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Implementation of a two-qubit controlled-rotation gate based on unconventional geometric phase with a constant gating time

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ABSTRACT

We propose an alternative scheme to implement a two-qubit controlled-R (rotation) gate in the hybrid atom-*CCA* (coupled cavities array) system. Our scheme results in a constant gating time and, with an adjustable qubit-bus coupling (atom-resonator), one can specify a particular rotation R on the target qubit. We believe that this proposal may open promising perspectives for networking quantum information processors and implementing distributed and scalable quantum computation.

1. Introduction

For distributed quantum information processing in a quantum computer with practical applications, the coupling between different sub-systems (a large ensemble of qubits) is essential for realizing an efficient quantum communication and for implementing controllable and distributed quantum gates.

Cavity quantum electrodynamics systems (cQED), which combine atomic and photonic quantum bits, have attracted much attention because their low decoherence rates and feasibility to scale up. Furthermore, coupled cavities array has the advantage of easily addressing single lattice sites via optical lasers.

Schemes have been proposed for quantum communication [1] and generation of maximally entangled states [2] between two atoms trapped in distant optical cavities connected by an optical fiber.

Moreover, quantum logic gates based on cavity QED system have been extensively investigated over the recent years via resonant or dispersive interactions of the atoms with a cavity mode [3–6]. In particular, the scheme proposed in Ref. [6] has already been realized experimentally with long living Rydberg atoms in the microwave domain [7]. However, such schemes are based on dynamic evolution which is very sensitive to the parameter fluctuations and thereby difficult for scaling.

In order to have a high-fidelity and scalable quantum gating, proposals that adopt an engineering reservoir [8,9], a decoherence-free subspace [10] or a geometric operation (which results in geometric phases) [11–13] have proved to be promising approaches for the implementation of built-in fault-tolerant quantum computation.

Geometric phases may offer some practical advantages since they are determined only by some global geometric features, being insensitive to the initial state distribution. Therefore, they may be more robust against dephasing and the fidelity of the geometric gates might be significantly higher than that based on dynamic evolution.

In fact there are two kinds of geometric quantum gates (GQGs): conventional GQGs [11] and unconventional ones [12,13]. In the first one some further operations is required to remove the effects from the corresponding dynamic phases, which may result in additional errors. In the second one, it is unnecessary to eliminate the dynamic phase because it is proportional to the corresponding geometric phase by a constant. For this reason, the unconventional GQGs (which have already been done experimentally with trapped ions [14]) are better than the conventional ones.

On the other hand, controlled-U gates (in which U stands for a unitary transformation) are very useful in quantum information processing [15,16]. Two-qubit controlled-U gates represent a kind of controlled gate which performs a specific unitary transformation on a target qubit subject to a control one. As it is well known, such gates are essential to simplify quantum circuits as they can: (i) create a general two-qubit gate, (ii) prepare an arbitrary pure quantum state and (iii) produce entangled states.

In this work, we present an alternative implementation of a two-qubit controlled-R (rotation) gate in the hybrid atom-*CCA* system. The proposal is based on single qubit operations and

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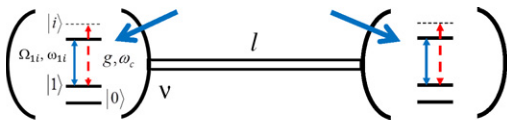

Fig. 1. Two distant atoms (and level configuration) in separate cavities connected by an optical fiber.

unconventional geometric phases on two identical three-level atoms, strongly driven by a resonant classical field [17], trapped in distant cavities connected by an optical fiber.

Our proposal has the following main advantages: (i) it is implemented in a constant gating time (which depends on the fixed experimental parameters) independent of the rotation R (ii) and, with an adjustable qubit-bus coupling (atom-resonator), one can specify a particular rotation R on the target qubit.

We stress that this proposal is quite general and can be applied to any type of three-level physical system (qubit plus intermediate state) interacting with a coupled cavities system (bus). One just has to have a deterministic tunable qubit-bus coupling.

