Available at www.ElsevierMathematics.com
POWERED BY SCIENCE @ DIRECT®

An International Journal
computers & mathematics with applications

ELSEVIER Computers and Mathematics with Applications 46 (2003) 1095–1101
www.elsevier.com/locate/camwa

An Optimal Maintenance Time of Automatic Monitoring System of ATM with Two Kinds of Breakdowns

S. NAKAMURA
College of Human Life and Environment, Department of Human Life and Information
Kinjo Gakuin University, 1723 Omori 2-chome, Moriyama-ku, Nagoya, Aichi 463-8521, Japan
snakam@kinjo-u.ac.jp

C. QIAN
Department of Industrial Engineering, Aichi Institute of Technology
1247 Yachigusa, Yakusa-cho, Toyota 470-0392, Japan
qch64317@ie.aitech.ac.jp

I. HAYASHI
Hardware Application Engineering Division, Hitachi Chubu Software, Ltd.
Owariasahi, Aichi 488, Japan
hayashi@ts.hitachi.co.jp

T. NAKAGAWA
Department of Industrial Engineering, Aichi Institute of Technology
1247 Yachigusa, Yakusa-cho, Toyota 470-0392, Japan
nakagawa@ie.aitech.ac.jp

Abstract—All automatic teller machines (ATMs) in a bank operate unmanned on weekends and holidays, and an automatic monitoring system continuously watches the operation of ATMs through the telecommunication network. There are two kinds of troubles according to the installed places of ATMs. One is the trouble which occurs inside the branch of a bank where ATMs operate manned except on weekends and holidays, and the other is the one which occurs outside the branch where ATMs always operate unmanned. Two kinds of breakdowns are introduced, and the expected cost for an unmanned operation period is obtained. A maintenance policy which minimizes the expected cost is analytically derived. Finally, a numerical example is given and some useful discussions are made.

Keywords—ATM of bank, Two breakdowns, Expected cost, Maintenance policy.

1. INTRODUCTION

Automatic teller machines (ATMs) have spread among daily life through the country, and also, their operational times have greatly increased. Recently, some ATMs are usually operating even on weekends and holidays. Further, ATMs have various kinds of facilities such as the transfer of cash, the contract and cancellation of deposit and account, the reception of loan, and so on. Most ATMs are connected with the online system of a bank and increase the efficiency of business. Moreover, it is now planned to connect ATMs with other organizations, so that their networks

0898-1221/03/$ - see front matter Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX
doi: 10.1016/S0898-1221(03)00315-8

would be expanded on every nook and corner, and they would become one of the indispensable infrastructures of life in society. In such situations, adequate and prompt maintenances for troubles and breakdowns of ATMs have to be done from both viewpoints of a customer's trust and service. Therefore, it is very important to adopt a monitoring system of ATMs and to previously form its maintenance policy. There are roughly two kinds of ATMs in accordance with their installed places. One is an ATM which is set up in the branch of a bank, which is called an inside branch ATM, and the other is in department stores, stations, supermarkets, or other public facilities, which is called an outside branch ATM. A bank consigns the replenishment of cash, and the check and maintenance of outside branch ATMs and inside branch ATM except weekdays, to a security company [1].

An automatic monitoring system continuously watches the operation of outside branch ATMs because they always operate unmanned. On the other hand, an inside branch ATM is watched by a bank employee in this branch on weekdays, and is done at the control center on holidays. A bank employee checks an ATM at the beginning time of the next day after holidays. Even if some troubles have occurred in an ATM on holidays, they are removed by a bank employee on the next day and it is restored to a normal condition. A monitoring system at the control center can display troubles for outside branch ATMs in the terminal unit and output them. Moreover, there might sometimes be phone calls for users in ATMs to report the trouble situation. If the troubles are displayed in the terminal unit, a member at the control center can remove some of them, by remotely operating the terminal unit according to their states. Otherwise, a member reports such a fact to a security company, which can promptly remove troubles or breakdowns of ATMs.

It is assumed in this paper that there exist two kinds of breakdowns by which an ATM breaks down after trouble occurrence and directly. We propose a stochastic model of an inside branch ATM with two breakdowns, which operates unmanned on a weekend and is checked at time after trouble occurrence. This is one kind of modified inspection model [2]. The probability distributions of the time to each occurrence of two breakdowns are given, and the checking cost and the loss costs suffered for breakdowns are introduced. Then, the expected cost of an ATM for an unmanned operating period is obtained, and an optimal maintenance policy, which minimizes it, is analytically derived. Finally, a numerical example is given and some useful discussions are made.

2. MODEL

An automatic monitoring system watches an inside branch ATM on holidays by the polling selecting method through a telephone line, and can display the state of an ATM. The state is roughly classified into the following five small ones.

