Chinese Journal of Aeronautics, (2014),27(4): 821–828

Chinese Society of Aeronautics and Astronautics
& Beihang University

Chinese Journal of Aeronautics

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FULL LENGTH ARTICLE

Performance deterioration modeling and optimal preventive maintenance strategy under scheduled servicing subject to mission time

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Received 16 June 2013; revised 17 February 2014; accepted 17 April 2014
Available online 23 June 2014

KEYWORDS

Dependability;
Imperfect maintenance;
Preventive maintenance;
Reliability;
Scheduled servicing

Abstract Servicing is applied periodically in practice with the aim of restoring the system state and prolonging the lifetime. It is generally seen as an imperfect maintenance action which has a chief influence on the maintenance strategy. In order to model the maintenance effect of servicing, this study analyzes the deterioration characteristics of system under scheduled servicing. And then the deterioration model is established from the failure mechanism by compound Poisson process. On the basis of the system damage value and failure mechanism, the failure rate refresh factor is proposed to describe the maintenance effect of servicing. A maintenance strategy is developed which combines the benefits of scheduled servicing and preventive maintenance. Then the optimization model is given to determine the optimal servicing period and preventive maintenance time, with an objective to minimize the system expected life-cycle cost per unit time and a constraint on system survival probability for the duration of mission time. Subject to mission time, it can control the ability of accomplishing the mission at any time so as to ensure the high dependability. An example of water pump rotor relating to scheduled servicing is introduced to illustrate the failure rate refresh factor and the proposed maintenance strategy. Compared with traditional methods, the numerical results show that the failure rate refresh factor can describe the maintenance effect of servicing more intuitively and objectively. It also demonstrates that this maintenance strategy can prolong the lifetime, reduce the total lifetime maintenance cost and guarantee the dependability of system.

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Peer review under responsibility of Editorial Committee of CJA.

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1. Introduction

Maintenance as a necessary part for some important and expensive systems can keep the system in good condition or restore it to a state in which it can perform its required

http://dx.doi.org/10.1016/j.cja.2014.06.002

function. Especially for the aircraft, because of its often fatal and costly consequences of failure, one of the most important goals of the maintenance is to ensure safety, characterized by reliability or availability indexes generally.[1,2] With the aim of postponing the reliability decrease speed and restoring system's operational performance, the aeronautic facility is usually subject to periodical servicing during its life cycle, such as engine tune-up and aeronautical instrument emendation. As a result, the preventive maintenance strategy considering scheduled servicing is widely used to decline failures and reduce high maintenance cost.

In the past decades, the analysis and modeling of imperfect operations have been extensively discussed in Refs.[3–6] There are many different methods to model imperfect maintenance. According to different points of view, the methods for treating imperfect maintenance can be classified into two categories as follows. In one category, parameters are used to describe imperfect degree from various reliability indexes. For example, the improvement factor method[3,4] is in terms of failure rate, the virtual age method[5,6] is in terms of lifetime. These methods have the advantages of modeling imperfect degree intuitively and simplifying the process. So there are many applications in engineering practice. Because of intuitional supposition, this kind of viewpoint also has some subjectivity. It is generally relies on an expert judgment to estimate the improvement factor or virtual age factor in practice. The other category can be defined as shock model which treats as imperfect from failure mechanism.[7,8] Compared with the parameter model proposed above, it is more objective, because the existing researches focus on microcosmic analysis and shock types. The shock model has complicate expressions that imperfect degree cannot be comprehend intuitively, such as δ-shock model.[9] An extensive review and in-depth analysis of imperfect maintenance model exist in much literature.[10–14]

In order to achieve different purposes, there are many studies on the subject of the preventive maintenance strategy. Those studies take into consideration several criteria such as cost, risk (safety), reliability, availability, or a combination of the above-stated criteria. Chien and Chen[15] used the cost-effectiveness to determine the preventive replacement strategy which describes the maintenance degree by parameter model. Based on the improvement factor, Zhang and Gao[16] proposed the maintenance optimization model considering reliability requirement. Lina et al.[17] studied the maintenance method for comparing the effect of different maintenance strategies on system reliability and cost. In the above literature no consideration is, however, given to the case of mission time or dependability. To some extent, the operators prefer reliability requirements to be based on mission. Therefore, this paper takes into account the mission time effect on maintenance strategy which is useful to extend the scope of application.