This Letter is organized as follows. In Section 2 we introduce the basic theories necessary for the gate implementation. In Section 3 we present an alternative multi-step two-qubit controlled-R gate implementation. Imperfections such as dissipation due cavity decay and errors during the procedure execution are discussed in Section 4. Finally, some discussions and conclusions are given in Section 5.

2. Basic theory

Before the controlled-R gate implementation, it is worth discussing some of the basic theories.

2.1. CCA and the Jaynes–Cummings model strongly driven

Consider two identical three-level atoms trapped in distant cavities connected by an optical fiber, as shown in Fig. 1.

The number of modes of the fiber that significantly interact with the corresponding cavities is on the order of $l\bar{\nu}/2\pi c$ where l is the length of the fiber, $\bar{\nu}$ is the decay rate of the cavity fields into the continuum of the fiber modes and c is the speed of light. In the short limit, only one fiber mode essentially interacts with the cavity modes and the coupling Hamiltonian is given by [1]

$$H_{cf} = \hbar\nu\hat{b}\left(\hat{a}_1^\dagger + e^{i\varphi}\hat{a}_2^\dagger\right) + h.c., \tag{1}$$

where $\hat{b}$ is the annihilation operator for the fiber mode, $\hat{a}_j^\dagger$ is the creation operator for the jth cavity mode, ν is the cavity fiber coupling strength, and φ is a phase due to propagation of the field through the fiber.

Each atom has one excited (intermediate) state $|i\rangle$ and two ground states $|1\rangle$ and $|0\rangle$ (the logical qubits). The transition $|i\rangle \leftrightarrow |1\rangle$ (frequency ω_{1i}) is coupled to the cavity mode with the coupling constant g and detuning $\delta = \omega_c - \omega_{1i}$. Furthermore, the same transition is driven by a resonant classical field with Rabi frequency Ω_{1i}. As the state $|0\rangle$ is not affected during the interaction, the atom-field interaction in cavity j in the interaction picture is described by the Hamiltonian

$$H_{acj} = \hbar\left(g\hat{a}_j e^{-i\delta t} + \Omega_{1i}\right)\sigma_{+j} + h.c., \tag{2}$$

where $\sigma_{+j} = |i\rangle_{jj}\langle 1|$.

Let us consider the normal modes $\hat{c} = 1/\sqrt{2}(\hat{a}_1 - e^{i\varphi}\hat{a}_2)$ and $\hat{c}_\pm = 1/2(\hat{a}_1 + e^{i\varphi}\hat{a}_2 \pm \sqrt{2}\hat{b})$ with frequency ω_c and $\omega_c \pm \sqrt{2}\nu$ [18]. The whole Hamiltonian in the interaction picture can be rewritten as

$$H = H_0 + H_1, \tag{3}$$

where

$$H_0 = \hbar\Omega_{1i}\sum_{j=1}^{2}(\sigma_{+j} + \sigma_{-j}),$$

$$H_1 = \hbar g\left[\frac{1}{2}\left(\hat{c}_+ e^{-i\sqrt{2}\nu t} + \sqrt{2}\hat{c} + \hat{c}_- e^{i\sqrt{2}\nu t}\right)\sigma_{+1}e^{-i\delta t} + \frac{1}{2}\left(\hat{c}_+ e^{-i\sqrt{2}\nu t} - \sqrt{2}\hat{c} + \hat{c}_- e^{i\sqrt{2}\nu t}\right)\sigma_{+2}e^{-i\delta t} + h.c.\right]. \tag{4}$$

