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de Sitter space and extremal surfaces for spheres

K. Narayan

Chennai Mathematical Institute, SIPCOT IT Park, Siruseri 603103, India

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ABSTRACT

Following arXiv:1501.03019 [hep-th], we study de Sitter space and spherical subregions on a constant boundary Euclidean time slice of the future boundary in the Poincaré slicing. We show that as in that case, complex extremal surfaces exist here as well: for even boundary dimensions, we isolate the universal coefficient of the logarithmically divergent term in the area of these surfaces. There are parallels with analytic continuation of the Ryu–Takayanagi expressions for holographic entanglement entropy in AdS/CFT. We then study the free energy of the dual Euclidean CFT on a sphere holographically using the dS/CFT dictionary with a dual de Sitter space in global coordinates, and a classical approximation for the wavefunction of the universe. For even dimensions, we again isolate the coefficient of the logarithmically divergent term which is expected to be related to the conformal anomaly. We find agreement including numerical factors between these coefficients.

1. Introduction

Generalizations of gauge/gravity duality [1–4] to de Sitter space, or dS/CFT [5–7], involve a hypothetical dual Euclidean CFT on the future timelike infinity $\mathcal{I}^+$ boundary. The late time wavefunction of the universe with appropriate boundary conditions is equated with the partition function of the dual CFT. Further work on dS/CFT including higher spin realizations appears in *e.g.* [8–15].

Ideas pertaining to entanglement entropy have been of great interest in recent times. In AdS/CFT, the Ryu–Takayanagi prescription [16,17] (see [18,19] for reviews) maps entanglement entropy of a field theory subsystem to the area (in Planck units) of a bulk minimal surface (more generally extremal surface [20]) anchored at the subsystem interface and dipping into the bulk, in the gravity approximation. Similar ideas were explored in [21] in de Sitter space with a view to exploring entanglement entropy in the dual CFT with dS/CFT in mind. For strip-shaped subregions on a constant Euclidean time slice of the future boundary, it was found that the area of certain complex extremal surfaces has structural resemblance with entanglement entropy of dual Euclidean CFTs (reviewed in sec. 2). The coefficients of the leading divergent "area law" terms in dS_{d+1} resemble the central charges $\mathcal{C}_d \sim i^{1-d}\frac{R_{dS}^{d-1}}{G_{d+1}}$ of the CFT$_d$s appearing in the $\langle TT\rangle$ correlators in [7]. The areas of these surfaces obtained thus essentially amount to analytic continuation from the Ryu–Takayanagi expressions for holographic entanglement entropy in AdS/CFT. Note that the areas are in general not real-valued or positive definite and are distinct from the entanglement entropy of bulk fields in de Sitter space *e.g.* [22].

Towards exploring this further, we study spherical subregions in this paper. As in [21], similar complex extremal surfaces can be shown to exist in this case too (sec. 3). The area of these surfaces exhibits a similar leading area law divergence as well as subleading terms: for dS_{d+1} with even d, this includes a logarithmically divergent term whose coefficient is analogous to the "universal" terms in AdS/CFT related to the conformal anomaly [17]. In the present case also, we expect this anomaly to arise in the free energy of the CFT on a curved space. With this in mind, we then calculate in the present case (sec. 4), the free energy of the Euclidean CFT on a sphere holographically using the dS/CFT dictionary $Z_{CFT} = \Psi$ [7] with an auxiliary de Sitter space in global coordinates (whose constant time slices are spheres). In the classical regime, we approximate the wavefunction of the universe Ψ in terms of the bulk action S of this auxiliary de Sitter space: this gives $-F = \log Z_{CFT} = \log\Psi \sim iS$. We find precise agreement between the coefficients of the logarithmic terms in the complex extremal surfaces and those in the free energy via the wavefunction of the universe, including numerical factors. Since the coefficient of the logarithmic term in the free energy is related to the trace anomaly for any CFT, this supports the idea that the area of these complex extremal surfaces encodes entanglement entropy of the dual Euclidean CFT in dS/CFT, as we discuss in sec. 5.

E-mail address: narayan@cmi.ac.in.

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2. Reviewing de Sitter extremal surfaces: strips

Here we review the study [21] of bulk de Sitter extremal surfaces anchored over strip-shaped subregions on the future boundary $\mathcal{I}^+$ in the Poincaré slicing. de Sitter space dS_{d+1} in the Poincaré slicing or planar coordinate foliation is given by the metric

$$ds^2 = \frac{R_{dS}^2}{\tau^2}(-d\tau^2 + dw^2 + d\sigma_{d-1}^2)\,, \tag{1}$$

where half of the spacetime, *e.g.* the upper patch, has $\mathcal{I}^+$ at $\tau = 0$ and a coordinate horizon at $\tau = -\infty$. This may be obtained by analytic continuation of a Poincaré slicing of *AdS*,

$$z \to -i\tau\,, \qquad R_{AdS} \to -iR_{dS}\,, \qquad t \to -iw\,, \tag{2}$$

where w is akin to boundary Euclidean time, continued from time in *AdS* (with z the bulk coordinate). The dual Euclidean CFT is taken as living on the future $\tau = 0$ boundary $\mathcal{I}^+$. We assume translation invariance with respect to the boundary Euclidean time direction w, and consider a subregion on a $w = const$ slice of $\mathcal{I}^+$. One might imagine that tracing out the complement of this subregion then gives entropy in some sense stemming from the information lost. In the bulk, we study de Sitter extremal surfaces on the $w = const$ slice, analogous to the Ryu–Takayanagi prescription in *AdS/CFT*. Operationally these extremal surfaces begin at the interface of the subsystem (or subregion) and dip into the bulk time direction.

