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D3–D5 holography with flux

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ABSTRACT

It is shown that the Berezinski–Kosterlitz–Thouless phase transition that has been found in D3–D5 brane systems with nonzero magnetic field and charge density can also be found by tuning an extra-dimensional magnetic flux. We find numerical solutions for the probe D5-brane embedding and discuss properties of the solutions. We also demonstrate that the nontrivial embeddings include those which can be regarded as spontaneously breaking chiral symmetry.

The AdS/CFT duality of an appropriately oriented probe D5-brane embedded in $AdS_5 \times S^5$ space–time and a supersymmetric defect conformal field theory is a well-studied example of holography [1–11]. In the limit of large N and large radius of curvature, the D5-brane geometry is found as an extremum of the Dirac–Born–Infeld action with appropriate Wess–Zumino terms added. Its world-volume is the product space $AdS_4(\subset AdS_5) \times S^2(\subset S^5)$ which preserves an $OSp(4|4)$ subgroup of the $SU(2,2|4)$ superconformal symmetry of the $AdS_5 \times S^5$ background. The superconformal field theory which is dual to this D3–D5 system, and which is described by it in the strong coupling limit, has a co-dimension one membrane that is embedded in $3+1$-dimensional flat space. The bulk of the $3+1$-dimensional space is occupied by $\mathcal{N}=4$ supersymmetric Yang–Mills theory with $SU(N)$ gauge group. A bi-fundamental chiral hypermultiplet lives on the membrane defect and its field theory is dual to the low energy modes of open strings connecting the D5-branes and the D3-branes. These fields transform in the fundamental representation of the $SU(N)$ bulk gauge group and in the fundamental representation of the global $U(N_5)$, where N_5 is the number of D5-branes (in the probe limit, $N_5 \ll N$ and we will take $N_5 = 1$). The defect field theory preserves half of the supersymmetries of the bulk $\mathcal{N}=4$ theory, resulting in the residual $OSp(4|4)$ super-conformal symmetry. It is massless with a hypermultiplet mass operator which breaks an $SU(2)$ R-symmetry [3].

An external magnetic field has a profound effect on this system. In the quantum field theory, the magnetic field is constant and is perpendicular to the membrane defect. In the string theory, the magnetic field destabilizes the conformal symmetric state to one which spontaneously breaks the SU(2) R-symmetry and generates a mass gap for the D3–D5 strings [5]. The only solution for the D5-brane embedding has it pinching off before it reaches the Poincaré horizon of AdS_5. As a result, the D3–D5 strings which, when excited, must reach from the D5-brane to the Poincaré horizon, have a minimum length and an energy gap. This occurs for any value of the magnetic field, in fact, since the theory has conformal invariance, the magnetic field is the only dimensional parameter and there is no distinction between large field and small field. A mass and a mass operator condensate for the D3–D5 strings can readily be identified (the conformal dimensions of their field theory duals are protected by supersymmetry) and there is simply no solution of the probe D5-brane embedding problem with a magnetic field when both the mass and the condensate are zero. There can be a solution when one of those parameters vanishes and the other does not vanish. Such a solution can be interpreted as presence of a condensate in the absence of a mass operator, that is, as dynamical symmetry breaking. This phenomenon is regarded as a holographic realization of the "magnetic catalysis" of chiral symmetry breaking that has been studied in $2+1$-dimensional quantum field theories [14–20]. The field theory studies rely on weak coupling expansions and re-summation of Feynman diagrams. Whether the phenomenon can persist at strong coupling is an interesting question which appears to have an affirmative answer in the context of this construction. It and many other aspects of the phase diagram of the D5-brane have been well studied in what is by now an extensive literature [5–13].

This interesting behavior becomes more complex when a $U(1)$ charge density, as well as the magnetic field, is introduced. The state then has a nonzero density of D3–D5 strings. There is also a tuneable dimensionless parameter, the ratio of charge density to the field, the "filling fraction" $\nu = \frac{2\pi\rho}{B}$. In this case, there is no charge gap. The D5-brane must necessarily reach the Poincaré horizon. This is due to the fact that, to have a nonzero charge density, there must be a density of fundamental strings suspended between the D5-brane and the Poincaré horizon. However, the fundamental

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string tension is always greater than the D5-brane tension [6] and such strings would therefore pull the D5-brane to the horizon. The result is a gapless state: the D3–D5 strings could have zero length, and therefore have no energy gap. At weak coupling, the dual process is the formation of a Fermi surface and a gapless metallic state when the charge density is nonzero.