The main objective of this article is to give an approach to determine the optimal preventive maintenance strategy based on servicing analysis and mission time. It is well-known that servicing is an imperfect maintenance action that just restores the system damage and cannot eliminate the system failure cause totally. Based on the system failure mechanism, Sections 2 and 3 are devoted to present an improved method to model the system damage process and describe the maintenance effect of servicing. And the purpose of the improved method is to describe intuitively and objectively the damage process of system on scheduled servicing and lays the foundation for the maintenance strategy research. It deals with preventive maintenance strategy modeling considering scheduled servicing and mission time in Section 4. Therefore, the results are helpful in determining the servicing period and preventive maintenance time with the aim of ensuring the system dependability and decreasing the maintenance cost. A numerical example is presented to illustrate the method of maintenance effect of servicing and the proposed preventive maintenance strategy in Section 5. Finally, there are some conclusions in Section 6.

2. Scheduled servicing process analysis

It is well-known that the system damage can be modeled as a stochastic cumulative process with increasing sample paths, such as fatigue, creep, corrosion and wear.[18] If there is no maintenance, the system damage gradually cumulates. A failure occurs if the total damage exceeds a critical threshold L_s. The damage process can be described in Fig. 1(a), in which $X(t)$ is defined as the total damage at time t.

From Fig. 1(a), it shows a continuous and monotone damage process.

For some important and expensive system, the daily servicing is usually used to restore the system damage to the prescribed level that is generally defined as zero value. Fig. 1(b) shows the damage process considering scheduled servicing, in which Δt is the servicing period.

It can be seen from Fig. 1(b) that there is significant discontinuity at any servicing time, because the servicing restores the system damage to be zero. Fig. 1(b) also shows that the system damage has a progressive increase considering schedule servicing. Furthermore, the system damage shows substantial increases at some time. For the system reliability, the damage change rule considering scheduled servicing can make it decline slowly at the beginning time, but after a moment it decreases very fleetly, just because servicing restores the system damage

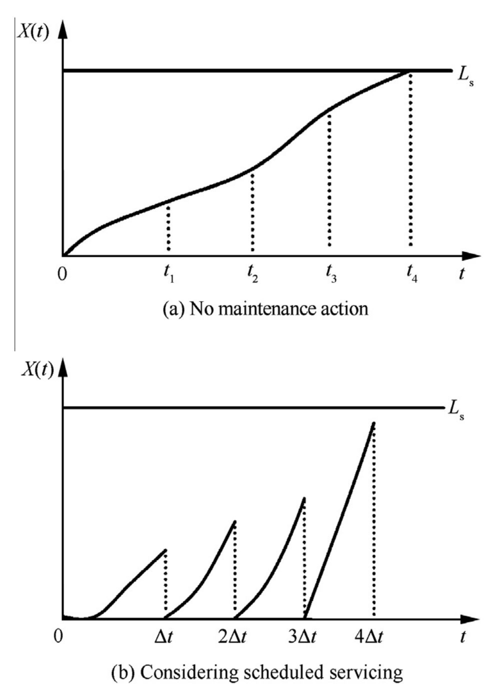

Fig. 1 System damage process.

zero does not mean that servicing can be seen as a perfect maintenance. Servicing does not totally change the system deterioration speed, so the scope of damage increases gradually. Then, servicing is an imperfect maintenance action which restores the system operating state to some extent between as good as new and as bad as old and has a positive effect on the system damage. In order to analyze the maintenance effect of servicing and optimize the maintenance strategy, the damage process must be modeled based on the two characteristics above.

3. Modeling deterioration and scheduled servicing

3.1. Deterioration model

Consider the system which suffers a random damage caused by stochastic shock. Let $N_{t,t+s}$ be the total number of shocks occurring at the time interval $(t,\ t+s]$. Although the shock process $N_{t,t+s}$ can be modeled as a general stochastic process, it is supposed here as a non-homogeneous Poisson process with intensity $m(t,t+s)=\int_t^{t+s} at^{b-1}\mathrm{d}t$. a and b are the parameters of the intensity function. Because of this, the definition of shock process is given as

$$P(N_{t,t+s}=n)=\frac{[a(t+s)^b-at^b]}{b^n n!}\exp\left\{-\frac{1}{b}[a(t+s)^b-at^b]\right\} \quad (n=1,2,\cdots) \tag{1}$$

where n is a positive integer.

The damage after each shock is denoted by Y. The extent of the damage has common relations with the system material and process of manufacture.[19] Then, Y_1, $Y_2,\cdots$ is independent and identically normal distribution, showing as

$$Y_i \sim N(\mu,\sigma) \quad (i=1,2,\cdots) \tag{2}$$

where μ and σ can be seen as the system performance parameters.