For a strip-shaped subregion on $\mathcal{I}^+$ (with width say along x), parameterizing the spatial part in (1) as $d\sigma_{d-1}^2 = \sum_{i=1}^{d-1} dx_i^2$, the dS_{d+1} area functional on a $w = const$ slice is

$$S_{dS} = \frac{R_{dS}^{d-1} V_{d-2}}{4G_{d+1}} \int \frac{d\tau}{\tau^{d-1}} \sqrt{\left(\frac{dx}{d\tau}\right)^2 - 1}\,,$$
$$\dot{x}^2 = \frac{-A^2\tau^{2d-2}}{1 - A^2\tau^{2d-2}}\,, \tag{3}$$

with $\frac{dx}{d\tau} \equiv \dot{x}$, and the constant A^2 is the conserved quantity obtained in the extremization. First let us consider dS_4 with the bulk time parametrized by real τ with the range $-\infty < \tau < 0$ and correspondingly real surfaces, described in [21]. These surfaces are obtained by taking $A^2 < 0$, which gives $\dot{x}^2 = \frac{|A|^2\tau^4}{1+|A|^2\tau^4}$. For real τ, we note that a crucial sign difference from the *AdS* case implies the absence of a turning point where $\dot{x} \to \infty$. These are timelike surfaces with $\dot{x}^2 \to 0$ as $\tau \to 0$ (anchored on the subregion boundary at $\mathcal{I}^+$, dipping into the past): as $|\tau| \to \infty$, we have $\dot{x}^2 \to 1$ asymptoting to a null surface. An extremal surface can then be constructed by taking two half-extremal-surfaces bending "inward" and joining them with a cusp (see Fig. 1 in [21]). Since these are real surfaces, it is natural to take $S_{dS} = \frac{R_{dS}^{d-1}V_{d-2}}{4G_{d+1}} \int \frac{d\tau}{\tau^{d-1}}\sqrt{1-\dot{x}^2}$ as for a real timelike surface. The area decreases as $|A|^2$ increases: as $|A|^2 \to \infty$, these real surfaces become null with $\dot{x}^2 \to 1$ and are the analogs of surfaces with minimal (zero) area. They are restrictions (to a boundary Euclidean time slice) of the boundary of the past lightcone wedge of the boundary subregion, with vanishing area, and no bearing on entanglement. One can also consider half-extremal-surfaces bending "outward" from the interface: again minimal area surfaces are null with zero area. Taking $A^2 = 0$ gives disconnected surfaces $x(\tau) = const$, again with no turning point: it is then natural to take τ to extend all the way to $|\tau| \to \infty$ which gives $S_{dS} \sim \frac{R_{dS}^{d-1}}{G_{d+1}} \frac{V_{d-2}}{\epsilon^{d-2}}$ with no cutoff-independent terms (encoding the interesting finite size-dependent part of entanglement). Thus real dS extremal surfaces do not give interesting entanglement structure. Real codim-1 surfaces have similar behavior.

With dS/CFT in mind, we now consider $A^2 > 0$: this gives a complex surface.[1] For dS_4, we have $x(\tau) \sim \pm iA\tau^3 + x(0)$ as $\tau \sim 0$, so that $x(\tau)$ representing a spatial direction in the CFT is real-valued only if τ is pure imaginary. More generally, requiring that the width $\Delta x = l$ be real-valued suggests that τ takes imaginary values, parametrized as $\tau = iT$ with T real. There is now a turning point $\tau_* = \frac{i}{\sqrt{A}}$ which is the "deepest" location this (smooth) complex surface dips upto in the bulk (with $|\dot{x}| \to \infty$): in other words, $|\tau| \le |\tau_*|$. The width condition $\Delta x = l$ can be shown to give $\tau_* = il$, so that large width $l \to \infty$ implies $|\tau_*| \to \infty$. It is worth noting that the τ-parametrization here lies outside the original de Sitter parametrization where τ was a real coordinate: instead for this complex solution to the extremization (with width l), τ runs along the imaginary axis, ranging from $i\epsilon$ to il. The corresponding surface $x(\tau)$ does not directly correspond to any real bulk dS_4 subregion: instead, complexified τ suggests an effective analytic continuation of (1) to Euclidean *AdS* and a corresponding extremal surface. For even d, similar analysis can be done [21] with complex surface saddle points of the area functional arising by taking $A^2 < 0$ and similar paths $\tau = iT$ (the details are different from dS_4). The area of these surfaces is

$$S_{dS} = -i\frac{R_{dS}^{d-1}}{4G_{d+1}} V_{d-2} \int_{\tau_{UV}}^{\tau_*} \frac{d\tau}{\tau^{d-1}} \frac{1}{\sqrt{1 - (-1)^{d-1}A^2\tau^{2d-2}}}$$
$$= i^{1-d}\frac{R_{dS}^{d-1}}{2G_{d+1}} V_{d-2}\left(\frac{1}{\epsilon^{d-2}} - c_d\frac{1}{l^{d-2}}\right), \tag{4}$$

where $\tau_{UV} = i\epsilon$ and $\tau_* = il$, and the integral is as in *AdS* (with corresponding constant c_d). Note that here we have used the relation $\tau_{UV} = i\epsilon$ for the ultraviolet cutoff in the dual Euclidean field theory suggested by previous investigations in dS/CFT (see *e.g.* [7–9, 13]) with time evolution mapping to renormalization group flow.

