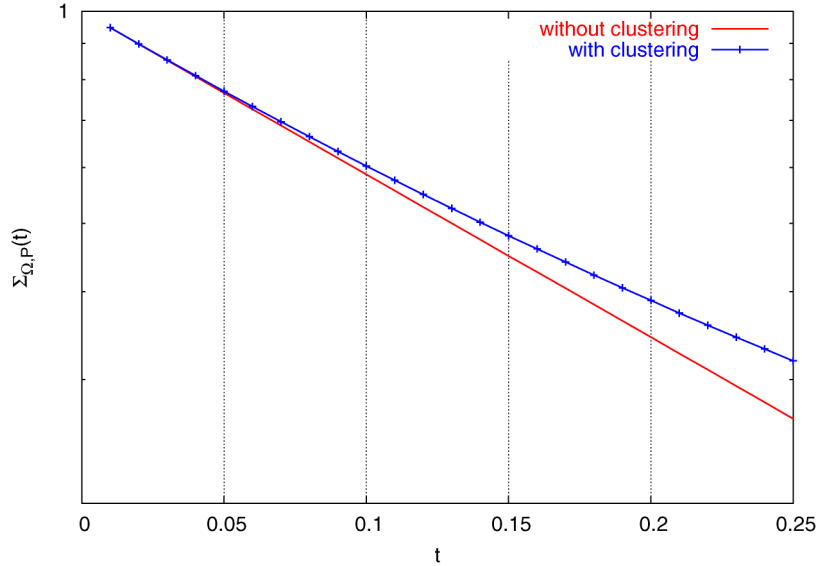


Fig. 3. The results for the primary emission resummation with and without k_t clustering for $R = 1$, $\Delta\eta = 1$, using an adaptation of the program used for Ref. [8]. As can be seen, the clustering affects the primary emission term and the effect for $t = 0.25$ is an increment of over 30%.

ing. In these cases further studies are therefore required to account correctly for the missing primary emission terms before one can argue that use of the clustering method mitigates the uncertainty involved in the theoretical predictions, by reducing the non-global component significantly. This is currently work in progress [16].

Acknowledgements

We would like to thank Mike Seymour for useful discussions concerning this Letter and the work described in Ref. [8] and Rob Appleby for supplying us with the numerical code used in Ref. [8]. We also thank Gavin Salam for helpful comments.

References

- [1] Yu.L. Dokshitzer, G. Marchesini, B.R. Webber, Nucl. Phys. B 469 (1996) 93.
- [2] G. Marchesini, B.R. Webber, Phys. Rev. D 38 (1988) 3419; CDF Collaboration, Phys. Rev. D 70 (2004) 072002.
- [3] M. Dasgupta, G.P. Salam, Phys. Lett. B 512 (2001) 323.
- [4] M. Dasgupta, G.P. Salam, JHEP 0203 (2002) 17.
- [5] A. Banfi, G. Marchesini, G.E. Smye, JHEP 0208 (2002) 006.
- [6] C.F. Berger, T. Kucs, G. Sterman, Phys. Rev. D 68 (2003) 014012.
- [7] Yu.L. Dokshitzer, G. Marchesini, JHEP 0303 (2003) 040.
- [8] R.B. Appleby, M.H. Seymour, JHEP 0212 (2002) 063.
- [9] R.B. Appleby, M.H. Seymour, JHEP 0309 (2003) 056.
- [10] S. Catani, Yu.L. Dokshitzer, M.H. Seymour, B.R. Webber, Nucl. Phys. B 406 (1993) 187; S.D. Ellis, D.E. Soper, Phys. Rev. D 48 (1993) 3160; J.M. Butterworth, J.P. Couchman, B.E. Cox, B.M. Waugh, Comput. Phys. Commun. 153 (2003) 85.
- [11] H1 Collaboration, C. Adloff, et al., Eur. Phys. J. C 24 (2002) 517.
- [12] ZEUS Collaboration, M. Derrick, et al., Phys. Lett. B 369 (1996) 55.
- [13] C.F. Berger, T. Kucs, G. Sterman, Phys. Rev. D 65 (2002) 094031.
- [14] S. Catani, M.H. Seymour, Phys. Lett. B 378 (1996) 287; S. Catani, M.H. Seymour, Nucl. Phys. B 485 (1997) 291.
- [15] Yu.L. Dokshitzer, G. Marchesini, G. Oriani, Nucl. Phys. B 387 (1992) 675.
- [16] A. Banfi, G. Corcella, M. Dasgupta, in preparation.