The shock process $N_{t,t+s}$ and the cumulative damage are assumed to be independent. The total damage in the time interval $(t,\ t+s]$, $X_{t,t+s}$, can be described as

$$X_{t,t+s}=\sum_{i=1}^{N_{t,t+s}} Y_i$$

The stochastic process $X_{t,t+s}$ is said to be a compound Poisson process. It follows that

$$P(X_{t,t+s}<x)=\sum_{i=1}^{\infty}P(N_{t,t+s}=i)P\left(\sum_{j=1}^{i}Y_j<x\right) \tag{3}$$

where x is a rational number which less than critical threshold L_s

It can be noted that the total damage distribution can be defined as the combination of infinite normal distribution. Based on the characteristic function of $X_{t,t+s}$ and the moment generating function of $X_{t,t+s}$,[20] the expected value and variance of $X_{t,t+s}$ are obtained:

$$\begin{cases} E(X_{t,t+s})=\dfrac{a\mu}{b}[(t+s)^b-t^b] \\ D(X_{t,t+s})=\dfrac{a(\mu^2+\sigma^2)}{b}[(t+s)^b-t^b] \end{cases} \tag{4}$$

It is too complex to relatively easy to evaluate Eq. (3) numerically. With the aim of easy calculation and description, the stochastic process $X_{t,t+s}$ can be almost a normal distribution by using central limit theorem. Thus $X_{t,t+s}$ can be rewritten as

$$X_{t,t+s}\sim N(\mu_{t,t+s},\sigma^2_{t,t+s})$$

then based on Eq. (4) above, $\mu_{t,t+s}=E(X_{t,t+s})$ and $\sigma^2_{t,t+s}=D(X_{t,t+s})$.

The servicing is a limited maintenance action, such as lubrication, tune-up and clean. So the servicing does not change the failure mechanism. It is supposed that the servicing affects only the damage process, not the shock process. That is to say, the shock intensity is unchanged after each servicing, but the total damage is restored to zero at the servicing time. Hence, if the system has a constant servicing period Δt, the total damage $X_{i\Delta t,t}$ in an arbitrary time $t\in(i\Delta t,\ i\Delta t+\Delta t]$ is

$$P(X_{\Delta t,t}<x)=\begin{cases}\Phi\left(\dfrac{x-\mu_{0,t}}{\sigma_{0,t}}\right) & i=0\\ \Phi\left(\dfrac{x-\mu_{\Delta t,t}}{\sigma_{\Delta t,t}}\right) & i=1\\ \vdots & \\ \Phi\left(\dfrac{x-\mu_{k\Delta t,t}}{\sigma_{k\Delta t,t}}\right) & i=k\end{cases} \tag{5}$$

where $\Phi(\bullet)$ is the cumulative distribution function of standard normal distribution, $k\in\mathbf{N}^+$.

With the given critical threshold L_s, the system reliability function $R(t)$ at time t can be calculated as

$$\begin{aligned} R(t) &= P(X_{i\Delta t,t}<L_s, X_{0,i\Delta t}<L_s, X_{\Delta t,i\Delta t}<L_s,\cdots,X_{(i-1)\Delta t,i\Delta t}<L_s) \\ &= P(X_{i\Delta t,t}<L_s)\prod_{j=0}^{i}P(X_{j\Delta t,(j+1)\Delta t}<L_s) \\ &= \Phi\left(\frac{L_s-\mu_{i\Delta t,t}}{\sigma_{i\Delta t,t}}\right)\prod_{j=0}^{i}\Phi\left(\frac{L_s-\mu_{j\Delta t,(j+1)\Delta t}}{\sigma_{j\Delta t,(j+1)\Delta t}}\right)\end{aligned} \tag{6}$$

In particular, when $i=0$, Eq. (6) is

$$R(t)=P(X_{0,t}<L_s)=\Phi\left(\frac{L_s-\mu_{0,t}}{\sigma_{0,t}}\right)$$

Noting that Eq. (6) is conditional probability, it proves that the system reliability function of scheduled servicing has relations with both working time t and servicing period Δt. So it makes a great difference from the one without servicing.

3.2. Maintenance effect of servicing

According to the above statistical analysis, this study focuses on the maintenance effect of servicing. The system failure rate $\lambda(t)$ at time t is defined as

$$\lambda(t)=\lim_{\varepsilon\to 0}\frac{R(t)-R(t+\varepsilon)}{R(t)\varepsilon} \tag{7}$$

where ε is the time of system continues to work.