S_{dS} in (4) bears structural resemblance to entanglement entropy in a dual CFT with central charge $\mathcal{C}_d \sim i^{1-d}\frac{R_{dS}^{d-1}}{G_{d+1}}$. The first term $S_{dS}^{div} \sim i^{1-d}\frac{R_{dS}^{d-1}}{G_{d+1}}\frac{V_{d-2}}{\epsilon^{d-2}}$ resembles an area law divergence [25, 26], proportional to the area of the interface between the subregion and the environment, in units of the ultraviolet cutoff. It appears independent of the shape of the subregion, expanding (3) and assuming that $\dot{x}$ is small near the boundary τ_{UV}. Written as $\mathcal{C}_d\frac{V_{d-2}}{\epsilon^{d-2}}$, we see that it is also proportional to the central charge $\mathcal{C}_d \sim i^{1-d}\frac{R_{dS}^{d-1}}{G_{d+1}}$ representing the number of degrees of freedom in the dual (non-unitary) CFT: these arose in the $\langle TT\rangle$ correlators obtained in [7]. In dS_4, the central charge $\mathcal{C} \sim -\frac{R_{dS}^2}{G_4}$ is real and negative, while in dS_3, dS_5, it is imaginary. The second term is a finite cutoff-independent piece. Unlike dS_4, note that S_{dS} in dS_{d+1} with even d is not real-valued: *e.g.* in dS_3, we obtain $S_{dS} \sim -i\frac{R_{dS}}{G_3}\log\frac{l}{\epsilon}$ while in dS_5, we have $S_{dS} \sim i\frac{R_{dS}^3}{G_5}V_2(\frac{1}{\epsilon^2} - c_4\frac{1}{l^2})$. Similar complex surfaces can be studied in the dS black brane [15] which are dual to the CFT at uniform energy density: then the finite part resembles an extensive thermal entropy, again with a coefficient central charge as above. It is interesting to note that a replica calculation of entanglement entropy in a free 3d $Sp(N)$ theory for the half-plane [27] gives behavior similar to the leading area law divergence here (although the $Sp(N)$ theory is dual to the higher

[1] Complex geodesics and surfaces have also appeared in *e.g.* [23,24].

spin dS_4 theory [9] and it is unclear if geometric objects such as extremal surfaces are of relevance).

While there is structural resemblance with entanglement entropy, there are questions. Since these are bulk complex extremal surfaces, changing the sign in the square root branch in (3) introduces an overall $\pm i$ factor. Fixing this in (4) as $-i$ makes the leading divergence to be of the form of the area law $\mathcal{C}_d \frac{V_{d-2}}{\epsilon^{d-2}}$ with $\mathcal{C}_d$ the central charges in [7]. The resulting expressions are corroborated by and essentially amount to analytic continuation from the Ryu–Takayanagi expressions for holographic entanglement entropy in AdS/CFT. While this is suggestive, it would be useful to explore this further with a view to associating these complex extremal surfaces and corresponding area with entanglement entropy in dS/CFT.

Here we study spherical subregions and the corresponding complex extremal surfaces. For dS_{d+1} with even d, there is a term in the area with logarithmic dependence on the cutoff ϵ whose coefficient can be compared with that obtained from the conformal anomaly appearing in the free energy of the CFT on a sphere holographically using dS/CFT. We obtain agreement between both sides: this vindicates the signs we have used above in defining these complex surfaces, and analytic continuation.

3. Spherical extremal surfaces in de Sitter space

Building on [21] for strip-shaped subregions, here we consider spherical subregions on the boundary $\mathcal{I}^+$, with radius l parametrized as $0 \le r \le l$. Since we are interested in spherical entangling surfaces, we will parametrize $d\sigma_{d-1}^2$ in (1) in polar coordinates. Then the $w = const$ surface (*i.e.* a constant boundary Euclidean time surface) is a bulk d-dim subspace with metric

$$ds^2 = \frac{R_{dS}^2}{\tau^2}\Big(-d\tau^2 + dr^2 + r^2 d\Omega_{d-2}^2\Big). \tag{5}$$

The bulk surface on the $w = const$ slice bounding this subregion and dipping into the τ-direction is bulk codim-2: let us parametrize this as $r = r(\tau)$. Its area functional in Planck units is

$$S_{dS} = \frac{1}{4G_{d+1}} \int \prod_{i=1}^{d-2} \frac{R_{dS} r d\Omega_i}{\tau} \frac{R_{dS}}{\tau} \sqrt{dr^2 - d\tau^2}$$
$$= \frac{R_{dS}^{d-1}\Omega_{d-2}}{4G_{d+1}} \int \frac{d\tau}{\tau^{d-1}} r^{d-2} \sqrt{\left(\frac{dr}{d\tau}\right)^2 - 1}\ . \tag{6}$$

The variational equation of motion for an extremum $\frac{\partial}{\partial\tau}(\frac{\partial\mathcal{L}}{\partial\dot{r}}) = \frac{\partial\mathcal{L}}{\partial r}$ gives

$$\frac{\partial}{\partial\tau}\left(\frac{r^{d-2}}{\tau^{d-1}} \frac{\dot{r}}{\sqrt{\dot{r}^2-1}}\right) = \frac{d-2}{\tau^{d-1}} r^{d-3}\sqrt{\dot{r}^2-1}\,, \tag{7}$$

where $\frac{dr}{d\tau} \equiv \dot{r}$. It can be seen that

$$r(\tau) = \sqrt{l^2+\tau^2} \tag{8}$$

is an extremal surface that solves (7), thus extremizing S_{dS}, using

$$\dot{r} = \frac{\tau}{\sqrt{l^2+\tau^2}}\,, \qquad \dot{r}^2 - 1 = \frac{-l^2}{l^2+\tau^2}\,. \tag{9}$$

