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Logarithmic corrections to the Bekenstein–Hawking entropy for five-dimensional black holes and de Sitter spaces

Y.S. Myung

Relativity Research Center and School of Computer Aided Science, Inje University, Gimhae 621-749, South Korea

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Abstract

We calculate corrections to the Bekenstein–Hawking entropy formula for the five-dimensional topological AdS (TAdS)-black holes and topological de Sitter (TdS) spaces due to thermal fluctuations. We can derive all thermal properties of the TdS spaces from those of the TAdS black holes by replacing k by $-k$. Also we obtain the same correction to the Cardy–Verlinde formula for TAdS and TdS cases including the cosmological horizon of the Schwarzschild–de Sitter (SdS) black hole. Finally we discuss the AdS/CFT and dS/CFT correspondences and their dynamic correspondences.

1. Introduction

Recently there are several works which show that for a large class of black holes (AdS–Schwarzschild one), the Bekenstein–Hawking entropy receives logarithmic corrections due to thermodynamic fluctuations [1–5]. The corrected formula takes the form

$$S = S_0 - \frac{1}{2}\ln C_v + \cdots, \tag{1}$$

where C_v is the specific heat of the given system at constant volume and S_0 denotes the uncorrected Bekenstein–Hawking entropy. Here an important point is that for Eq. (1) to make sense, C_v should be positive. However, the d-dimensional Schwarzschild black hole which is asymptotically flat has a negative specific heat of $C_v^{\text{Sch}} = -(d-2)S_0$ [6]. This means that the Schwarzschild black hole is never in thermal equilibrium and it evaporates according to the Hawking radiation. But the Schwarzschild black hole could be thermal equilibrium with a radiation in a bounded box. This is because the black hole has a negative specific heat while the radiation has a positive one. The two will be in thermal equilibrium if the box is bounded. The AdS–Schwarzschild black hole belongs to this category. On the contrary, if the box is unbounded as the Schwarzschild black hole, the black hole evaporates completely. Furthermore we note that a cosmological horizon in five-dimensional de Sitter space has a positive

E-mail address: ysmyung@physics.inje.ac.kr (Y.S. Myung).

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specific heat of $C_v^{\rm dS} = 3S_0$. Hence we can use the Eq. (1) to calculate the correction to the Bekenstein–Hawking entropy of the cosmological horizon.

In this Letter, we find new five-dimensional AdS-black holes and de Sitter spaces which give us positive specific heats and thus logarithmic corrections to the entropy are achieved. These are the topological AdS-black holes and topological de Sitter spaces. For completeness, we study thermal properties of the Schwarzschild–de Sitter black hole. Further we make corrections to the Cardy–Verlinde formula which is a higher-dimensional version of the two-dimensional Cardy formula. This formula realizes the holography principle through the A(dS)/CFT correspondences.

2. Topological AdS black holes

It is believed that black holes in asymptotically flat spacetime should have spherical horizon. When introducing a negative cosmological constant, a black hole can have non-spherical horizon. We call this the topological black hole [7]. The topological AdS black holes in five-dimensional spacetimes are given by

$$ds^2_{\rm TAdS} = -h(r)\,dt^2 + \frac{1}{h(r)}dr^2 + r^2\big[d\chi^2 + f_k(\chi)^2\big(d\theta^2 + \sin^2\theta\,d\phi^2\big)\big], \tag{2}$$

where k describes the horizon geometry with a constant curvature. $h(r)$ and $f_k(\chi)$ are given by

$$h(r) = k - \frac{m}{r^2} + \frac{r^2}{\ell^2}, \qquad f_0(\chi) = \chi, \qquad f_1(\chi) = \sin\chi, \qquad f_{-1}(\chi) = \sinh\chi. \tag{3}$$

Here we define $k = 1, 0$, and -1 cases as the Schwarzschild–AdS (SAdS) black hole [8], flat-AdS (FAdS) black hole, and hyperbolic-AdS (HAdS) black hole [9], respectively. In the case of $k = 1$, $m = 0$, we have an exact AdS_5-space with its curvature radius ℓ. However, $m \neq 0$ generates the topological AdS black holes. The only event horizon is given by

$$r^2_{\rm EH} = \frac{\ell^2}{2}\Big(-k + \sqrt{k^2 + 4m/\ell^2}\Big). \tag{4}$$