We now switch to a new atomic basis $|+\rangle_j = 1/\sqrt{2}(|1\rangle_j + |i\rangle_j)$ and $|-\rangle_j = 1/\sqrt{2}(|1\rangle_j - |i\rangle_j)$ and perform the unitary transformation $U = e^{-iH_0 t/\hbar}$, that results in [17]

$$\begin{aligned} H' &= U^\dagger H U - i\hbar U^\dagger \dot{U} \\ &= \hbar g\bigg[\frac{1}{2}\left(\hat{c}_+ e^{-i(\delta+\sqrt{2}\nu)t} + \sqrt{2}\hat{c}e^{-i\delta t} + \hat{c}_- e^{-i(\delta-\sqrt{2}\nu)t}\right) \\ &\quad \times \frac{1}{2}\left(\tilde{\sigma}_{z1} - \tilde{\sigma}_{-1}e^{-i2\Omega_{1i}t} + \tilde{\sigma}_{+1}e^{i2\Omega_{1i}t}\right) \\ &\quad + \frac{1}{2}\left(\hat{c}_+ e^{-i(\delta+\sqrt{2}\nu)t} - \sqrt{2}\hat{c}e^{-i\delta t} + \hat{c}_- e^{-i(\delta-\sqrt{2}\nu)t}\right) \\ &\quad \times \frac{1}{2}\left(\tilde{\sigma}_{z2} - \tilde{\sigma}_{-2}e^{-i2\Omega_{1i}t} + \tilde{\sigma}_{+2}e^{i2\Omega_{1i}t}\right) + h.c.\bigg], \end{aligned} \tag{5}$$

where $\tilde{\sigma}_{zj}|\pm\rangle_j = \pm|\pm\rangle_j$ and $\tilde{\sigma}_{\pm j}|\mp\rangle_j = |\pm\rangle_j$.

For a strong cavity-fiber coupling $\nu \gg g$ and intense driving regime $\Omega \gg g, \delta$, we can neglect the terms oscillating fast

$$H'_{eff} = \frac{\hbar g}{2\sqrt{2}}\left(\hat{c}e^{-i\delta t} + \hat{c}^\dagger e^{i\delta t}\right)(\tilde{\sigma}_{z1} - \tilde{\sigma}_{z2}). \tag{6}$$

The evolution operator for Hamiltonian (6) can be written as [19]

$$U' = e^{-iA(t)(\tilde{\sigma}_{z1}-\tilde{\sigma}_{z2})^2}e^{-iB(t)\hat{c}(\tilde{\sigma}_{z1}-\tilde{\sigma}_{z2})} \times e^{-iC(t)\hat{c}^\dagger(\tilde{\sigma}_{z1}-\tilde{\sigma}_{z2})}, \tag{7}$$

in order to find the time dependent functions A and B we can use the Schrödinger equation and obtain

$$A(t) = -\frac{g^2}{8\delta}\left[t - \frac{1}{i\delta}\left(e^{i\delta t} - 1\right)\right],$$

$$B(t) = C^*(t) = -\frac{g}{2\sqrt{2}i\delta}\left(e^{-i\delta t} - 1\right). \tag{8}$$

When the interaction time satisfies $t = \tau = 2\pi/\delta$, the whole evolution operator of the system can be expressed as

$$\begin{aligned} U(\tau) &= e^{-iH_0\tau/\hbar}U'(\tau) \\ &= e^{-i\Omega_{1i}\tau(\tilde{\sigma}_{z1}+\tilde{\sigma}_{z2})}e^{i\lambda\tau(\tilde{\sigma}_{z1}-\tilde{\sigma}_{z2})^2}, \end{aligned} \tag{9}$$

with $\lambda = g^2/8\delta$.

It is evident that such an operator is independent of the cavity mode, which means that the evolution gets insensitive to the initial field state (in the ideal case).