This satisfies the boundary conditions which require the surface to be anchored at the subregion interface, *i.e.* $r \to l$ as $\tau \to 0$. Unlike the strip case, there are no parameters for the surface in this case[2] for $d > 2$. We see from (8), (9), that for τ real, there is no bulk turning point where $\frac{dr}{d\tau} \to \infty$ with the surface turning around smoothly: instead the surface asymptotically approaches $r^2 \to \tau^2$. Furthermore this surface has $r(\tau) \ge l$ whereas all interior points within the subregion satisfy $0 \le r \le l$ with $r \to l$ near $\tau \sim 0$, so that this surface bends "outwards" from the subregion boundary. This is a real timelike surface with $\dot{r} \le 1$. It is then more natural to consider, rather than (6), the area as $S_{dS} = \frac{R_{dS}^{d-1}\Omega_{d-2}}{4G_{d+1}} \int \frac{d\tau}{\tau^{d-1}} r^{d-2}\sqrt{1-\dot{r}^2}$ which is real-valued. Since the surface does not "end" at any finite τ, we consider the whole τ-range and obtain $S_{dS} = \frac{R_{dS}^{d-1}\Omega_{d-2} l}{4G_{d+1}} \int_{-\infty}^{-\epsilon} \frac{d\tau}{\tau^{d-1}} (l^2+\tau^2)^{(d-3)/2}$, taking $\tau_{UV} = -\epsilon$. This gives $S_{dS} = \frac{R_{dS}^2}{4G_4}\frac{A_1}{\epsilon}$ [for dS_4, with $A_1 = 2\pi l$ the interface area of the circular subregion], and $S_{dS} = \frac{R_{dS}^3}{8G_5}\frac{A_2}{\epsilon^2} + \frac{\pi R_{dS}^3}{2G_5}\log\frac{l}{\epsilon}$ [for dS_5, with $A_2 = 4\pi l^2$ the 2-sphere interface area]. Note that there are no interesting finite cutoff-independent pieces for these surfaces since those contributions die at $|\tau| \to \infty$.

From the point of view of the dual Euclidean CFT, we expect that the central charge coefficients in the extremal surface area (interpreted as entanglement entropy) must match those of the CFT. For the leading area law divergence (which is not sensitive to the detailed geometry of the subregion) of a spherical subregion, the scaling must be the same as for the strip, matching the central charges obtained in [7] which are negative or imaginary. In addition there are expected to be universal coefficients for the sphere case which should match CFT anomaly coefficients. Further we would also intuitively expect that there exist interesting finite cutoff-independent parts which are size-dependent measures of entanglement entropy from the CFT point of view. Given these expectations, the real surfaces discussed above are unsatisfactory.

This suggests that we consider imaginary τ parametrized as $\tau = iT$ with T real, as for the strip case. Thus (8) becomes

$$r^2 = l^2 - T^2 \quad \Rightarrow \quad r_{min} = 0 \ \text{ at the turning point}$$
$$\tau_* = il \quad \Rightarrow \quad \Delta r = l\,. \tag{10}$$

Now $r(\tau)$ maps each point on the surface directly to a corresponding real-valued spatial location within the subregion in the dual CFT (*i.e.* the surface bends "inward"). We require that the boundary subregion radial parameter r be real-valued in (8) since this represents a spatial direction in the CFT: this excludes more general paths in complex τ-space. The range of τ is now restricted, and the subregion size given by $\Delta r \equiv r_{max} - r_{min}$ is bounded. (Perhaps more general complex paths and surfaces exist if both r, τ are complexified.)

Thus using (8), (9), (10), the area (6) in dS_{d+1} becomes

$$S_{dS} = \frac{R_{dS}^{d-1}\Omega_{d-2}}{4G_{d+1}} \int_{\tau_{UV}}^{\tau_*} \frac{d\tau}{\tau^{d-1}} (-il)(l^2+\tau^2)^{(d-3)/2}\,. \tag{11}$$

The integration is along the path $\tau = iT$, with $\tau_{UV} = i\epsilon$ and $\tau_* = il$. The leading divergence here is of the form of the area law (for $d > 2$), given by

[2] For $d = 2$, with just one spatial dimension, there is no difference between a strip and a sphere so the analysis is similar to that in [21]. In detail, from (7) we have $\dot{r}^2 = \frac{-B^2\tau^2}{1-B^2\tau^2}$. With $B^2 < 0$, these are real surfaces parametrized as $r(\tau) = \pm\sqrt{\tau^2 + (1/B^2)} + C$ subject to the boundary conditions *e.g.* $r \to \pm\frac{l}{2}$ as $\tau \to 0$. "Inward" bending half-extremal-surfaces appropriately joined (Fig. 1 in [21]) can be constructed asymptoting to null surfaces with zero area. Alternatively "outward" bending surfaces which extend all the way to $|\tau| \to \infty$ are represented by *e.g.* the two half-surfaces $r_L = -\sqrt{(l^2/4)+\tau^2}$, $r_R = \sqrt{(l^2/4)+\tau^2}$, with area $S_{dS} = 2\frac{R_{dS}}{4G_3}\int_{-\infty}^{-\epsilon}\frac{d\tau}{\tau}\frac{l/2}{\sqrt{(l^2/4)+\tau^2}} = \frac{R_{dS}}{2G_3}\log\frac{l}{2\epsilon}$.

$$S_{dS}^{div} = \frac{i}{d-2}\frac{R_{dS}^{d-1}\Omega_{d-2}}{4G_{d+1}}\frac{l^{d-2}}{\tau_{UV}^{d-2}} = \frac{i^{1-d}}{d-2}\frac{R_{dS}^{d-1}}{4G_{d+1}}\frac{\mathcal{A}_{d-2}}{\epsilon^{d-2}}\,, \tag{12}$$

with $\mathcal{A}_{d-2} \equiv l^{d-2}\Omega_{d-2}$ the interface area. There is an overal $\pm$ sign ambiguity in the choice of the square root branch in (6), (11), which we have fixed to be $+$ in (12), as in the strip subregions reviewed earlier. This sign corresponds to choosing $\sqrt{-l^2} = -il$ in (11). As for the strip [21], the leading divergence here has the form $\mathcal{C}_d \frac{\mathcal{A}_{d-2}}{\epsilon^{d-2}}$ with $\mathcal{C}_d \sim i^{1-d}\frac{R_{dS}^{d-1}}{G_{d+1}}$ of the form appearing in the $\langle TT\rangle$ correlators in [7]. We also see that analytic continuation using (2) from the leading area law divergence from the Ryu–Takayanagi expression in AdS/CFT gives $\frac{R^{d-1}}{4G_{d+1}}\frac{l^{d-2}\Omega_{d-2}}{(d-2)\epsilon^{d-2}} \longrightarrow \frac{i^{1-d}}{d-2}\frac{R_{dS}^{d-1}}{4G_{d+1}}\frac{l^{d-2}\Omega_{d-2}}{\epsilon^{d-2}}$ which is the sign above.