For $k = 1$, we have both a small black hole ($r^2_{\rm EH} \ll \ell^2, 4m \ll \ell^2$) with the horizon at $r = r_{\rm EH}$, where $r^2_{\rm EH} \simeq m$ and a large black hole ($r^2_{\rm EH} \gg \ell^2, 4m \gg \ell^2$) with the horizon at $r = r_{\rm EH}$ given by $r^2_{\rm EH} \simeq \sqrt{m}\,\ell$. For $k = 0$ case, one has the event horizon at $r = r_{\rm EH}$, where $r^2_{\rm EH} = \sqrt{m}\,\ell$. In the case of $k = -1$, for $4m \ll \ell^2$ one has the event horizon at $r = r_{\rm EH}$, where $r^2_{\rm EH} \simeq \ell^2 + m$ and for $4m \gg \ell^2$ one has the event horizon at $r = r_{\rm EH}$ given by $r^2_{\rm EH} \simeq \sqrt{m}\,\ell$. That is, one always finds $r^2_{\rm EH} > \ell^2$ for $k = -1$. This analysis is useful to justify whether the corresponding specific heat is or not positive.

The relevant thermodynamic quantities: reduced mass (m), free energy (F), Bekenstein–Hawking entropy (S_0), Hawking temperature ($T_{\rm H}$), and energy (ADM mass: $E = M$) are given by [10]

$$\begin{aligned} &m = r^2_{\rm EH}\left(\frac{r^2_{\rm EH}}{\ell^2} + k\right), \qquad F = -\frac{V_3 r^2_{\rm EH}}{16\pi G_5}\left(\frac{r^2_{\rm EH}}{\ell^2} - k\right), \qquad S_0 = \frac{V_3 r^3_{\rm EH}}{4G_5}, \\ &T_{\rm H} = \frac{k}{2\pi r_{\rm EH}} + \frac{r_{\rm EH}}{\pi\ell^2}, \qquad E = F + T_{\rm H}S_0 = \frac{3V_3 m}{16\pi G_5} = M, \end{aligned} \tag{5}$$

where V_3 is the volume of unit three-dimensional hypersurface and G_5 is the five-dimensional Newton constant. Using the above together with $C_v = (dE/dT)_V$, one finds

$$C_v = 3\frac{2r^2_{\rm EH} + k\ell^2}{2r^2_{\rm EH} - k\ell^2}S_0. \tag{6}$$

Here we obtain two positive specific heats for HAdS and FAdS black holes

$$C_v^{\mathrm{HAdS}} > 0, \qquad C_v^{\mathrm{FAdS}} = 3S_0 > 0, \quad \text{for any } r_{\mathrm{EH}}. \tag{7}$$

On the other hand one finds a condition for positive specific heat for SAdS black hole [8]

$$C_v^{\mathrm{SAdS}} > 0, \quad \text{for } r_{\mathrm{EH}}^2 > \ell^2/2. \tag{8}$$

In the limit of $\ell \to \infty$, we recover the negative specific heat ($C_v^{\mathrm{Sch}} = -3S_s$) of the Schwarzschild black hole. On the other hand, in the limit of $\ell \to 0$ one finds a positive value of $C_v^{\ell\to 0} = 3S_0$ for the large SAdS-black hole.

3. Schwarzschild–de Sitter black hole

In order to find thermal property of a black hole in de Sitter space, we consider Schwarzschild–de Sitter (SdS) black hole in five-dimensional spacetimes [11]

$$ds_{\mathrm{SdS}}^2 = -h(r)\,dt^2 + \frac{1}{h(r)}\,dr^2 + r^2\big[d\chi^2 + \sin^2\chi\big(d\theta^2 + \sin^2\theta\,d\phi^2\big)\big], \tag{9}$$

where $h(r)$ is given by

$$h(r) = 1 - \frac{m}{r^2} - \frac{r^2}{\ell^2}. \tag{10}$$

In the case of $m = 0$, we have an exact de Sitter space with its curvature radius ℓ. However, $m \neq 0$ generates the SdS black hole. Here we have two horizons. The cosmological and event horizons are given by

$$r_{\mathrm{CH/EH}}^2 = \frac{\ell^2}{2}\Big(1 \pm \sqrt{1 - 4m/\ell^2}\Big). \tag{11}$$