2.2. Microwave and optical pulses

Consider a particular atomic transition $|i\rangle \leftrightarrow |j\rangle$ ($|i\rangle$ is the lower energy level) driven by a resonant classical (optical or microwave) pulse. The interaction Hamiltonian in the interaction picture is then given by

$$H = \hbar\left(\Omega_{ij}e^{i\phi}|i\rangle\langle j| + h.c.\right), \tag{10}$$

in which Ω_{ij} and ϕ are the Rabi frequency and the initial phase of the pulse, respectively. From the Hamiltonian (10) it is easy to find the following state rotation due a pulse of duration t [20]

$$|i\rangle = \cos\Omega_{ij}t|i\rangle - ie^{-i\phi}\sin\Omega_{ij}t|j\rangle,$$
$$|j\rangle = \cos\Omega_{ij}t|j\rangle - ie^{i\phi}\sin\Omega_{ij}t|i\rangle. \tag{11}$$

3. Controlled-R gate implementation

Previously we have introduced two types of interaction of qubit systems with the cavity mode and/or the pulses. The results presented earlier will be employed for the gate implementation discussed in this section.

Initially the atoms are in one of the computational base states $\{|0\rangle_j, |1\rangle_j\}$ (the first qubit is the control and the second one is the target qubit) and the *CCA* system is prepared in the vacuum state $|0\rangle_c$. Indeed, in the ideal case, the *CCA* system can be in any state.

We propose that a controlled-R gate can be implemented through the following operations:

- STEP 1: Apply a microwave pulse (with a frequency ω_{01} and $\phi = -\pi/2$) in the target qubit for $\Omega_{01}t_1 = \pi/4$. Such a single qubit operation create a superposition state,

$$|0\rangle_2 \to \frac{1}{\sqrt{2}}\big(|0\rangle_2 + |1\rangle_2\big),$$
$$|1\rangle_2 \to \frac{1}{\sqrt{2}}\big(-|0\rangle_2 + |1\rangle_2\big). \tag{12}$$

- STEP 2: Apply an optical pulse (with a frequency ω_{1i} and $\phi = \pi/2$) in both qubits for $\Omega_{1i}t_2 = \pi/4$. After the pulse, we have $|1\rangle_j \to |-\rangle_j$.
- STEP 3: Turn on the atom-field interaction as described in Section 2.1 for $t_3 = \tau = 2\pi/\delta$ and $\Omega_{1i} = 100\delta$

$$|0\rangle_1|0\rangle_2|0\rangle_c \to |0\rangle_1|0\rangle_2|0\rangle_c,$$
$$|0\rangle_1|-\rangle_2|0\rangle_c \to e^{i\lambda\tau}|0\rangle_1|-\rangle_2|0\rangle_c,$$
$$|-\rangle_1|0\rangle_2|0\rangle_c \to e^{i\lambda\tau}|-\rangle_1|0\rangle_2|0\rangle_c,$$
$$|-\rangle_1|-\rangle_2|0\rangle_c \to |-\rangle_1|-\rangle_2|0\rangle_c. \tag{13}$$

- STEP 4: Repeat the operation of step 2 which now results in $|-\rangle_j \to -|i\rangle_j$.
- STEP 5: Apply an optical pulse (with a frequency ω_{1i} and $\phi = -\lambda\tau - \pi/2$) in both qubits for $\Omega_{1i}t_4 = \pi/2$ which results

$$|i\rangle_j \to -e^{-i\lambda\tau}|1\rangle_j. \tag{14}$$

- STEP 6: Repeat the operation of step 1 but with $\phi = \pi/2$.

The states of the two qubits (atoms) after such a procedure are

$$|0\rangle_1|0\rangle_2 \to |0\rangle_1|0\rangle_2,$$
$$|0\rangle_1|1\rangle_2 \to |0\rangle_1|1\rangle_2,$$
$$|1\rangle_1|0\rangle_2 \to |1\rangle_1 e^{-i\Theta(g)}\big(\cos\Theta(g)|0\rangle_2 - i\sin\Theta(g)|1\rangle_2\big),$$
$$|1\rangle_1|1\rangle_2 \to |1\rangle_1 e^{-i\Theta(g)}\big(\cos\Theta(g)|1\rangle_2 - i\sin\Theta(g)|0\rangle_2\big), \tag{15}$$