There are subleading terms: *e.g.* for dS_4, the area (11) gives

$$S_{dS} = \frac{R_{dS}^2\Omega_1}{4G_4}(-il)\int_{\tau_{UV}}^{\tau_*}\frac{d\tau}{\tau^2} = -\frac{\pi R_{dS}^2}{2G_4}\Big(\frac{l}{\epsilon}-1\Big)\,. \tag{13}$$

The finite constant cutoff-independent piece $\frac{\pi R_{dS}^2}{2G_4}$ is a universal term. For d even, one of the subleading terms is the universal logarithmic term. Expanding (11), this logarithmic term can be seen to be

$$-i\begin{pmatrix}\frac{d-3}{2}\\ \frac{d-2}{2}\end{pmatrix}\frac{\Omega_{d-2}}{4}\frac{R_{dS}^{d-1}}{G_{d+1}}\log\frac{l}{\epsilon}\,, \tag{14}$$

where $\binom{\nu}{k}$ is the (generalized) binomial coefficient of the x^k-term in the expansion of $(1+x)^\nu$ and $\Omega_d = \frac{2\pi^{(d+1)/2}}{\Gamma((d+1)/2)}$ is the d-dim sphere volume. The argument in the logarithmic term is obtained as $\frac{\tau_*}{\tau_{UV}} = \frac{il}{i\epsilon}$. Explicitly, the coefficients (14) for dS_3, dS_5, dS_7 are

$$-i\frac{R_{dS}}{2G_3}\quad [dS_3]\,,\qquad -i\frac{\pi R_{dS}^3}{2G_5}\quad [dS_5]\,,$$
$$-i\frac{\pi^2 R_{dS}^5}{4G_7}\quad [dS_7]\,. \tag{15}$$

These coefficients resemble those arising in the $\langle TT\rangle$ correlators in [7], except that the numerical factors are unambiguously fixed here. For dS_3, the area contains only the logarithmic term and the coefficient can be calculated directly[3] from (6), (11): writing this area as $\frac{c}{3}\log\frac{l}{\epsilon}$ gives the central charge $c = -i\frac{3R_{dS}}{2G_3}$ which can be seen to be the analytic continuation of the known AdS_3 central charge $\frac{3R_{AdS}}{2G_3}$.

Note that $-i\frac{R_{dS}^{d-1}}{G_{d+1}}$ under the analytic continuation (2) becomes $(-1)^{\frac{d}{2}-1}\frac{R_{AdS}^{d-1}}{G_{d+1}}$ which we recall arises in the universal coefficient of the logarithmic term in the AdS case for spherical surfaces [17] (and the numerical factors also corroborate). This coefficient is proportional to the a central charge appearing in the trace anomaly of the CFT on a sphere (for even d): note that in Einstein gravity, the central charges a, c are the same, with $a \sim \frac{R_{AdS}^{d-1}}{G_{d+1}}$ [28–31].

This suggests that in dS/CFT, the coefficients of the logarithmic term for these complex extremal surfaces are the analogs of these a-type central charges in the Einstein gravity approximation. These coefficients match with those in the logarithmically divergent terms in the CFT free energy evaluated using dS/CFT as we discuss now.

4. $\Psi \sim e^{iS}$, CFT on sphere and conformal anomaly

For what follows, it is useful to recall the dS/CFT correspondence for de Sitter space. A version of dS/CFT [5–7] states that quantum gravity in de Sitter space is dual to a Euclidean CFT living on the boundary $\mathcal{I}^+$. More specifically, the CFT partition function with specified sources $\phi_{i0}(\vec{x})$ coupled to operators $\mathcal{O}_i$ is identified with the bulk wavefunction of the universe as a functional of the boundary values of the fields dual to $\mathcal{O}_i$ given by $\phi_{i0}(\vec{x})$. In the classical regime this becomes $Z_{CFT} = \Psi[\phi_{i0}(\vec{x})] \sim e^{iS_{cl}[\phi_{i0}]}$ where we need to impose regularity conditions on the past cosmological horizon $\tau \to -\infty$: *e.g.* scalar modes satisfy $\phi_k(\tau) \sim e^{ik\tau}$, which are Hartle–Hawking (or Bunch–Davies) initial conditions. Operationally, certain dS/CFT observables can be obtained by analytic continuation (2) from AdS (see *e.g.* [7], as well as [8]).