We classify three cases: (1) $4m = \ell^2$, (2) $4m > \ell^2$, (3) $4m < \ell^2$. The case of $4m = \ell^2$ corresponds to the maximum black hole and the minimum cosmological horizon in asymptotically de Sitter space (that is, Nariai black hole). In this case we have $r_{\mathrm{EH}}^2 = r_{\mathrm{CH}}^2 = \ell^2/2 = 2m$. The case of $4m > \ell^2$ is not allowed for the black hole in de Sitter space. The case of $4m < \ell^2$ corresponds to a small black hole within the cosmological horizon. In this case we have the cosmological horizon at $r = r_{\mathrm{CH}}$, where $r_{\mathrm{CH}}^2 \simeq \ell^2 - m$ and the event horizon at $r = r_{\mathrm{EH}}$ given by $r_{\mathrm{EH}}^2 \simeq m$. Hence we have two relations for the SdS solution:

$$m \leqslant r_{\mathrm{EH}}^2 \leqslant \ell^2/2, \qquad \ell^2/2 \leqslant r_{\mathrm{CH}}^2 \leqslant \ell^2 - m \tag{12}$$

which means that as m increases from a small value to the maximum of $m = \ell^2/4$, a small black hole increases up to the Nariai black hole. On the other hand the cosmological horizon decreases from the maximum of $(\ell^2 - m)$ to the minimum of $\ell^2/2$.

The relevant thermodynamic quantities for two horizons are given by [12,13]

$$\begin{aligned} &m = r_{\mathrm{EH/CH}}^2\left(-\frac{r_{\mathrm{EH/CH}}^2}{\ell^2} + 1\right), \qquad F_{\mathrm{EH/CH}} = \pm\frac{V_3 r_{\mathrm{EH/CH}}^2}{16\pi G_5}\left(\frac{r_{\mathrm{EH}}^2}{\ell^2} + 1\right), \qquad S_0 = \frac{V_3 r_{\mathrm{EH/CH}}^3}{4G_5}, \\ &T_{\mathrm{H}}^{\mathrm{EH/CH}} = \pm\frac{1}{2\pi r_{\mathrm{EH/CH}}} \mp \frac{r_{\mathrm{EH/CH}}}{\pi\ell^2}, \qquad E = F_{\mathrm{EH/CH}} + T_{\mathrm{H}}^{\mathrm{EH/CH}} S_0 = \pm\frac{3V_3 m}{16\pi G_5}, \end{aligned} \tag{13}$$

where V_3 is the volume of unit three-dimensional sphere. Using the above relations, one finds

$$C_v^{\mathrm{EH/CH}} = 3\frac{2r_{\mathrm{EH/CH}}^2 - \ell^2}{2r_{\mathrm{EH/CH}}^2 + \ell^2} S_0. \tag{14}$$

Making use of Eq. (12), one finds negative specific heat for the event horizon of the SdS black hole (ESdS) and positive specific heat for the cosmological horizon (CSdS)

$$C_v^{\mathrm{ESdS}} \leqslant 0, \qquad C_v^{\mathrm{CSdS}} \geqslant 0, \quad \text{for any } r_{\mathrm{EH}}. \tag{15}$$

This means that the cosmological horizon is thermodynamically stable while the event horizon is unstable. The equality sign (that is, zero specific heat) holds for the Nariai black hole. In the limit of $\ell \to \infty$, we recover the negative specific heat ($C_v^{\mathrm{Sch}} = -3S_s$) of the Schwarzschild black hole. On the other hand, in the limit of $\ell \to 0$ one finds a positive value of $C_v^{\mathrm{dS}} = 3S_0$ for the exact de Sitter space.