with

$$\Theta(g) = \lambda\tau = \frac{g^2\pi}{4\delta^2}, \tag{16}$$

which implies that if and only if the control qubit is in the state $|1\rangle$ a unitary transformation (that can be appropriately chosen varying the atom-field coupling strength) is performed on the target qubit and nothing happens otherwise. Moreover, such an operation is implemented in a constant time given by

$$t_{tot} = \sum_{j=1}^{6} t_j = \frac{\pi}{2\Omega_{01}} + \frac{\pi}{\Omega_{1i}} + \frac{2\pi}{\delta} \tag{17}$$

that depends only on experimental parameters.

4. Imperfections in the gate implementation

The relevant cavity parameters in cQED experiments are the atom-field coupling rate g, the cavity decay rate κ and the atomic spontaneous emission rate γ.

In order to maximize g and to enter as far as possible into the strong coupling regime, one has to make cavities with a small mode volume, that is, reduce the mirror spacing in Fabry–Perot cavities. However, doing so it also increases the cavity decay rate and typically one has $\kappa > \gamma$ [21]. For this reason, we will only consider the dissipation effects due the cavity decay. Although it is important to note that only the spontaneous emission of the intermediate state $|i\rangle$ would be relevant in such a dissipative channel.

In our procedure only during step 3 the mode in each cavity is populated. So, including the cavity decay, Eq. (6) can be written as

$$H'_{eff} = \frac{\hbar g}{2\sqrt{2}}\big(\hat{c}e^{-i\delta t} + \hat{c}^\dagger e^{i\delta t}\big)(\tilde{\sigma}_{z1} - \tilde{\sigma}_{z2}) - i\frac{\hbar\kappa}{2}\hat{c}^\dagger\hat{c}, \tag{18}$$

where κ is the cavity decay rate (for both), and we have neglected the fast oscillating normal modes. Considering that $\kappa \ll \delta$ (to make sure that no quantum jump happens during the procedure), the evolution operator for Hamiltonian (18) can be written as

$$U' = e^{-iA(t)(\tilde{\sigma}_{z1}-\tilde{\sigma}_{z2})^2} e^{-iB(t)\hat{c}(\tilde{\sigma}_{z1}-\tilde{\sigma}_{z2})} \times e^{-iC(t)\hat{c}^\dagger(\tilde{\sigma}_{z1}-\tilde{\sigma}_{z2})} e^{-\frac{\kappa t}{2}\hat{c}^\dagger\hat{c}}, \tag{19}$$

with now

$$A(t) = -i\frac{g^2}{4}\frac{1}{(\kappa+2i\delta)}\left[t - \left(\frac{2e^{(\kappa+2i\delta)t/2} - 2}{\kappa+2i\delta}\right)\right],$$
$$B(t) = \frac{e^{\kappa t/2} - e^{-i\delta t}}{\sqrt{2}(2i\delta+\kappa)},$$
$$C(t) = \frac{-e^{-\kappa t/2} + e^{i\delta t}}{\sqrt{2}(2i\delta+\kappa)}. \tag{20}$$

In addition, assuming that the cavities and the fiber are initially in the vacuum state (the operation is no longer insensitive to the initial field state), the evolution of the system due to step 3 is

$$|0\rangle_1|0\rangle_2|0\rangle_c \to |0\rangle_1|0\rangle_2|0\rangle_c,$$
$$|0\rangle_1|-\rangle_2|0\rangle_c \to e^{iP_\kappa}e^{-D_\kappa}|0\rangle_1|-\rangle_2\big|-iC(\tau)\big\rangle_c,$$
$$|-\rangle_1|0\rangle_2|0\rangle_c \to e^{iP_\kappa}e^{-D_\kappa}|-\rangle_1|0\rangle_2\big|iC(\tau)\big\rangle_c,$$
$$|-\rangle_1|-\rangle_2|0\rangle_c \to |-\rangle_1|-\rangle_2|0\rangle_c, \tag{21}$$

with the geometric phase given by

$$P_\kappa = \mathrm{Im}\left[-iA(\tau) - B(\tau)C(\tau) + \frac{|C(\tau)|^2}{2}\right]. \tag{22}$$