What is more, if the filling fraction is large enough, the state with no mass term and mass operator condensate equal to zero exists and is stable. In this state, the $SU(2)$ R-symmetry is not broken. As the filling fraction is lowered from large values where the system takes up this symmetric phase, as pointed out in the beautiful paper [10], the system undergoes a Berezinski–Kosterlitz–Thouless-like (BKT) phase transition. This phase transition has BKT scaling and is one of the rare examples on non-mean field phase transitions in holographic systems. When the filling fraction is less than the critical value, again, even though the D5-brane world-volume now reaches the Poincaré horizon, there is no solution of the theory unless either the mass operator or mass operator condensate or both are turned on. This state breaks the $SU(2)$ R-symmetry but still has no charge gap.

In this Letter, we shall observe that, as well as density, there is a second parameter which can drive the BKT-like transition. The parameter is the value of a magnetic flux which forms a $U(1)$ monopole bundle on the D5-brane world-volume 2-sphere. The possibility of adding this flux was suggested by Myers and Wapler [6]. They found that the idea could be used to construct stable D3–D7 systems, in particular, and a modification of their idea was subsequently used to study holography in D3–D7 systems [21–23]. In the limit where the string theory is classical, the problem of embedding a D5-brane in the $AdS_5 \times S^5$ geometry reduces to that of finding an extremum of the Dirac–Born–Infeld and Wess–Zumino actions,

$$S = \frac{T_5}{g_s}\int d^6\sigma\left[-\sqrt{-\det\left(g+2\pi\alpha' F\right)} + C^{(4)}\wedge 2\pi\alpha' F\right] \tag{1}$$

where g_s is the closed string coupling constant, which is related to the $\mathcal{N}=4$ Yang–Mills coupling by $4\pi g_s = g_{YM}^2$, $g_{ab}(\sigma)$ is the induced metric of the D5-brane, $C^{(4)}$ is the 4-form of the $AdS_5\times S^5$ background, F is the world-volume gauge field and $T_5 = \frac{1}{(2\pi)^5\alpha'^3}$. We shall use the metric of $AdS_5\times S^5$ and 4-form

$$\begin{aligned} ds^2 = L^2\Big[& r^2\left(-dt^2+dx^2+dy^2+dx^2\right)+\frac{dr^2}{r^2}+ \\ &+d\psi^2+\cos^2\psi\left(d\theta^2+\sin^2\theta\, d\phi^2\right) \\ &+\sin^2\psi\left(d\tilde\theta^2+\sin^2\tilde\theta\, d\tilde\phi^2\right)\Big] \end{aligned} \tag{2}$$

$$\begin{aligned} C^{(4)} = {}& L^4 r^4\, dt\wedge dx\wedge dy\wedge dz \\ &+ L^4\frac{c(\psi)}{2}\, d\cos\theta\wedge d\phi\wedge d\cos\tilde\theta\wedge d\tilde\phi \end{aligned} \tag{3}$$

with $\partial_\psi c(\psi) = 8\sin^2\psi\cos^2\psi$. In (2), the 5-sphere is represented by two 2-spheres fibered over the interval $\psi\in[0,\frac{\pi}{2}]$. The radius of curvature of AdS is L and $L^2=\sqrt{\lambda}\alpha'$ with $\lambda = g_{YM}^2 N$. The embedding of the D5-brane is mostly determined by symmetry. The dynamical variables are $\{x(\sigma), y(\sigma), z(\sigma), t(\sigma), r(\sigma), \psi(\sigma), \theta(\sigma), \phi(\sigma), \tilde\theta(\sigma), \tilde\phi(\sigma)\}$. We look for a solution of the form