4. Topological de Sitter space

The topological de Sitter (TdS) solution was originally introduced to check the mass bound conjecture in de Sitter space: any asymptotically de Sitter space with the mass greater than exact de Sitter space has a cosmological singularity [14]. For our purpose, we consider the topological de Sitter solution in five-dimensional spacetimes

$$ds^2_{\mathrm{TdS}} = -h(r)\,dt^2 + \frac{1}{h(r)}dr^2 + r^2\big[d\chi^2 + f_k(\chi)^2\big(d\theta^2 + \sin^2\theta\,d\phi^2\big)\big], \tag{16}$$

where $k = 0, \pm 1$. $h(r)$ and $f_k(\chi)$ are given by

$$h(r) = k + \frac{m}{r^2} - \frac{r^2}{\ell^2}, \qquad f_0(\chi) = \chi, \qquad f_1(\chi) = \sin\chi, \qquad f_{-1}(\chi) = \sinh\chi. \tag{17}$$

Requiring $m > 0$, the black hole disappears and instead a naked singularity occurs at $r = 0$. Here we define $k = 1, 0, -1$ cases as the Schwarzschild-topological de Sitter (STdS) space, flat-topological de Sitter (FTdS) space, and hyperbolic-topological de Sitter (HTdS) space, respectively. In the case of $k = 1, m = 0$, we have an exact de Sitter space with its curvature radius ℓ. However, $m > 0$ generates the topological de Sitter spaces. The only cosmological horizon exists as

$$r^2_{\mathrm{CH}} = \frac{\ell^2}{2}\Big(k + \sqrt{k^2 + 4m/\ell^2}\Big). \tag{18}$$

For $k = -1$ case we have both a small cosmological horizon ($r^2_{\mathrm{CH}} \ll \ell^2, 4m \ll \ell^2$) with the horizon at $r = r_{\mathrm{CH}}$, where $r^2_{\mathrm{CH}} \simeq m$ and a large cosmological horizon ($r^2_{\mathrm{CH}} \gg \ell^2, 4m \gg \ell^2$) with the horizon at $r = r_{\mathrm{CH}}$ given by $r^2_{\mathrm{CH}} \simeq \sqrt{m}\,\ell$. For $k = 0$ case, one has the cosmological horizon at $r = r_{\mathrm{CH}}$, where $r^2_{\mathrm{CH}} = \sqrt{m}\,\ell$. In the case of $k = 1$, for $4m \ll \ell^2$ one has the cosmological horizon at $r = r_{\mathrm{CH}}$, where $r^2_{\mathrm{CH}} \simeq \ell^2 + m$ and for $4m \gg \ell^2$, one has the cosmological horizon at $r = r_{\mathrm{CH}}$, where $r^2_{\mathrm{CH}} \simeq \sqrt{m}\,\ell$. Here we have $r^2_{\mathrm{CH}} > \ell^2$ for $k = 1$ case. This analysis is useful to justify whether the specific heat of the cosmological horizon is or not positive.

The relevant thermodynamic quantities for the cosmological horizon are calculated as [11]

$$m = r^2_{\mathrm{CH}}\left(\frac{r^2_{\mathrm{CH}}}{\ell^2} - k\right), \qquad F = -\frac{V_3 r^2_{\mathrm{CH}}}{16\pi G_5}\left(\frac{r^2_{\mathrm{CH}}}{\ell^2} + k\right), \qquad S_0 = \frac{V_3 r^3_{\mathrm{CH}}}{4G_5},$$
$$T_{\mathrm{H}} = -\frac{k}{2\pi r_{\mathrm{CH}}} + \frac{r_{\mathrm{CH}}}{\pi\ell^2}, \qquad E = F + T_{\mathrm{H}}S = \frac{3V_3 m}{16\pi G_5} = M, \tag{19}$$

where V_3 is the volume of unit three-dimensional hypersurface. Using the above relations, one finds

$$C_v = 3\frac{2r^2_{\mathrm{CH}} - k\ell^2}{2r^2_{\mathrm{CH}} + k\ell^2}S_0. \tag{20}$$

Here we have positive specific heats for STdS and FTdS spaces

$$C_v^{\rm STdS} > 0, \qquad C_v^{\rm FTdS} = 3S_0 > 0, \quad \text{for any } r_{\rm CH}. \tag{21}$$

On the other hand one finds the positive specific heat for HTdS space,

$$C_v^{\rm HTdS} > 0 \quad \text{when } r_{\rm CH}^2 > \ell^2/2. \tag{22}$$

We note that all results of the TdS solution can be recovered from the TAdS solution by replacing k by $-k$. This relation will play an important role in understanding de Sitter space in terms of AdS solution.