State 0: An ATM is normal. There is no trouble in the ATM.

State 1: There are some troubles in an ATM such that the cash and the receipts may be running out soon, or an ATM may be choked up with the card and the cash. An ATM will break down soon by these troubles. If a member at the control center can remove the troubles, they are not included in State 1.

State 2: An ATM is checked at time t_0 after State 1. A security company member goes to the ATM location directly and can remove troubles before it breaks down. This is an easy job which changes the cashbox or replenishes the receipts and the journal form.

State 3: An ATM breaks down until time t_0 after State 1 (Breakdown 1); i.e., it breaks down before a security company member arrives at the ATM location. He recovers an ATM by changing the cashbox or replenishing the receipts and the journal form.

State 4: An ATM breaks down by mechanical factors (Breakdown 2). For example, the power supply stops or an ATM is choked up with the cash and the card. A security company member goes to the ATM location and recovers the ATM. Therefore, an ATM cannot be used from the breakdown to the arrival time of a security company member. The maintenance time of Breakdown 2 would usually be longer than that of Breakdown 1 in State 3.

Figure 1 shows the transition relation between the above five states.

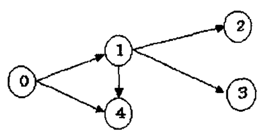

Figure 1. Figure of transition between five states.

In the operation of an ATM, some troubles associated with the cash, the receipt forms, and the journal form would occur at most one time for a short time span such as a weekend and holidays. It is supposed that an ATM has to operate during the interval $[0,T]$ and the trouble occurs only at most one time in this interval.

It is assumed that some troubles occur according to a general distribution $F_0(t)$, and after trouble occurrence, the time to Breakdown 1 has a general distribution $F_1(t)$. Further, the time to Breakdown 2 is independent of the occurrences of troubles and Breakdown 1, and has a general distribution $F_2(t)$. If there are two or more ATMs in the same booth, five states are defined as the state of the last operating ATM.

We give the following probabilities that events such as troubles and breakdowns occur during $[0,T]$, where $\bar{F}_i \equiv 1 - F_i$ $(i = 0, 1, 2)$.

(i) The probability that any troubles and Breakdown 2 do not occur during $[0,T]$ is

$$\bar{F}_0(T)\bar{F}_2(T). \tag{1}$$

(ii) The probability that Breakdown 2 occurs before trouble occurrence during $[0,T]$ is

$$\int_0^T \bar{F}_0(x)\,dF_2(x). \tag{2}$$

(iii) The probability that an ATM is checked at T without breakdowns after trouble occurrence is

$$\bar{F}_2(T)\int_{T-t_0}^T \bar{F}_1(T-x)\,dF_0(x). \tag{3}$$

(iv) The probability that Breakdown 1 occurs after trouble occurrence (see Figure 2) is

$$\int_{T-t_0}^T dF_0(x)\int_0^{T-x} \bar{F}_2(x+y)\,dF_1(y). \tag{4}$$

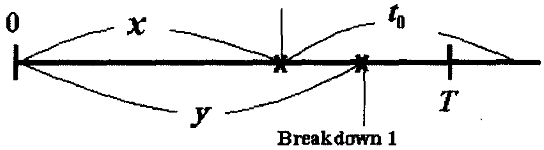

Figure 2. Breakdown 1 occurrence.

(v) The probability that Breakdown 2 occurs after trouble occurrence is

$$\int_{t-t_0}^{T} dF_0(x) \int_x^T \bar{F}_1(y-x)\, dF_2(y). \tag{5}$$

(vi) The probability that an ATM is checked at time t_0 after trouble occurrence is

$$\bar{F}_1(t_0) \int_0^{T-t_0} \bar{F}_2(t_0+x)\, dF_0(x). \tag{6}$$

(vii) The probability that Breakdown 1 occurs until time t_0 after trouble occurrence is

$$\int_0^{T-t_0} dF_0(x) \int_0^{t_0} \bar{F}_2(x+y)\, dF_1(y). \tag{7}$$

(viii) The probability that Breakdown 2 occurs until time t_0 after trouble occurrence (see Figure 3) is

$$\int_0^{T-t_0} dF_0(x) \int_x^{x+t_0} \bar{F}_1(y-x)\, dF_2(y). \tag{8}$$

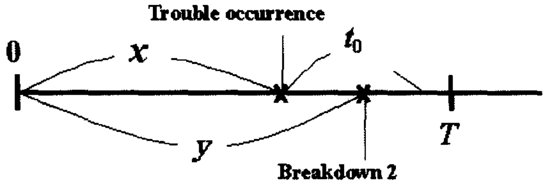

Figure 3. Breakdown 2 occurrence.