$$\begin{aligned} &\sigma_1 = x, \quad \sigma_2 = y, \quad \sigma_3 = t, \quad \sigma_4 = r, \quad \sigma_5 = \theta - \frac{\pi}{2}, \\ &\sigma_6 = \phi, \quad \tilde\theta = 0, \quad \tilde\phi = 0 \end{aligned} \tag{4}$$

and the remaining coordinates depending only on $\sigma_4 = r$, $(z(r), \psi(r))$.[1] With this Ansätz, the D5-brane world-volume metric is

$$\begin{aligned} ds^2 = L^2\Big[& r^2\left(-dt^2+dx^2+dy^2\right)+\frac{dr^2}{r^2}\left(1+r^2\psi'^2+r^4z'^2\right) \\ &+\cos^2\psi\left(d\theta^2+\sin^2\theta\, d\phi^2\right)\Big] \end{aligned} \tag{7}$$

where prime denotes derivative by r and the world-volume gauge fields are

$$\begin{aligned} F = {}& \frac{L^2}{2\pi\alpha'}a'(r)\, dr\wedge dt + \frac{L^2}{2\pi\alpha'} b\, dx\wedge dy \\ &+ \frac{L^2}{2\pi\alpha'}\frac{f}{2}\, d\cos\theta\wedge d\phi \end{aligned} \tag{8}$$

Here, f is the strength of the monopole bundle.[2] b is a constant magnetic field which is proportional to a constant magnetic field in the field theory dual. $a(r)$ is the temporal world-volume gauge field which must be nonzero in order to have a uniform charge density in the field theory dual. The bosonic part of the R-symmetry is $SU(2)\times SU(2)$. One $SU(2)$ is the isometry of the S^2 which is wrapped by the D5-brane (7) and is also a symmetry of the background fields (8). The other is the rotation in the transverse $S^2 \subset S^5$ with S^5 coordinates $\tilde\theta, \tilde\phi$. This is a symmetry of the embedding only when the former S^2 is maximal, that is, when $\psi(r) = 0$ for all r. If $\psi(r)$ deviates from zero, it must choose a direction in the transverse space, and the choice breaks the second $SU(2)$. The hypermultiplet mass shows up in the D5-brane embedding as

$$M \sim m \equiv \lim_{r\to\infty} r\sin\psi(r), \qquad \psi(r\to\infty) = \frac{m}{r}+\frac{c}{r^2}+\cdots \tag{9}$$

and deviation of $\psi(r)$ from the constant $\psi=0$ so that the parameter m is nonzero is a signal of having switched on a hypermultiplet mass operator in the dual field theory. The parameter c is dual to the chiral condensate, although in an alternative quantization these could be interchanged [24].

With (7) and (8), the action (1) is

$$\begin{aligned} S = \mathcal{N}\int & d^3x\, dr \\ &\times\left[-\sqrt{\left(f^2+4\cos^4\psi\right)\left(b^2+r^4\right)\left(1+r^2\psi'^2+r^4z'^2\right)-a'^2} \right.\\ &\left. + f r^4 z'\right] \end{aligned} \tag{10}$$

[1] This ansatz is symmetric under space–time parity which can be defined for the Wess–Zumino terms

$$\int d^6\sigma\, \epsilon^{\mu_1\mu_2\ldots\mu_6}\partial_{\mu_1}x(\sigma)\partial_{\mu_2}y(\sigma)\partial_{\mu_3}z(\sigma)\partial_{\mu_4}t(\sigma)r^4(\sigma)\partial_{\mu_5}A_{\mu_6}(\sigma) \tag{5}$$

$$\int d^6\sigma\, \epsilon^{\mu_1\mu_2\ldots\mu_6}\partial_{\mu_1}\cos\theta(\sigma)\partial_{\mu_2}\phi(\sigma)\partial_{\mu_3}\cos\tilde\theta(\sigma)\partial_{\mu_4}\tilde\phi(\sigma)c(\psi)\partial_{\mu_5}A_{\mu_6}(\sigma) \tag{6}$$