5. Correction to entropy and Cardy–Verlinde formula

In this section we make corrections to the Bekenstein–Hawking entropy according to the formula of Eq. (1). For the FAdS black hole and FTdS solution one finds $C_v = 3S_0$ without any approximation. However, other cases (HAdS and SAdS black holes, CSdS, STdS and HTdS spaces) lead to $C_v \simeq 3S_0$ when choosing large black holes ($r_{\rm EH}^2 \gg \ell^2$) and large cosmological horizons ($r_{\rm CH}^2 \gg \ell^2$). As far as $C_v \simeq 3S_0$ is guaranteed, the logarithmic correction to the Bekenstein–Hawking entropy is given by

$$S^{\rm TAdS,CSdS,TdS} = S_0 - \frac{1}{2}\ln S_0 + \cdots. \tag{23}$$

Note that there is no correction to the SdS black hole horizon (ESdS): $S_{\rm EH}^{\rm ESdS} = S_0^{\rm ESdS}$. Thus we do not consider this case for correction.

The holographic principle means that the number of degrees of freedom associated with the bulk gravitational dynamics is determined by its boundary spacetime. The AdS/CFT correspondence represents a realization of this principle [15]. Further, for a strongly coupled CFT with its AdS dual, one obtains the Cardy–Verlinde formula [16]. Indeed, this formula holds for various kinds of asymptotically AdS spacetimes including the TAdS black holes [9]. Also this formula holds for a few of asymptotically de Sitter spacetimes including the SdS black hole and TdS spacetimes [11]. Hence it needs to correct the Cardy–Verlinde formula if possible. For this purpose, we have to define thermodynamic quantities described by the boundary CFT through the A(dS)/CFT correspondences [17]. The relation between the five-dimensional bulk and four-dimensional boundary quantities is given by $E_4 = (\ell/R)E$, $T = (\ell/R)T_{\rm H}$ where R satisfies $T > 1/R$ but one has the same entropy: $S_4 = S_0$. We note that the boundary physics is described by the CFT-radiation matter with the equation of state: $p = E_4/3V_3$. Then a logarithmic correction is being determined by the Casimir energy defined by $E_c = 3(E_4 + pV_3 - TS_0)$. We obtain

$$E_c^{\rm TAdS} = k\frac{3\ell r_{\rm EH}^2 V_3}{8\pi G_5 R} + \frac{3}{2}T\ln S_0, \tag{24}$$

$$E_c^{\rm CSdS} = -\frac{3\ell r_{\rm CH}^2 V_3}{8\pi G_5 R} + \frac{3}{2}T\ln S_0, \tag{25}$$

$$E_c^{\rm TdS} = -k\frac{3\ell r_{\rm CH}^2 V_3}{8\pi G_5 R} + \frac{3}{2}T\ln S_0. \tag{26}$$

Substituting this expression into the Cardy–Verlinde formula, one finds

$$\text{TAdS}: \quad \frac{2\pi R}{3\sqrt{|k|}}\sqrt{|E_c(2E_4 - E_c)|} \simeq S_0 + \frac{\pi R\ell T}{2kr_{\rm EH}^3}\left(\frac{r_{\rm EH}^4}{\ell^2} - kr_{\rm EH}^2\right)\ln S_0, \tag{27}$$

$$\text{CSdS}: \quad \frac{2\pi R}{3}\sqrt{|E_c(2E_4 - E_c)|} \simeq S_0 - \frac{\pi R\ell T}{2r_{\rm CH}^3}\left(\frac{r_{\rm CH}^4}{\ell^2} + r_{\rm CH}^2\right)\ln S_0, \tag{28}$$

$$\text{TdS:} \qquad \frac{2\pi R}{3\sqrt{|k|}}\sqrt{|E_c(2E_4-E_c)|} \simeq S_0 - \frac{\pi R\ell T}{2kr_{\rm CH}^3}\left(\frac{r_{\rm CH}^4}{\ell^2}+kr_{\rm CH}^2\right)\ln S_0. \tag{29}$$