Evidently, we have

$$\begin{aligned}
&(3)+(4)+(5)\\
&= \int_{T-t_0}^{T} dF_0(x) \left[\bar{F}_2(T)\bar{F}_1(T-x) + \int_0^{T-x} \bar{F}_2(x+y)\, dF_1(y) + \int_x^T \bar{F}_1(y-x)\, dF_2(y) \right]\\
&= \int_{T-t_0}^{T} \bar{F}_2(x)\, dF_0(x),
\end{aligned} \tag{9}$$

$$\begin{aligned}
&(6)+(7)+(8)\\
&= \int_0^{T-t_0} dF_0(x) \left[\bar{F}_2(t_0+x)\bar{F}_1(t_0) + \int_0^{t_0} \bar{F}_2(x+y)\, dF_1(y) + \int_x^{x+t_0} \bar{F}_1(y-x)\, dF_2(y) \right]\\
&= \int_0^{T-t_0} \bar{F}_2(x)\, dF_0(x).
\end{aligned} \tag{10}$$

Hence, it is proved that

$$(1)+(2)+(9)+(10) = \bar{F}_0(T)\bar{F}_2(T) + \int_0^T \bar{F}_0(x)\, dF_2(x) + \int_0^T \bar{F}_2(x)\, dF_0(x) = 1.$$

3. EXPECTED COST

We introduce the following costs.

$c_0 =$ Cost at T. An ATM stops at time T. A bank employee checks an ATM before it begins to operate on the next day, and replenishes the cash, the journal, and receipt forms.

$c_1 =$ Checking cost at time t_0. A security company member refills the cashbox, and if necessary, replenishes the journal and receipt forms. A cost c_1 is higher than c_0 because a security company member has to go to the ATM location.

$c_2 =$ Cost for Breakdown 1. An ATM has stopped until a security company member arrives at time t_0 after Breakdown 1 occurrence. Customers cannot use it and have to use ATMs of other banks. In this case, not only customers pay the commission to other banks, but also a bank pays the commission for customers' usage. A cost c_2 includes the whole cost which is the sum of cost c_1 and the loss cost for Breakdown 1.

$c_3 =$ Cost for Breakdown 2. An ATM breaks down directly, and has stopped until a security company member arrives at the ATM location. The maintenance time and cost for Breakdown 2 would usually be longer and higher than those of Breakdown 1, respectively. It could be seen in general that $c_3 > c_2 > c_1 > c_0$.

From the notations of the above costs, the total expected cost of an ATM during $[0, T]$ is given by

$$\begin{aligned} C(t_0) = c_0\bar{F}_2(T)\left[\bar{F}_0(T) + \int_{T-t_0}^{T} \bar{F}_1(T-x)\,dF_0(x)\right] + c_1\bar{F}_1(t_0)\int_0^{T-t_0} \bar{F}_2(t_0+x)\,dF_0(x) \\ + c_2\left[\int_{T-t_0}^{T} dF_0(x)\int_0^{T-x} \bar{F}_2(x+y)\,dF_1(y) \right. \\ \left. + \int_0^{T-t_0} dF_0(x)\int_0^{t_0} \bar{F}_2(x+y)\,dF_1(y)\right] \\ + c_3\left[\int_0^{T} \bar{F}_0(x)\,dF_2(x) + \int_{T-t_0}^{T} dF_0(x)\int_x^{T} \bar{F}_1(y-x)\,dF_2(y) \right. \\ \left. + \int_0^{T-t_0} dF_0(x)\int_x^{x+t_0} \bar{F}_1(y-x)\,dF_2(y)\right], \qquad 0 \le t_0 \le T. \end{aligned} \tag{11}$$

4. OPTIMAL POLICY

It is a problem to determine when a security company member goes to the ATM location after trouble occurrence. For example, if troubles occur near time T, it would be unnecessary to send a security company member. We find an optimal time t_0^* $(0 \le t_0^* \le T)$ which minimizes the expected cost $C(t_0)$ in (11). In the particular case of $t_0 = 0$, i.e., when an ATM is maintained immediately after trouble occurrence, the expected cost is

$$C(0) = c_0\bar{F}_2(T)\bar{F}_0(T) + c_1\int_0^{T} \bar{F}_2(x)\,dF_0(x) + c_3\int_0^{T} \bar{F}_0(x)\,dF_2(x). \tag{12}$$

In the particular case of $t_0 = T$, i.e., when an ATM is not maintained until time T even if troubles occur, the expected cost is

$$\begin{aligned} C(T) = c_0\bar{F}_2(T)\left[\bar{F}_0(T) + \int_0^{T} \bar{F}_1(T-x)\,dF_0(x)\right] \\ + c_2\int_0^{T} dF_0(x)\int_0^{T-x} \bar{F}_2(x+y)\,dF_1(y) \\ + c_3\left[\int_0^{T} \bar{F}_0(x)\,dF_2(x) + \int_0^{T} dF_0(x)\int_x^{T} \bar{F}_1(y-x)\,dF_2(y)\right]. \end{aligned} \tag{13}$$