in the following way. The world-volume coordinates transform as $\{\sigma'_1, \sigma'_2, \ldots, \sigma'_6\} = \{-\sigma_1, \sigma_2, \ldots, \sigma_6\}$ and the embedding functions as $x'(\sigma') = -x(\sigma)$, $\tilde\theta'(\sigma') = \pi - \theta(\sigma)$, $A'_1(\sigma') = -A_1(\sigma)$ with all other variables obeying $\chi(\sigma') = \chi(\sigma)$. This is a symmetry of the Wess–Zumino terms and the Ansätz (4) is invariant. Charge conjugation flips the sign of all gauge fields, $A\to -A$ and we augment it by $\{\sigma'_1, \ldots, \sigma'_5, \sigma'_6\} = \{\sigma_1, \ldots, -\sigma_5, \sigma_6\}$. The Wess–Zumino terms are invariant. The background field $f\, d\cos\theta\wedge d\phi$ is also invariant once we choose $\sigma_5 = \theta - \frac{\pi}{2}$. The fields $a(r)$ breaks C and preserves P. b breaks C and P and preserves CP.

[2] A monopole bundle has quantized flux. Here the number of quanta is very large in the strong coupling limit $n_D \sim \sqrt{\lambda}$, so that it is to a good approximation a continuously variable parameter. b and q are related to the physical magnetic field and charge density as $b = \frac{2\pi}{\sqrt{\lambda}}B$, $q = \frac{4\pi^3}{\sqrt{\lambda}N}\rho$ so that $\frac{q}{b} = \frac{\pi}{N}\frac{2\pi\rho}{B} \equiv \frac{\pi}{N}\nu$ where the dimensionless parameter ν is the filling fraction. A Landau level would have degeneracy N. The filling fraction of a set of N degenerate levels naturally scales like N to give order one b and q in the large N limit.

where $\mathcal{N} = \frac{2\pi T_5 L^6}{g_s} = \frac{\sqrt{\lambda} N}{4\pi^3}$. The factor of 2π in the numerator comes from half of the volume of the unit 2-sphere (the other factor of 2 is still in the action). The Wess–Zumino term gives a source for $z(r)$.

Now, we must solve the equations of motion for the functions $\psi(r)$, $a(r)$ and $z(r)$ which result from (10) and the variational principle. The variables $a(r)$ and $z(r)$ are cyclic and they can be eliminated using their equations of motion,

$$\frac{d}{dr}\frac{\delta S}{\delta z'(r)} = 0 \rightarrow \frac{\sqrt{(f^2 + 4\cos^4\psi)(b^2 + r^4)}\, r^4 z'}{\sqrt{1 + r^2\psi'^2 + r^4 z'^2 - a'^2}} - f r^4 = p_z \tag{11}$$

$$\frac{d}{dr}\frac{\delta S}{\delta a'(r)} = 0 \rightarrow \frac{\sqrt{(f^2 + 4\cos^4\psi)(b^2 + r^4)}\, a'}{\sqrt{1 + r^2\psi'^2 + r^4 z'^2 - a'^2}} = -q \tag{12}$$

where p_z and q are constants of integration. If these equations are to hold near $r \rightarrow 0$, we must set $p_z = 0$. q is proportional to the charge density in the field theory dual. Then, we can solve for z' and a',

$$z' = \frac{f\sqrt{1 + r^2\psi'^2}}{\sqrt{4\cos^4\psi\,(b^2 + r^4) + f^2 b^2 + q^2}} \tag{13}$$

$$a' = \frac{-q\sqrt{1 + r^2\psi'^2}}{\sqrt{4\cos^4\psi\,(b^2 + r^4) + f^2 b^2 + q^2}} \tag{14}$$

We must then use the Legendre transformation

$$\mathcal{R} = S - \int a'(r)\frac{\partial L}{\partial a'(r)} - \int z'(r)\frac{\partial L}{\partial z'(r)}$$

to eliminate z' and a'. We obtain the Routhian

$$\mathcal{R} = -\mathcal{N}\int d^3x\, dr \sqrt{4\cos^4\psi\left(b^2 + r^4\right) + b^2 f^2 + q^2}\sqrt{1 + r^2\psi'^2} \tag{15}$$

which must now be used to find an equation of motion for $\psi(r)$,

$$\frac{\ddot\psi}{1 + \dot\psi^2} + \dot\psi\left[1 + \frac{8r^4\cos^4\psi}{4(b^2 + r^4)\cos^4\psi + f^2 b^2 + q^2}\right] + \frac{8(b^2 + r^4)\cos^3\psi\sin\psi}{4(b^2 + r^4)\cos^4\psi + f^2 b^2 + q^2} = 0 \tag{16}$$

where the overdot is the logarithmic derivative $\dot\psi = r\frac{d}{dr}\psi$.