Moreover, different from the ideal case, we have the corresponding amplitude damping factor

$$D_\kappa = -\mathrm{Re}\left[-iA(\tau) - B(\tau)C(\tau) + \frac{|C(\tau)|^2}{2}\right]. \tag{23}$$

Without cavity decay ($\kappa = 0$) we resume to the ideal case in which $P_0 = \lambda\tau$, $D_0 = 0$ and $|\alpha| = |C(\tau)| = 0$. On the other hand,

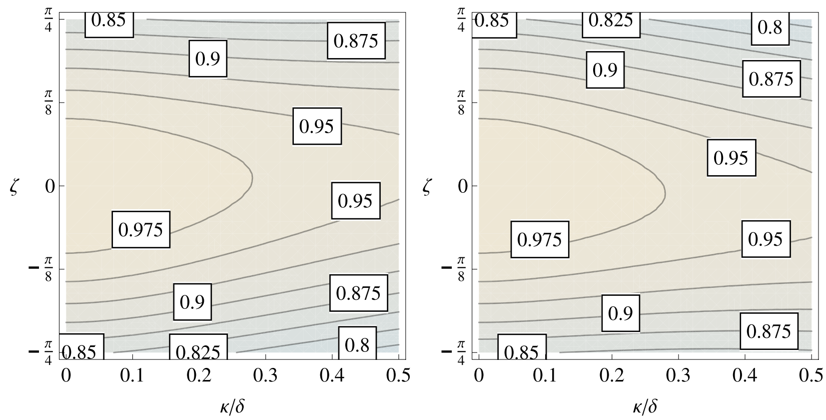

Fig. 2. Fidelity of the two-qubit controlled-R gate for $\Theta = \pi/4$ ($g = \delta$). In the left for the initial states $\{|00\rangle, |01\rangle\}$ (F_0) and in the right for $\{|10\rangle, |11\rangle\}$ (F_1).

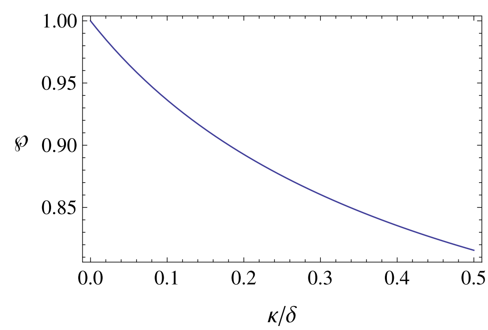

Fig. 3. Success probability $\wp$ of the two-qubit controlled-R gate for $\Theta = \pi/4$ ($g = \delta$).

one can see that we no longer have an exactly closed path in the presence of cavity decay. In such a case, a small decay rate follows a heating effect in the cavities [22] and a residual non-vacuum field state entangled with the atomic states can still be observed after step 3. Nevertheless, for $g = \delta$ ($\Theta = \pi/4$) and $\kappa = 0.05\delta$ (in agreement with current experimental values [21]) we have $|\alpha| = 0.05$ which corresponds to an approximately closed path in the parameter phase space.

Apart the dissipation issues in step 3, perhaps the most trick part of the procedure is step 5. In this stage we must set the exactly phase field to implement the controlled-R gate. Now we will analyze how a mismatch field phase in step 5 affects the fidelity of the quantum gate.