All coefficients in front of $\ln S_0$ in the above equations are transformed as [10,13]

$$\frac{\pi R\ell T}{2kr_{\rm EH}^3}\left(\frac{r_{\rm EH}^4}{\ell^2}-kr_{\rm EH}^2\right)=\frac{(4E_4-E_c)(E_4-E_c)}{2(2E_4-E_c)E_c}, \tag{30}$$

$$-\frac{\pi R\ell T}{2r_{\rm CH}^3}\left(\frac{r_{\rm CH}^4}{\ell^2}+r_{\rm CH}^2\right)=\frac{(4E_4-E_c)(E_4-E_c)}{2(2E_4-E_c)E_c}, \tag{31}$$

$$-\frac{\pi R\ell T}{2kr_{\rm CH}^3}\left(\frac{r_{\rm CH}^4}{\ell^2}+kr_{\rm CH}^2\right)=\frac{(4E_4-E_c)(E_4-E_c)}{2(2E_4-E_c)E_c}. \tag{32}$$

Finally we obtain the same corrected formula for the Cardy–Verlinde formula as

$$S_{\rm CV}^{\rm TAdS,CSdS,TdS} \simeq \frac{2\pi R}{3\sqrt{|k|}}\sqrt{|E_c(2E_4-E_c)|} - \frac{(4E_4-3E_c)E_4}{2(2E_4-E_c)E_c}\ln\left(\frac{2\pi R}{3\sqrt{|k|}}\sqrt{|E_c(2E_4-E_c)|}\right). \tag{33}$$

6. Discussion

First of all we summarize our result. As is shown in Table 1, $C_v = 3S_0$ for FAdS and FTdS cases without any approximation. Also we have $C_v > 0$ for HAdS, STdS and CSdS cases whereas $C_v > 0$ if $r_{\rm EH/CH}^2 > \ell^2/2$ for SAdS black holes and HTdS space. Note that $C_v^{\rm ESdS} < 0$ for the black hole in de Sitter space. However, choosing large black holes and large de Sitter spaces ($C_v \simeq 3S_0$) except ESdS case leads to the same corrected formulae for the Bekenstein–Hawking entropy Eq. (23) and the Cardy–Verlinde formula Eq. (33).[1] Concerning the A(dS)/CFT correspondences, we remind that the boundary CFT energy (E_4) should be positive in order for it to make sense. However, one finds from table that $E_4^{\rm CSdS} < 0$ for the cosmological horizon of the SdS black hole. It suggests that the dS/CFT correspondence is not valid for this case. Also the Casimir energy (E_c) is related to the central charge of the corresponding CFT. Hence, if it is negative, one may obtain the non-unitary CFT. In this sense, HAdS, STdS, and CSdS cases are problematic. Further we comment on the extension of this approach to the dynamic A(dS)/CFT correspondence by introducing a moving domain wall in the bulk background (brane world cosmology). Although there is no problem in the AdS-back hole background [19,20], there remains problem in interpreting the cosmic energy density in compared with the static energy like E_4 in the de Sitter background [21].

Table 1
Summary of specific heats, boundary CFT energy and uncorrected Casimir energy for 5D TAdS black holes, TdS spaces and SdS black hole

Thermodynamical system	C_v	$E_4(E_c)$
HAdS	$+$	$+(-)$
FAdS	$+(3S_0)$	$+(0)$
SAdS	$+$ if $r_{\rm EH}^2 > \ell^2/2$	$+(+)$
STdS	$+$	$+(-)$
FTdS	$+(3S_0)$	$+(0)$
HTdS	$+$ if $r_{\rm CH}^2 > \ell^2/2$	$+(+)$
ESdS	$-$	$+(+)$
CSdS	$+$	$-(-)$

[1] Also a similarly corrected Cardy–Verlinde formula for the TNRdS space appeared in [18].

Finally we wish to mention that through this work, we can derive all thermal properties of the topological de Sitter (TdS) spaces from the topological anti-de Sitter (TAdS) black holes by replacing k by $-k$.

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