First, we note that, if $\psi(r)$ is to be finite at $r \rightarrow \infty$, its logarithmic derivatives should vanish. Then, the only boundary condition which is compatible with the equation of motion is $\psi(r \rightarrow \infty) = 0$. The asymptotic solution of (16) at large r is given in (9).

If we set $b = 0$, f does not appear in the Routhian (15) or in the equation of motion (16). $\psi(r)$ which is then f-independent. In fact, when $b = 0$, the constant solution $\psi = 0$ is a stable solution of (16). $z(r)$ is f- and r-dependent. Eq. (13) has the solution $z(r) = \int dr \frac{f}{\sqrt{4r^4 + f^2}} = r_2F_1(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -\frac{4r^4}{f^2})$. The world-volume is $AdS_4 \times S^2$, where the radii of the two spaces differ, the S^2 has radius L whereas AdS_4 has radius $L\sqrt{1 + \frac{f^2}{4}}$. The field theory dual of this system was discussed in Ref. [6]. It has a planar defect dividing three-dimensional space into two half-spaces with $\mathcal{N} = 4$ Yang–Mills theory with gauge group $SU(N + n_D)$ on one side of the defect and $\mathcal{N} = 4$ Yang–Mills theory with gauge group $SU(N)$ on the other side. Here n_D is the number of Dirac monopole quanta in f. The r-dependence of the embedding function $z(r)$ can be viewed as an energy-scale dependent position of the defect in the field theory.

When b is not zero, scaling $r \rightarrow \sqrt{b}r$, removes b from most of Eq. (16), the dependence which remains is only in the parameter $f^2 + (\frac{q}{b})^2$. If this parameter is large enough, the solution $\psi(r) = 0$ is still a stable solution of (16). When $f^2 + (\frac{q}{b})^2$ is lowered to a critical value, the $\psi = 0$ solution becomes unstable. At that point, the BKT-like phase transition occurs. That transition was found in Ref. [10] where they adjusted $\frac{q}{b}$ (they had $f = 0$) with the critical value being $(\frac{q}{b})^2|_{\text{crit.}} = 28$. The onset of instability of the symmetric solution $\psi = 0$ at that point is easily seen by looking at solutions of the linearized equation which, at small r, must be $\psi \sim c_1 r^{\nu_+} + c_2 r^{\nu_-}$ with $\nu_\pm = -\frac{1}{2} \pm \frac{1}{2}\sqrt{1 - 32/(4 + f^2 + \frac{q}{b})^2}$. The instability sets in when the exponents become complex, that is, at $[f^2 + (\frac{q}{b})^2]_{\text{crit.}} = 28$. The complex exponents are due to the fact that, in the $r \sim 0$ regime, the fluctuations obey a wave equation for AdS_2 with a mass that violates the Breitenholder–Freedman bound. Since, in the stable regime, $f^2 + (\frac{q}{b})^2 > 28$ both of the exponents in the fluctuations are negative, deviation from $\psi(r) = 0$ is not allowed, it is an isolated solution. Here we point out that we can find the phase transition even when $\frac{q}{b}$ vanishes by varying f, stability occurs where $f^2 > 28$ and the phase transition at $f^2_{\text{crit}} = 28$. In particular, this allows us to study the theory in the charge neutral state where $q = 0$.

When the symmetric solution $\psi = 0$ is unstable, we must find another solution of Eq. (16) for $\psi(r)$, where we now assume that it depends on r. $\psi = 0$ was an isolated solution, there are no other solutions closeby. As soon as it depends on r, if $\psi(r)$ is to remain finite in the small r region, it must go to the other solution of (16) at small r, $\psi(r \rightarrow 0) = \frac{\pi}{2}$. At this point, the S^2 which the D5-brane wraps has collapsed to a point and the D5-brane is effectively a D3-brane with world-volume oriented in the x, y, t, r-directions.