For a system requiring scheduled servicing, the parameters of reliability function $R(t)$ change with working time and servicing period from Eq. (6). The most important is that the left-hand limit does not equal right-hand limit at any servicing time because of the maintenance effect of servicing. These mean that the failure rate of scheduled servicing is not a continuous function of time. It is difficult to analyze the failure rate by Eq. (7).

To describe the failure rate compactly and intuitively, the average failure rate is given by considering fully servicing. Let $\varepsilon=\Delta t$, it can be derived as

$$\lambda(t) = \frac{R(t) - R(t+\Delta t)}{R(t)\Delta t} \tag{8}$$

Eq. (8) has an engineering signification, which is used to describe the failure proportion per time in the time interval $(t, t + \Delta t]$; the system still works at time t.

Therefore, substituting Eq. (6) into Eq. (8), the average failure rate is

$$\lambda^{+}(i\Delta t) = \frac{1 - \Phi\left(\frac{L_{\mathrm{s}} - \mu_{i\Delta t,(i+1)\Delta t}}{\sigma_{i\Delta t,(i+1)\Delta t}}\right)}{\Delta t} \tag{9}$$

where $\lambda^{+}(i\Delta t)$ is the average failure rate that system is serviced at time $i\Delta t$.

If there is no servicing at time $i\Delta t$, the average failure rate $\lambda^{-}(i\Delta t)$ is

$$\lambda^{-}(i\Delta t) = \frac{R(i\Delta t) - R^{-}(i\Delta t + \Delta t)}{R(i\Delta t)\Delta t} = \frac{\Phi\left(\frac{L_{\mathrm{s}} - \mu_{(i-1)\Delta t,i\Delta t}}{\sigma_{(i-1)\Delta t,i\Delta t}}\right) - \Phi\left(\frac{L_{\mathrm{s}} - \mu_{(i-1)\Delta t,(i+1)\Delta t}}{\sigma_{(i-1)\Delta t,(i+1)\Delta t}}\right)}{\Phi\left(\frac{L_{\mathrm{s}} - \mu_{(i-1)\Delta t,i\Delta t}}{\sigma_{(i-1)\Delta t,i\Delta t}}\right)\Delta t} \tag{10}$$

where $R^{-}(i\Delta t + \Delta t)$ is the reliability function that system is not serviced at time $i\Delta t$.

Evidently, there is a progressive decline in the average failure rate after each servicing, i.e., $\lambda^{-}(i\Delta t) > \lambda^{+}(i\Delta t)$. To analyze the maintenance effect of servicing, the failure rate refresh factor β_i is given as

$$\beta_i = \frac{\lambda^{-}(i\Delta t) - \lambda^{+}(i\Delta t)}{\lambda^{-}(i\Delta t)} \tag{11}$$

where $\beta_i \in [0, 1]$. In particular, when $\beta_i = 0$, the average failure rate is unchanged after servicing, i.e., the system is returned to the "as bad as old" state. Contrariwise, when $\beta_i = 1$, the average failure rate is restored to zero after servicing, i.e., the system is returned to the "as good as new" state. Noting that the failure refresh factor is a function of servicing time, it can represent intuitively the maintenance effect of servicing.

4. Evaluation of preventive maintenance strategy

4.1. Preventive maintenance strategy description

In this section, preventive maintenance strategy considering scheduled servicing is proposed. The maintenance model is studied under the following assumptions:

(1) It is supposed that the system is periodically serviced at an interval Δt.

(2) At T_{s} time, the preventive maintenance action is applied to the system before the total damage reaches the critical threshold L_{s}.

(3) The failure is reported immediately as soon as the total damage exceeds the critical threshold L_{s} and the corrective maintenance action is required consequently.

(4) The system can be restored to a state "as good as new" after a preventive or corrective maintenance action.

4.2. Maintenance strategy analysis considering mission time and cost

According to the proposed maintenance strategy, the life cycle of the system can be defined as the duration beginning with a renewal and ending with a preventive or corrective maintenance action. The engineers hope the system can have high dependability during the life cycle. That is to say, the system should keep the operating state for the duration of mission time. To control the level of dependability at all times, a limit α is set to the work time over the life cycle of the system. It can be determined as

$$P(X(t+T_{\mathrm{m}}) < L_{\mathrm{s}} | X(t) < L_{\mathrm{s}}) = \frac{R(t+T_{\mathrm{m}})}{R(t)} \geqslant \alpha \tag{12}$$

where T_{m} is defined as the mission time. Evidently, Eq. (12) requires the probability that the system continues to carry out mission cannot be less than α at any time.