For even dimensions d, the free energy of the CFT_d on a sphere is expected to contain a logarithmic divergence whose coefficient is related to the integrated conformal anomaly of the CFT. Since the (nonunitary) CFT here is that dual to de Sitter space, this can be calculated holographically using the dS/CFT dictionary $Z_{CFT} = \Psi$ [7] with an auxiliary de Sitter space in global coordinates whose constant time slices are spheres. In the classical regime, we approximate the Hartle–Hawking wavefunction of the universe Ψ in terms of the bulk action S of this auxiliary de Sitter space: this gives $-F = \log Z_{CFT} = \log\Psi \sim iS$. We can then calculate the coefficient of the logarithmic term in the classical approximation.

de Sitter space dS_{d+1} in global coordinates, with scale R_{dS}, is

$$ds^2 = -dt^2 + R_{dS}^2\Big(\cosh\frac{t}{R_{dS}}\Big)^2 d\Omega_d^2\,. \tag{16}$$

The spatial slices are d-spheres, with minimum radius R_{dS} at $t = 0$. This is a solution to Einstein gravity $R_{MN} = \frac{d}{R_{dS}^2}g_{MN}$ with cosmological constant $\Lambda = \frac{d(d-1)}{2R_{dS}^2}$. The on-shell bulk action is

$$\begin{aligned} S &= \frac{1}{16\pi G_{d+1}}\int d^{d+1}x\sqrt{-g}(R-2\Lambda) \\ &= \frac{1}{16\pi G_{d+1}}\int dt d^d\Omega_d\, R_{dS}^d\Big(\cosh\frac{t}{R_{dS}}\Big)^d\frac{2d}{R_{dS}^2}\,, \end{aligned} \tag{17}$$

where $R - 2\Lambda = \frac{d(d+1)-d(d-1)}{R_{dS}^2}$ and Ω_d the d-dim sphere volume. We have suppressed writing the surface terms and counterterms for canceling the leading divergences in this action since the logarithmic term we are interested in arises solely from the bulk action: this is motivated by similar arguments in AdS/CFT (see *e.g.* [28–31] and the review [4]). This gives

$$\begin{aligned} S &= \frac{2d\,\Omega_d R_{dS}^{d-1}}{16\pi G_{d+1}}\int\frac{dt}{R_{dS}}\Big(\cosh\frac{t}{R_{dS}}\Big)^d \\ &= \frac{R_{dS}^{d-1}}{16\pi G_{d+1}}\frac{2d\,\Omega_d}{2^d}\int d\Big(\frac{t}{R_{dS}}\Big)\,e^{dt/R_{dS}}(1+e^{-2t/R_{dS}})^d\,. \end{aligned} \tag{18}$$

With $\tau = -2R_{dS}e^{-t/R_{dS}}$, the metric (16) at asymptotically late times becomes of Poincaré form $ds^2 \sim \frac{R_{dS}^2}{\tau^2}(-d\tau^2 + R_{dS}^2 d\Omega_d^2)$. It is useful to write the bulk action by redefining $y = e^{t/R_{dS}} = \frac{2R_{dS}}{-\tau}$ and we obtain

$$S = \frac{R_{dS}^{d-1}}{16\pi G_{d+1}}\frac{2d\,\Omega_d}{2^d}\int^{y_{UV}} dy\, y^{d-1}\Big(1+\frac{1}{y^2}\Big)^d\,. \tag{19}$$

[3] Explicitly the surface is parametrized by the two half-surfaces $x_L = -\sqrt{(l^2/4)-T^2}$, $x_R = \sqrt{(l^2/4)-T^2}$ satisfying the boundary conditions $x_L \to -l/2$, $x_R \to l/2$ as $\tau = iT \to 0$ and $x_L, x_R \to 0$ as $\tau \to \tau_* = il/2$. It is easy to check that these join smoothly at τ_*: the resulting surface can be recognized as a continuation of the AdS_3 case. The area is $S_{dS} = 2\frac{R_{dS}}{4G_3}\int_{i\epsilon}^{il/2}\frac{d\tau}{\tau}(-il)\frac{1}{\sqrt{l^2+\tau^2}}$, with log-coefficient as above.

The upper limit of integration here is at large t *i.e.* the future cutoff τ_{UV}. The lower limit will not be important in what follows as long as some regularity conditions are satisfied (see *e.g.* the Hartle–Hawking prescription [32]).

The expansion of this action has a logarithmic term for d even. Now the wavefunction of the universe in the classical approximation is $\Psi = e^{iS}$ and the free energy is $-F \equiv \log Z = \log\Psi \equiv iS$ [7]. Thus the logarithmic term in the free energy can be found by expanding the action (19), which arises as

$$iS = \ldots + i\binom{d}{\frac{d}{2}}\frac{2d\ \Omega_d}{16\pi\ 2^d}\frac{R_{dS}^{d-1}}{G_{d+1}}\log\frac{R_{dS}}{\epsilon} + \ldots$$
$$= -i\binom{d}{\frac{d}{2}}\frac{2d\ \Omega_d}{16\pi\ 2^d}\frac{R_{dS}^{d-1}}{G_{d+1}}\log\epsilon + \ldots, \qquad (20)$$