When q, b and f are all nonzero, it is interesting that the embedding problem depends only on the combination $\sqrt{f^2 + (\frac{q}{b})^2}$, reminiscent of bound states of F-strings and D-branes [25]. When either or both of q and f are nonzero, the D5-brane must reach the Poincaré horizon. Otherwise, the charge density q and magnetic monopole flux f would have to have sources on the D5-brane world-volume. The appropriate source would be n_D D3-branes carrying electric charge density q suspended between the D5 world-volume and the Poincaré horizon. However, as in the case of fundamental strings when there was only charge present, it is possible to show that the appropriate D3-brane tension is always greater than the D5-brane tension. The suspended D3-branes would pull the D5-brane to the horizon. The D5-brane world-volume could still reflect this behavior with a spike or funnel-like configuration which emulates suspended strings and D3-branes in the $r < 1/\sqrt{b}$ regime.

We expect to find solutions of (16) which interpolate between $\psi = 0$ at $r \rightarrow \infty$ to $\psi = \frac{\pi}{2}$ at $r \rightarrow 0$. Indeed, for generic asymptotic behavior, such solutions are easy to find by a shooting technique. Examples are given in Fig. 1. Indeed we see that, as a function of $\ln(\sqrt{b}r)$ and when $f^2 + (\frac{q}{b})^2 < [f^2 + (\frac{q}{b})^2]^2_{\text{crit.}} = 28$, they exhibit a rapid soliton-like crossover between $\psi = 0$ at large $\sqrt{b}r$, which is a D5-brane, and $\psi \sim \frac{\pi}{2}$ at small $\sqrt{b}r$, which is like n_D D3-branes with electric charge q dissolved into them. In the third plot in Fig. 1, where $f^2 + (\frac{q}{b})^2 > [f^2 + (\frac{q}{b})^2]^2_{\text{crit.}}$, the funnel is much more diffuse.

It is also possible to find solutions that can be interpreted as chiral symmetry breaking, although the D5-brane still reaches the Poincaré horizon and we expect that the D3–D5 strings are still gapless. In the region of large r, the linearized equation for $\psi(r)$ is solved by (9). The two asymptotic behaviors have power laws associated with the ultraviolet conformal dimensions of the mass and the chiral condensate in the dual field theory. These are the

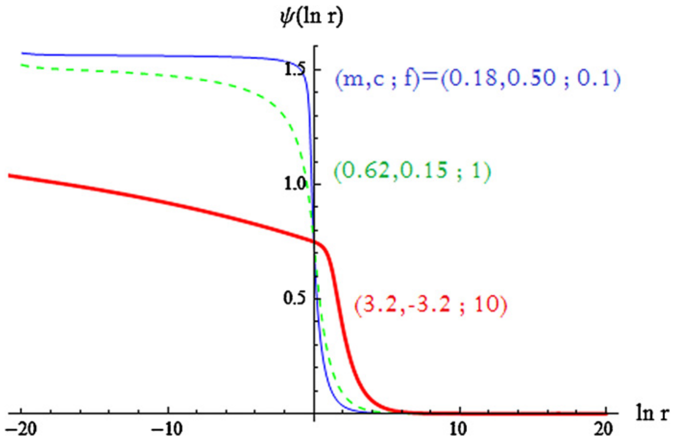

Fig. 1. Eq. (16) integrated with $q=0$ and $f^2=0.01$ $f^2=1$ and $f^2=100$. The solutions interpolate between the correct asymptotic values, $\psi(r=\infty)=0$ and $\psi(r=0)=\frac{\pi}{2}$. The AdS radius r is measured in units of $1/\sqrt{b}$. For smaller values of f, the transition is sharp and occurs at $\sqrt{b}r\sim 1$.