As stated before, the servicing is an imperfect maintenance. Thus, there exists a finite time T_{s} that cannot meet Eq. (12). It has to take preventive maintenance action that restores the system "as good as new" at that time. The preventive maintenance time can be confirmed with the given mission maintenance time T_{m} and limit α.

The life cycle of the system will be ended in two cases. In Case 1, the system survives before the preventive maintenance time. The preventive maintenance action is implemented at time T_{s}. The expected length of the life cycle in this case, $E_1(t)$, is calculated as

$$E_1(t) = \begin{cases} T_{\mathrm{s}} P(X_{k\Delta t,T_{\mathrm{s}}} < L_{\mathrm{s}}) \prod_{i=1}^{k} P(X_{(i-1)\Delta t,i\Delta t} < L_{\mathrm{s}}) & T_{\mathrm{s}} - k\Delta t \geqslant T_{\mathrm{m}} \\ k\Delta t \prod_{i=1}^{k} P(X_{(i-1)\Delta t,i\Delta t} < L_{\mathrm{s}}) & T_{\mathrm{s}} - k\Delta t < T_{\mathrm{m}} \end{cases}$$

where $k = \left[\frac{T_{\mathrm{s}}}{\Delta t}\right]$.

In Case 2, the system fails in an arbitrary time before the preventive maintenance time. The corrective maintenance action is implemented immediately after the failure. The expected length of the life cycle in this case, $E_2(t)$, is calculated as

$$E_2(t) = \begin{cases} \int_0^{\Delta t} t\mathrm{d}F(t) + \sum_{i=1}^{k-1} \int_{i\Delta t}^{(i+1)\Delta t} t\mathrm{d}F(t) \prod_{j=1}^{i} P(X_{(j-1)\Delta t,j\Delta t} < L_{\mathrm{s}}) + \int_{k\Delta t}^{T_{\mathrm{s}}} t\mathrm{d}F(t) \prod_{j=1}^{k} P(X_{(j-1)\Delta t,j\Delta t} < L_{\mathrm{s}}) & T_{\mathrm{s}} - k\Delta t \geqslant T_{\mathrm{m}} \\ \int_0^{\Delta t} t\mathrm{d}F(t) + \sum_{i=1}^{k-1} \int_{i\Delta t}^{(i+1)\Delta t} t\mathrm{d}F(t) \prod_{j=1}^{i} P(X_{(j-1)\Delta t,j\Delta t} < L_{\mathrm{s}}) & T_{\mathrm{s}} - k\Delta t < T_{\mathrm{m}} \end{cases}$$

where $F(t) = P(X_{i\Delta t,t} < L_{\mathrm{s}})$, which is determined by Eq. (5).

As mentioned above, the expected available time of the system in a life cycle can be obtained by

$$E(t) = E_1(t) + E_2(t) \tag{13}$$

The total life cycle cost of the system includes servicing cost, preventive maintenance cost and corrective maintenance cost, denoted by C_s, C_p and C_m, respectively. In Case 1, the expected cost incurred from preventive maintenance action in a life cycle is

$$E_1(C) = \begin{cases} (C_p + kC_s)P(X_{k\Delta t, T_s} < L_s)\prod_{i=1}^{k} P(X_{(i-1)\Delta t, i\Delta t} < L_s) \\ \quad T_s - k\Delta t \geqslant T_m \\ [C_p + (k-1)C_s]\prod_{i=1}^{k} P(X_{(i-1)\Delta t, i\Delta t} < L_s) \\ \quad T_s - k\Delta t < T_m \end{cases}$$

In Case 2, the expected cost incurred from corrective maintenance action in a life cycle is

$$E_2(C) = \begin{cases} C_m\int_0^{\Delta t} dF(t) + \sum_{i=1}^{k-1}(C_m + iC_s)\int_{i\Delta t}^{(i+1)\Delta t} dF(t)\prod_{j=1}^{i} P(X_{(j-1)\Delta t, j\Delta t} < L_s) + (C_m + kC_s)\int_{k\Delta t}^{T_s} dF(t)\prod_{j=1}^{k} P(X_{(j-1)\Delta t, j\Delta t} < L_s) & T_s - k\Delta t \geqslant T_m \\ C_m\int_0^{\Delta t} dF(t) + \sum_{i=1}^{k-1}[C_m + (i-1)C_s]\int_{i\Delta t}^{(i+1)\Delta t} dF(t)\prod_{j=1}^{i} P(X_{(j-1)\Delta t, j\Delta t} < L_s) & T_s - k\Delta t < T_m \end{cases}$$