where the cutoff is $y_{UV} = \frac{2R_{dS}}{\epsilon}$ and $\binom{\nu}{k}$ is the binomial coefficient. In Euclidean AdS_{d+1} with metric $ds^2 = d\rho^2 + R_{AdS}^2\sinh^2(\frac{\rho}{R_{AdS}})d\Omega_d^2$ (the expected gravity dual for a conventional unitary Euclidean CFT on a sphere), a similar calculation yields the anomaly coefficient as is well known [28–31]: the Euclidean AdS action is $S^{EAdS} = \frac{1}{16\pi G_{d+1}}\int_{\epsilon_z} dz d^d x\sqrt{g}(R + 2|\Lambda|)$ (and $Z \sim e^{-S^{EAdS}}$), and the relevant terms arise in the expansion near the boundary. Under the analytic continuation $z \to -i\tau$, $R_{AdS} \to -iR_{dS}$, we have $-S^{EAdS} \to iS^{dS}$ and the $-i\frac{R_{dS}^{d-1}}{G_{d+1}}$ factor above continues to $(-1)^{\frac{d}{2}-1}\frac{R_{AdS}^{d-1}}{G_{d+1}}$ in $EAdS$. Near the boundary $z = \epsilon_z$, an asymptotically $EAdS_5$ space $ds^2 = \frac{R_{AdS}^2}{z^2}(dz^2 + \hat{g}_{\mu\nu}dx^\mu dx^\nu)$ gives $-S_{EAdS_5} \sim \frac{R_{AdS}^3}{G_5}(\frac{\#}{\epsilon_z^4} - \frac{\#}{\epsilon_z^2} - \#\log\epsilon_z + \ldots)$, with the # being positive coefficients for S^4 boundary (and $z = 2R_{AdS}e^{-\rho/R_{AdS}}$ near the boundary). Most terms analytically continue to give pure imaginary terms: with $\epsilon_z \to -i\epsilon_\tau$, the log-term continues as $\log\epsilon_z \to \log|\epsilon_\tau| + i\frac{\pi}{2}$, the final term giving a real factor in Ψ (see [33] for interesting discussions on relations of the coefficient of this logarithmic term to the Hartle–Hawking factor $|\Psi|^2$). From the point of view of the dS calculation (19), τ being real makes the action real and so iS is pure imaginary: a real part in Ψ is obtained by deforming the contour slightly in the far past, *e.g.* $y_{IR} \sim \frac{2R_{dS}}{|\tau_{IR}| - i\tilde{\epsilon}} \sim i\tilde{\epsilon}$ as $\tau_{IR} \to -\infty$. Then the logarithmic term gives a real term as $iS \sim \ldots - \frac{i\pi R_{dS}^3}{2G_5}\log y_{IR} \sim -\frac{i\pi R_{dS}^3}{2G_5}(\log i) = \frac{\pi^2 R_{dS}^3}{4G_5}$ and correspondingly the Hartle–Hawking factor $|\Psi|^2 \sim \exp[\frac{\pi^2 R_{dS}^3}{2G_5}]$ (this agrees with [33] using $M_{Pl}^3 = \frac{1}{8\pi G_5}$). We also note previous work [34] on the conformal anomaly in dS/CFT (which is however not based on the wavefunction of the universe).

The coefficient of this logarithmic term in (20) in the free energy via Ψ can be seen to be the same as that in the logarithmic term (14) in the complex spherical extremal surfaces. They appear to be the analogs of the a-type central charges in dS/CFT.

Further light is shed on this calculation in light of [35]. A conformal mapping was used there to transform the entanglement entropy of a spherical subsystem in AdS/CFT to the thermal entropy of the CFT in the static patch of an auxiliary de Sitter space. For AdS, this allows a precise comparison with the coefficient of the logarithmic term appearing in the extremal surface area. These coefficients are related to the conformal anomaly and the central charge a (which is also c) [17] in the Einstein gravity approximation (see also [36,37] for higher derivative theories).

It would appear that a similar argument is at play here modulo some caveats (below). The CFT in this case, intrinsically Euclidean, lives on the flat Euclidean space on the future boundary $\mathcal{I}^+$ of de Sitter space and is nonunitary. We use a conformal mapping to transform this flat d-dim Euclidean space $ds_E^2 = dt_E^2 + dr^2 + r^2 d\Omega_{d-2}^2$ to a sphere: this is given by the coordinate transformation

$$t_E = l\frac{\cos\theta\sin\frac{\rho}{l}}{1+\cos\theta\cos\frac{\rho}{l}}, \qquad r = l\frac{\sin\theta}{1+\cos\theta\cos\frac{\rho}{l}}\ :$$
$$ds_E^2 = \Omega^2[\cos^2\theta d\rho^2 + l^2(d\theta^2 + \sin^2\theta d\Omega_{d-2}^2)],$$
$$\Omega = \frac{1}{1+\cos\theta\cos\frac{\rho}{l}}. \qquad (21)$$

Removing the conformal factor Ω^2, this space becomes $d\tilde{s}^2 = \cos^2\theta d\rho^2 + l^2(d\theta^2 + \sin^2\theta d\Omega_{d-2}^2)$. Demanding that this space be smooth, we must avoid a conical singularity at $\theta = \frac{\pi}{2}$: then the coordinate ρ must be taken to be periodic with period $\Delta\rho = 2\pi l$. This space can then be seen to be a d-sphere $d\tilde{s}^2 = l^2(d\theta_1^2 + \sin^2\theta_1 d\theta_2^2 + \sin^2\theta_1\sin^2\theta_2 d\Omega_{d-2}^2)$, using a coordinate transformation $\sin\theta = \sin\theta_1\sin\theta_2$, $\tan\frac{\rho}{l} = \cos\theta_2\tan\theta_1$.

The free energy F of the Euclidean CFT on this sphere is expected to exhibit a logarithmically divergent term (in even dimensions) whose coefficient is related to the conformal anomaly. In general, we have an expansion $-F_{CFT} = \log Z_{CFT} =$ (non-universal terms) $+\ a\log\epsilon\ +$ (finite), with ϵ the ultraviolet cutoff. The CFT energy–momentum tensor [7] is defined as $T_{ij} = \frac{2}{\sqrt{h}}\frac{\delta Z_{CFT}}{\delta h^{ij}} = \frac{2}{\sqrt{h}}\frac{\delta\Psi}{\delta h^{ij}}$ which becomes $T_{ij} \sim \frac{2}{\sqrt{h}}\frac{\delta(-F_{CFT})}{\delta h^{ij}} \sim i\frac{2}{\sqrt{h}}\frac{\delta S}{\delta h^{ij}}$ in the classical approximation for Ψ. Under an infinitesimal conformal transformation $h_{ij} \to (1 + 2\delta\lambda)h_{\mu\nu}$, *i.e.* $\delta h^{ij} = -(2\delta\lambda)h^{ij}$, we have $\frac{\delta F_{CFT}}{\delta\lambda} = \int d^d x\sqrt{h}\langle T^k{}_k\rangle + (div)$, which is the integrated trace anomaly. Due to conformal invariance, this must be equivalent to simply shifting the ultraviolet cutoff $\epsilon \to (1 - \delta\lambda)\epsilon$. This gives the coefficient $a = \int\langle T^k{}_k\rangle$. This argument does not appear to require unitarity of the conformal field theory. We have calculated this free energy holographically assuming the nonunitary CFT has a de Sitter gravity dual and using the dS/CFT dictionary $Z_{CFT} = \Psi$ [7] with an auxiliary de Sitter space in global coordinates (where constant time slices are spheres). As we have seen, we find agreement with the coefficients of the logarithmic terms in the complex extremal surfaces earlier. The fact that these coefficients are pure imaginary is expected from the i in the relation $-F \sim iS_{bulk}$.