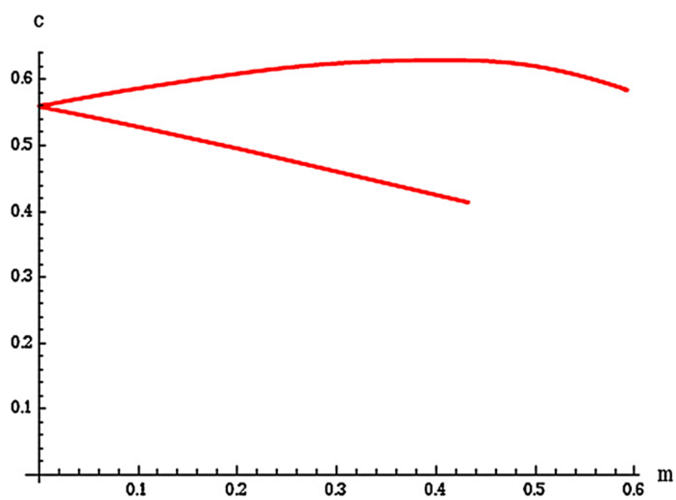

Fig. 2. The constants c versus m are plotted for a sequence of embeddings of the D5-brane in the region where the constant ψ solutions are unstable, $f^2=.01$.

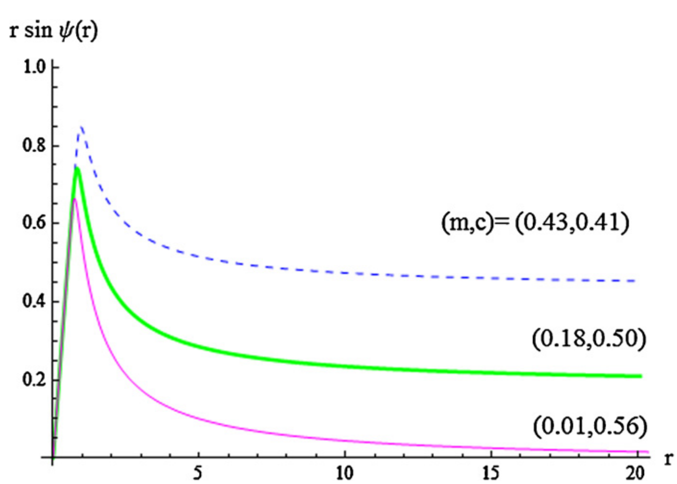

Fig. 3. The function $r\sin(\psi(r))$ is plotted versus r for some embeddings, parameterized by the asymptotic m and c, including the one which is close the solution with $m=0$ which is associated with dynamical symmetry breaking.

same as their classical dimensions since they are protected by supersymmetry. A symmetry breaking solution would have one of these equal to zero (and the other one interpreted as a condensate). Indeed, it is easy to find a family of solutions of (16) which, as we tune m, still exists and has nonzero c in the limit where m goes to zero. The c versus m behavior of this family of solutions is shown in Fig. 2. The behavior if $r\sin(\psi(r))$ which can be interpreted as the separation of the D5- and D3-branes is plotted in Fig. 3 for some values of m and c.

As an extension of our results, it would be interesting to analyze the electromagnetic properties of the solution with finite f and $q=0$. This is a charge neutral state and it has a mass operator condensate. It is possible to study Maxwell's equations for fluctuations of the world-volume gauge field and though it is difficult to fully derive even a formal solution, it is relatively straightforward to show that they have no solution when the field strength is a constant. This implies that the charged matter is still gapless and provides the singularities in response functions which make the theory singular at low energy and momentum.

From the point of view of the space–time symmetry, the flux f is charge conjugation symmetric, whereas the finite charge density state is not. In fact f itself does not violate any $2+1$-dimensional space–time symmetries associated with Lorentz, C, P or T invariance. The fact that states with finite f need to have a vanishing charge gap is somewhat mysterious from the field theory point of view. When q is nonzero, the absence of a charge gap is understandable as the theory should be in a finite density metallic state, even when it breaks chiral symmetry. When $q=0$ but f is nonzero, the system must also be gapless, even though the charge density is zero and the hypermultiplet should be massive. One possibility is that, in the field theory, the planar defect which separates spaces where $\mathcal{N}=4$ Yang–Mills theory has different gauge groups has a band of gapless edge states. It would be interesting to examine this further in the field theory.

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