Assume that the optical pulse applied in step 5 has a small variation in the phase such that

$$|i\rangle_j \to -e^{-i(\Theta-\zeta)}|1\rangle_j, \tag{24}$$

the fidelity of each output of the gate implementation, also considering the dissipation effects, is

$$F_0 = \frac{1}{2+2e^{-2D_\kappa}}\big[1 + e^{-2D_\kappa - |C(\tau)|^2} + 2e^{-D_\kappa - |C(\tau)|^2/2}\cos(P_\kappa - \Theta + \zeta)\big], \tag{25}$$

$$F_1 = \frac{1}{2+2e^{-2D_\kappa}}\big[1 + e^{-2D_\kappa - |C(\tau)|^2} + 2e^{-D_\kappa - |C(\tau)|^2/2}\cos(P_\kappa - \Theta - \zeta)\big], \tag{26}$$

in which the subscript refers to the control qubit, and success probability (i.e. no-jump probability) in both cases [23]

$$\wp = \frac{1 + e^{-2D_\kappa}}{2}. \tag{27}$$

Let us consider the realization of a two-qubit controlled-R gate for $\Theta = \pi/4$ ($g = \delta$). In Fig. 2 one can see that the present proposal works very well even in presence of a small cavity decay rate (which is currently achievable in experiments) and an error in the phase field of step 5. Deterministically controlling the atom-field coupling strength we can adjust the value of Θ and implement an arbitrary two-qubit controlled-R gate with a fixed gating time, high-fidelity and high success probability (see Fig. 3).

Choosing experimental parameters such as $\Omega_{01} = 10\delta$, $\Omega_{1i} = 100\delta$ and $\delta \cong 1$ GHz we can achieve a total operation time of $t_{tot} = 3.3$ ns (disregarding delays between the steps), which is considerably small compared to previous proposals [24,25].

Indeed there are other imperfection sources in the gate implementation such as: (i) error in pulse duration, (ii) delay between the steps (which is an issue if one considers the dissipation effects), (iii) the simultaneous interaction in step 3 (only) and (iv) the spontaneous emission of the excited atomic states.

However, we believe that the sources of imperfections analyzed in this work are the most significant ones.

5. Discussion and conclusion

In summary we have proposed a new multi-step protocol for implementing a controlled-R gate for two atoms in separate cavities connected by an optical fiber. The *CCA* system acts as a bus and the protocol can be implemented between distant qubits [26].

In contrast to previous proposal [24,25], our scheme is implemented in a constant gating time and, with an adjustable qubit-bus coupling (atom-resonator), one can specify a particular rotation R on the target qubit.

Two important issues deserved considerations in our work: (i) the influence of dissipation due cavity decay in step 3; (ii) an analysis of errors during the execution of the protocol in step 5. Even with such imperfections, we can still obtain a high-fidelity two-qubit controlled-R gate implementation with a high probability of success.

In our view, the most promising candidate to implement our proposal is made combining fiber-based Fabry–Perot cavities [21] with atom-chip technology [27]. In such a system, each atom (or atom cloud) can be strongly coupled to the cavity mode and positioned deterministically anywhere within the cavity that gives rise to a controlled, tunable coupling rate [28] with a high single-atom cooperativity factor of $g^2/2\kappa\gamma = 145$.

In fact, we just focus our analysis on the case of $\kappa \ll \delta$ which, as we have already mentioned, is achievable in current experimental techniques. For higher decay rates, the degree of non-hermiticity increases and a more rigorous treatment is required [29,30].

With a collective coupling, in which an ensemble of N atoms couples to the cavity mode as a single one, the atom-field coupling strength g increases by a factor of $\sqrt{N}$ which brings it up to tens of GHz and improves even more the cooperativity factor.

Another important consideration that we have not addressed deeply in our work is the condition $\nu \gg g$. In such a case we can neglect the fast oscillating normal modes in Eq. (5). In fact, we believe that with the technological advances one will be able to set ν greater than g easily.

Even without a tunable constant coupling g, one can still implement a controlled-R gate by varying the interaction time in step 3 ($t_3 = \tau_n = 2\pi n/\delta$) but then the set of available rotations is only a discrete set.

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