Hence, the expected total life cycle cost of the system can be inferred as

$$E(C) = E_1(C) + E_2(C) \tag{14}$$

4.3. Optimization model

In the proposed preventive maintenance strategy, an important problem is how to optimize servicing period Δt and the preventive maintenance time T_s. The expected life-cycle cost per unit time $C(t)$ has been considered as an objective for this optimization. Therefore, the optimization model subject to the constraint of dependability can be given as

$$\begin{aligned} &\min\ C(t) = \frac{E(C)}{E(T)} \\ &\text{s.t.}\ P(X(t+T_m) < L_s | X(t) < L_s) = \frac{R(t+T_m)}{R(t)} \geqslant \alpha \end{aligned} \tag{15}$$

There are many algorithms to compute the optimal maintenance strategy values $(\Delta t^*, T_s^*)$. In order to make maintenance strategy implement practically, it can search the servicing period Δt in the integral range to get the corresponding objective $C(t)$. It is easy to compare the calculated $C(t)$ with the aim of finding the minimum value $C(t)$. In the end, the values of Δt and T_s corresponding to minimum value $C(t)$ are the optimal maintenance strategy values.

5. Example application

5.1. Background

In this section, an example application of the model is presented. The work process of water pump rotor can be seen as a cumulative damage process. The impeller is the key part for water pump rotor. Its wear abrasion extend has an impact on the water pump rotor. The water pump rotor is on scheduled servicing in order to lubricate the impeller. Based on the failure time of the rotor and the servicing period Δt = 10 months, the parameters of composite Poisson process are fitted as follows. The number of shock $N(t)$ is modeled as a non-homogeneous Poisson process with intensity $m(0, t) = 0.12t^{1.75}$. The damage after each shock Y_i is modeled as a normal distribution with mean $\mu = 4.5 \times 10^{-4}$ and variance $\sigma^2 = 10^{-8}$. The critical threshold is chosen as $L_s = 0.04$. Based on Eq. (6), the reliability function of water pump rotor $R(t)$ at time t at $(0, \infty)$ is

$$R(t) = \Phi\left\{\frac{0.04 - 0.54[t^{1.75} - (10i-10)^{1.75}]}{1.6 \times 10^{-4}[t^{1.75} - (10i-10)^{1.75}]^{0.5}}\right\} \prod_{j=1}^{i-1}\Phi\left\{\frac{0.04 - 30.36[j^{1.75} - (j-1)^{1.75}]}{11.97 \times 10^{-4}[j^{1.75} - (j-1)^{1.75}]^{0.5}}\right\} \quad t \in (10i-10, 10i] \tag{16}$$

In Fig. 2, the reliability of water pump rotor obtained from Eq. (16) and real test data are shown considering the scheduled servicing.

The result shows that the reliability analytic values at servicing time are fairly close to the test value. The model seems to fit well for the rotor reliability. It can be noticed that after each servicing the rotor reliability values remain approximately steady for a while. This is due to the delay effect on reliability decrease speed from the scheduled servicing. Fig. 2 also shows that at some time the rotor reliability values decrease rapidly. For example, the reliability analytic values change from 0.827 to 0.595 when the working time increases from 137 months to 140 months. This can be explained by imperfect maintenance effect of servicing. The delay effect of servicing declines gradually firstly, when the working time increases, but after a moment it rapidly becomes worse. In practice,

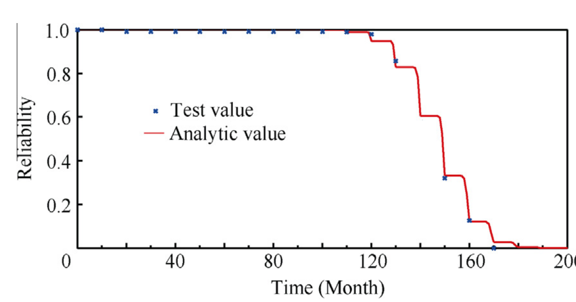

Fig. 2 Comparison of reliability between test and analytic method.

the failure number of rotor is few at the beginning of working time, but it will be apparently larger at some time. The reliability cure of the water rotor proves that the model in this paper meets more practical operational situations.

5.2. Analysis of failure rate refresh factor

To illustrate the maintenance effect of servicing, consider the water pump rotor described in Section 5.1. Failure rate refresh factors of each servicing time obtained from Eq. (11) are presented in the Table 1.