Finally, we expect that from the point of view of a CFT replica calculation, the entanglement entropy is $S_{CFT}^{EE} = -\lim_{n\to 1}\partial_n tr\rho_A^n$ where $tr\rho_A^n = \frac{Z_n}{(Z_1)^n}$ with Z_n the partition function on the n-sheeted replica space. A scale change is expected to be of the form $l\frac{\partial}{\partial l}S_{CFT}^{EE} \sim \int\langle T_\mu{}^\mu\rangle$ which is then related to the free energy F_{CFT} so that the logarithmic term coefficient in S_{CFT}^{EE} would be of the form $a\log\frac{l}{\epsilon}$. This argument does not depend on unitarity. Thus if S_{CFT}^{EE} is evaluated from the bulk side as the area S_{dS} of appropriate extremal surfaces, we expect the log-coefficients to match: this is vindicated for the complex surfaces we have been discussing.

5. Discussion

We have studied complex extremal surfaces for spherical subregions on a constant boundary Euclidean time slice of the future boundary of de Sitter space, building on [21]: as in that case, this ends up being quite different from the AdS case due to sign differences which makes the bulk quite different in structure. For even boundary dimensions, there is a logarithmically divergent term in the area of these surfaces whose coefficient is a universal term. Comparing this with a corresponding coefficient (related to the integrated conformal anomaly) in a logarithmically divergent term in the free energy of the dual Euclidean CFT on a sphere using the dS/CFT dictionary for a dual de Sitter space in global coordinates in a classical approximation for the wavefunction $\Psi \sim e^{iS}$, we find agreement including numerical factors. This coefficient is of the form $-i\nu_d \frac{R_{dS}^{d-1}}{G_{d+1}}$ where ν_d is a real positive numerical factor. Our analysis here and in [21] has effectively enlarged the original question of finding solutions to the extremization problem in de Sitter space (Poincaré slicing) with our boundary conditions, since the τ-parametrization being complex lies outside the original de Sitter τ-range: the eventual answers pass the checks of agreement of various central charges based on $Z_{CFT} = \Psi$. Perhaps this agreement is not surprising since both sides in this Einstein gravity approximation effectively amount to analytic continuation from the *AdS* case (where there is agreement), but it shows consistency between the two sides in the present case.

From the point of view of the dual Euclidean CFT, we expect that the central charge coefficients in the extremal surfaces area (interpreted as entanglement entropy) must match those in the dual Euclidean CFT obtained in [7] (which are negative or imaginary): this includes the leading area law divergence as well as subleading universal coefficients. We would also intuitively expect finite cutoff-independent parts which are size-dependent measures of entanglement entropy in the CFT. These expectations point to the complex extremal surfaces we have been considering which exhibit these features. The resulting analysis for these codim-2 complex extremal surfaces in de Sitter space in the end boils down to analytic continuation from the Ryu–Takayanagi formulation in *AdS* (and thus resembles known *AdS*/*CFT* results with *i*s or minus signs in appropriate places): however this was not obvious to begin with. Perhaps the other surfaces we have discussed are also of interest, in other contexts.

The investigations here and those in [21] thus support the idea that the areas of these complex extremal surfaces encode entanglement entropy of the dual Euclidean CFT in *dS*/*CFT*, using the formulation $Z_{CFT} = \Psi$ with Ψ the wavefunction of the universe [7]. It also suggests that in *dS*/*CFT*, the coefficients of the logarithmic terms for these complex extremal surfaces are perhaps the analogs of the *a*-type central charges. Relatedly it may be interesting to study analogs of [38] in the de Sitter case. It is clear however that all our calculations are in the bulk and so cannot clearly pinpoint the interpretation of entanglement entropy. (As an aside however, in the 2d CFT dual to dS_3 with central charge $\sim -i\frac{R_{dS}}{G_3}$, a replica calculation of entanglement entropy [39,17] appears to give $\sim -i\frac{R_{dS}}{G_3}\log\frac{l}{\epsilon}$ under certain assumptions, most importantly the existence of twist sector ground states.) In general the notion of entanglement entropy requires certain basic assumptions on the CFT Hilbert space, most importantly the existence of a ground state. The dual CFT for the pure de Sitter theory here appears to have pathologies in general such as complex conformal dimensions (the higher spin *dS*/*CFT* of [9] appears better-behaved in this regard, but geometric extremal surfaces may not be of relevance in this case). Also the CFT is intrinsically Euclidean, with no notion of time evolution (while the dual bulk time direction emerges). Thus the use of a conformal transformation along the lines of [35] to map the reduced density matrix to *e.g.* a thermal one appears more delicate, in such nonunitary CFTs. It would appear that this CFT entanglement entropy, assuming it exists, encodes CFT correlations and is thus likely, if only indirectly, to also encode bulk de Sitter expectation values which have intricate connections to the dual CFT correlation functions [7]. These issues would be interesting to explore further.

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