Next, suppose that distributions $F_0(t)$ and $F_2(t)$ are exponential, i.e., $F_0(t) = 1 - e^{-\lambda_0 t}$ and $F_2(t) = 1 - e^{-\lambda_2 t}$. Further, assume that $F_1(t)$ has a density $f_1(t)$, and define that $\gamma_1(t) \equiv f_1(t)/\bar{F}_1(t)$ with $\gamma_1(0) \equiv 0$ which represents the failure rate of time t_0 Breakdown 1. Differentiating $C(t_0)$ with respect to t_0 and setting it equal to zero,

$$[(c_2 - c_1)\gamma_1(t_0) + (c_3 - c_1)\lambda_2] \frac{e^{(\lambda_0+\lambda_2)(T-t_0)} - 1}{\lambda_0 + \lambda_2} = c_1 - c_0. \tag{14}$$

In general, it would be very difficult to derive an optimal time t_0^* analytically. However, we have the following results which are useful for computing t_0^* numerically.

(i) If there exists a solution to satisfy (14), then an optimal time is given by comparing $C(0)$ in (12), $C(T)$ in (13), and $C(t_0)$ in (11).

(ii) If there is no solution to satisfy (14), an optimal time is $t_0^* = T$, since $C(t_0)$ is a decreasing function of t_0.

5. NUMERICAL EXAMPLE

Suppose that the distribution $F_1(t)$ of time to Breakdown 1 has the IFR property [2]; i.e., $F_1(t) = 1 - e^{-\lambda_1 t^m}$ $(m > 1)$. Figure 4 draws the expected cost $C(t_0)$ for t_0 when $T = 16$ (hours), $\lambda_0 = 5/1000$ (1/hours), $\lambda_1 = 7/200$ (1/hours), $\lambda_2 = 5/200$ (1/hours), $c_0 = 4.5$, $c_1 = 6.0$, $c_2 = 7.0$, $c_3 = 8.5$.

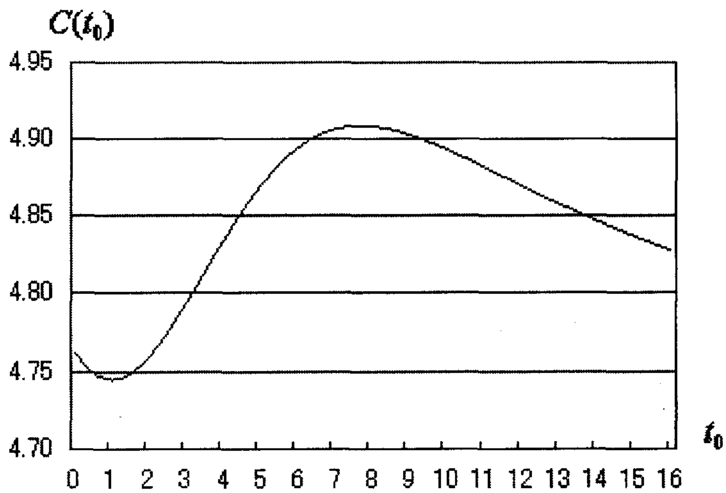

Figure 4. Graph of total expected cost $C(t_0)$.

It is shown from this figure that $t_0^* = 1.00$ (hours) and $C(t_0^*) = 4.745$. We should dispatch a security company member, who does the maintenance of an ATM, after 60 minutes from trouble occurrence. In practical operations, a security company member usually goes to the branch of ATMs from about 20 minutes to 60 minutes even if one of them in the booth breaks down, and sequentially does the maintenance of ATMs with troubles. The above model, where a security company member arrives there at about 60 minutes after trouble occurrence, would be suitable for the actual situation.

6. CONCLUSIONS

We have formulated the stochastic model of an automatic monitoring system for an unmanned ATM in a bank. It has been assumed that there exist two breakdowns in an ATM where one occurs after some troubles and the other occurs directly, and an ATM is checked at time after trouble occurrence. Then, we have derived the expected cost for an unmanned period and discussed numerically the optimal checking time t_0^* which minimizes it. The method and the result obtained in this paper would be applied to an actual monitoring system for an ATM by suitable modifications.

REFERENCES

1. S. Nakamura, T. Nakagawa and H. Sandoh, Optimal number of spare cashboxes for unmanned bank ATMs, *RAIRO-Operations Research* **32**, 389–398, (1998).
2. R.E. Barlow and F. Proschan, *Mathematical Theory of Reliability*, John Wiley & Sons, New York, (1965).