From Table 1, it can be seen that failure rate refresh factors of the first ten servicing times are 100% approximately. The servicing restores the rotor operating state to "as good as new" during early servicing time (from 10 months to 110 months in Table 1). This is because the failure rate of rotor is lower itself in early time and the incipient servicing with a better maintenance effect is able to restore the rotor operating state. Meanwhile, the reliability values before 110 months are approximately 1 in the Fig. 2. It also shows that the incipient servicing can restore the rotor "as good as new". In practice, the servicing can make the rotor work well and low the probability of failure. So this case matches the operations.

With the working time and servicing number increasing, it can be observed from Table 1 that the failure rate refresh factor decreases gradually (i.e., the maintenance effect becomes worse). Obviously, the servicing changes the rotor failure rate to some newer but not all the way to zero (not new). The failure rate refresh factor can describe the trend that maintenance effect changes with the working time and servicing number. Some traditional methods rely on an expert judgment to estimate the maintenance effect and the estimation value is commonly fixed.[5,6] Compared with these methods, the failure rate refresh factor proposed in this paper is on the basis of the system damage value and failure mechanism, which can represent the maintenance effect trend of servicing intuitively and objectively.

5.3. Maintenance strategy optimization

The section presents the water pump rotor numerical example to obtain the optimal preventive maintenance strategy proposed in Section 4.1. Let the servicing cost C_s = \$140, the preventive maintenance cost C_p = \$1000 and the corrective maintenance cost C_m = \$2000. It is assumed that the mission time of rotor T_m is 4 months. The probability that the rotor survive for the duration of T_m = 4 months is required to be not less than 0.8, i.e., $\alpha = 0.8$. Using the design parameters given above, the expected life-cycle cost per unit time $C(t)$ for different values of servicing period Δt is presented in Fig. 3.

Table 1 Failure rate refresh factor for water pump rotor.

Servicing time (Month)	10	20	30	40	50	60
Refresh factor (%)	100	100	100	100	100	100
Servicing time (Month)	70	80	90	100	110	120
Refresh factor (%)	100	100	99.99	99.89	99.07	95.74
Servicing time (Month)	130	140	150	160	170	180
Refresh factor (%)	87.38	72.99	54.74	21.97	11.90	5.88

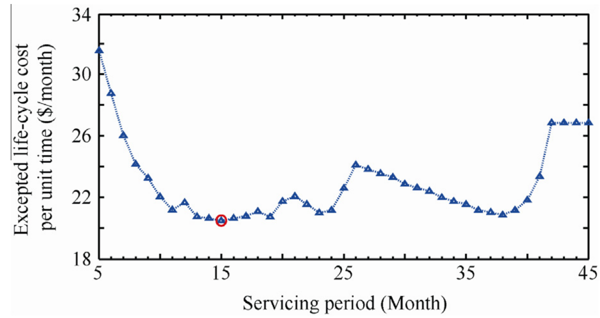

Fig. 3 Expected life-cycle cost per unit time.

As can be seen in the figure, the minimal cost is obtained for Δt = 15 months with $C^*(t)$ = 20.49 \$/month. Furthermore, the preventive maintenance time T_s is 85.5 months when the servicing period Δt is 15 months. In this case, the optimal servicing period Δt^* = 15 months and preventive maintenance time T_s^* = 85.5 months are determined. In practice, the water pump rotor is serviced periodically with 15 months and is preventively maintained at time 85.5 months. Fig. 3 shows that expected life-cycle cost per unit time changes with the integral servicing period, because the expected life-cycle cost per unit time is under the influence of servicing cost and average working time, and average working time also has a relationship with period and cost of servicing. The sensitivity of expected life-cycle cost per unit time to different servicing periods are not considered in this paper because of its complexity and limit paper space. Obviously, it is easier to search the optimal integral servicing period that makes the maintenance strategy implement practically.

To illustrate the effect of different strategy values (Δt, T_s) on the rotor operation condition, the failure probability p, the average working time $E(t)$, the average cost $E(C)$ and the average servicing cost $E(C_s)$ in one maintenance cycle are calculated. The results are summarized in Table 2.

From Table 2, we have the following observations:

(1) When the servicing period is small, the maintenance strategy in this paper cannot control the failure probability effectively, in this way the high corrective maintenance cost has to be paid. Meanwhile, the frequency of servicing is so great that the servicing cost is large. This result (when $\Delta t = 8, 9, 10$) implies that the cost has a significant impact on the expected cost when the servicing period is relatively small.

(2) As the servicing period increases, both the failure probability and servicing frequency are appropriate, and then the optimal expected cost can be derived. This is expected to occur, since the average working time is able to make expected cost optimal when the average working time changes a little. For instance, when Δt = 14, 15, 16 months, compared with the average working time, there is a greater different on the average time. However, when the servicing period is larger, the preventive maintenance action has to be ahead of time in order to meet the constraint Eq. (12). It is noticed that the rotor is serviced only once when servicing period is 36 months.

Table 2 Effect of different strategy values.

(Δt, T_s)	$C(t)$	p	$E(t)$	$E(C)$	$E(C_s)$
(8, 176)	24.18	0.2969	173.60	4196.83	3037.91
(9, 158)	23.25	0.2846	154.71	3597.83	2406.66
(10, 136)	22.04	0.1722	134.24	2959.06	1879.37
(14, 93.5)	20.64	0.0840	92.61	1911.89	926.12
(15, 85.5)*	20.49	0.0478	84.96	1741.04	793.01
(16, 76)	20.66	0.0091	75.89	1567.84	664.03
(36, 54)	21.18	0.0034	53.98	1143.29	209.93
(42, 37.5)	26.86	0.0072	37.49	1007.21	0
(43, 37.5)	26.86	0.0072	37.49	1007.21	0

* Note: represents the optimal values.

(3) As the servicing period continues to increase, it becomes too large to meet the constraint Eq. (12). The servicing has no influence on the rotor so that the preventive maintenance time is set as the same value. Obviously, the expected cost is a fixed value (i.e., 26.86 $/month).

All the numerical results are intuitive and reasonable. It can be seen that this study is appropriate for water pump rotor involving scheduled servicing.

5.4. Discussion

The section investigates the performance taking into account the servicing and mission time. The scheduled servicing can delay the reliability decrease speed in order to prolong the systems operation lifetime. It has a great operation performance improvement for the systems in practice, which cannot be neglected. Meanwhile, the proportion that system survives for the duration of the mission time can restrict the reliability decrease proportion in order to control the failure probability, so the delay effect and decrease proportion are jointly optimized to obtain the maintenance strategy in this paper. To investigate the benefits of the proposed maintenance strategy, some indexes of different strategies are provided in Table 3.

From Table 3, the average working time is 37.49 months without servicing. It means the lifetime of water pump rotor is 37.49 months. Considering scheduled servicing, the average working time of only servicing and combined strategy, respectively, are 98.21 months and 84.96 months. It is clear that the servicing makes the systems work longer. So the number of preventive (or corrective) maintenance action can be reduced and there is a large decrease in the total lifetime maintenance cost.[21] It can be observed that the expected life-cycle cost per unit time under servicing is less than 53.35 $/month. If there is only servicing, the system has to work until failure. Though the average working time is longer (i.e., 98.21 months), the expected life-cycle cost per unit time is higher. This is because the corrective maintenance cost must be paid. Compared with the strategy of only servicing, the combined strategy can lower effectively the failure probability (i.e., 0.0478). More importantly, it can guarantee the dependability of rotor. Based on the combined maintenance strategy proposed in this paper, the engineers know clearly that the probability of the rotor accomplish mission cannot be less than 0.8 at any time before the preventive maintenance time. Obviously, it is of great benefit to the engineers.

Table 3 Comparison of maintenance strategies.

Maintenance strategy	(Δt, T_s)	$E(t)$	p	$C(t)$
Without servicing		37.49	1.0000	53.35
Only servicing	(15, –)	98.21	1.0000	28.42
Combined strategy	(15, 85.5)	84.96	0.0478	20.49

6. Conclusions

(1) An improved method is presented to model the system damage process on servicing. And the failure rate refresh factor is proposed to measure the maintenance effect of servicing. Given the example of water pump rotor, the model can fit well the reliability rule. Meanwhile, the failure rate refresh factor can describe the characteristic that maintenance effect decreases with the working time and servicing number. Compared with the existing imperfect maintenance model, the result shows that it is helpful for operators to realize intuitively and objectively the maintenance effect of servicing by using the failure rate refresh factor.

(2) A preventive maintenance strategy which sets a limit to dependability over the life-cycle of the system is presented. Then the optimization model is developed to determine the optimal servicing period and preventive maintenance time, with an objective to minimize the system expected life-cycle cost per unit time. The result demonstrates that it can be used not only to prolong the lifetime of system aiming to reduce the total lifetime maintenance cost, but also to ensure the high dependability of system during the life-cycle. Under this study, the proposed maintenance strategy has practical significance for engineers and operators.

Acknowledgments

The authors are thankful to the referees for their valuable comments and suggestions, which help to significantly improve the paper. This study was supported by the National Defence Preresearch Foundation of China (Nos. 51327020105, 51304010206).

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