ELSEVIER

Theoretical Computer Science 233 (2000) 279–286

Theoretical Computer Science

www.elsevier.com/locate/tcs

Note

On slender 0L languages

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Received August 1997; revised September 1998
Communicated by G. Rozenberg

Abstract

We give a complete proof of Theorem 3.1 in [2]. A pathological exception of Theorem 4.3 in [2] is exhibited and a condition to remove it is mentioned.

Keywords: 0L system; Slender language; Factorization of word

1. Introduction

This note supplements a few insufficiencies of our previous paper [2]. The main part of this note is devoted to the proof of Theorem 3.1 in [2], whose original proof was insufficient by McNaughton's comment [1]. A few lemmas in [2] are changed because they are dependent on the insufficient proof. The formulation of the main theorem of [2], Theorem 4.3, is slightly changed to avoid pathological exceptions.

We use notations and definitions in [2] without explicit reference and the argument of this note begins at the beginning of Section 3 of [2]. As for the insufficiency of the proof of Lemma 1.6 in [2] pointed out by McNaughton [1], the value of j_1 which satisfies $|v|j_1 \geqslant |u|i_2$ if $i_2 \geqslant i_1$ or $|v|j_1 \leqslant |u|i_2$ if $i_2 < i_1$ suffices for the proof.

2. Complete proof of Theorem 3.1

The proof of Theorem 3.1 in [2] shows that different descendants $x, y \in \tau^n(a)$ of an unbounded letter a in a slender 0L system for some $n > 0$ must have the form

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0304-3975/00/$ - see front matter
PII: S0304-3975(99)00111-5

$x=(zz')^{i_1}z$ and $y=(zz')^{i_2}z$ for some $z, z' \in \Sigma^*$ and $i_1, i_2 \in \mathbb{N}$. But the proof does not tell whether or not a third word $u \in \tau^n(a)$ has the structure $u=(zz')^{i_3}z$.

To complete the proof of Theorem 3.1 in [2], we first investigate a pair of words which have repetitive structures of two common words. Then we separately consider self-productive, persistent non-self-productive, and nonpersistent letters.

Definition 1. Let $x, y \in \Sigma^+$ be words. Two words $z, z' \in \Sigma^*$ are said to be *prime* 2-*factors* of x and y if they satisfy

1. $z \neq 1$ (1 is the empty word),
2. $x=(zz')^{i_1}z$ and $y=(zz')^{i_2}z$ for some $i_1 \geqslant 0$ and $i_2 \geqslant 0$, and
3. If $x=(uu')^{j_1}u$ and $y=(uu')^{j_2}u$, then $i_1 \geqslant j_1$ and $i_2 \geqslant j_2$.

The next property shows that prime 2-factors of different words are unique.

Property 1. *Let $x, y \in \Sigma^+$ be words with $|x| < |y|$. If x and y have prime 2-factors z, z', then z and z' are unique.*

Proof. Let u and u' be prime 2-factors of x and y, i.e.,

$$x=(zz')^i z=(uu')^i u \quad \text{and} \quad y=(zz')^j z=(uu')^j u.$$

Then the lengths of these words satisfy

$$|zz'|i+|z|=|uu'|i+|u| \quad \text{and} \quad |zz'|j+|z|=|uu'|j+|u|,$$

so that

$$(i-j)|zz'|=(i-j)|uu'|.$$

Because $i \neq j$, we have $|zz'|=|uu'|$ and, thus, $|z|=|u|$ by the above equations. Therefore, $z=u$ and $z'=u'$. □

If an unbounded letter of a slender 0L system is nondeterministic, then the different descendants derived in the same steps have prime 2-factors. Indeed, the proof of Theorem 3.1 in [2] proves the next lemma.

Lemma 2. *If a 0L system $G=\langle \Sigma, \tau, \omega \rangle$ is slender, then for every non-deterministic unbounded letter $a \in \Sigma$; $x, y \in \tau^n(a)$ for some $n > 0$ with $x \neq y$ implies that x and y have prime 2-factors, that is, there are $z, z' \in \Sigma^*$ such that $x=(zz')^i z$ and $y=(zz')^j z$ for some $i \geqslant 0$ and $j \geqslant 0$.*

The descendants of an unbounded letter satisfy another important property: which says that the number of descendants of a specific length at given steps is at most one.

Property 3. *If a 0L system $G=\langle \Sigma, \tau, \omega \rangle$ is slender and $a \in \Sigma$ is an unbounded letter in G, then for every nonnegative integers n and l*

$$\text{card}(\{w \in \tau^n(a) \mid |w|=l\}, \leqslant 1.$$

Proof. Since a is unbounded, for every integer k $L(G)$ has a word $v = v_0 a v_1 a \dots a v_k$. If there are different words u_1 and u_2 in $\tau^n(a)$ with $|u_1| = |u_2|$, then $\tau^n(v)$ has at least $\binom{k}{k/2}$ different word of the same length

$$v_0' u_{i_1} v_1' u_{i_2} \dots u_{i_k} v_k'$$

where $k/2$ of u_{i_j}'s are equal to u_1 and the others u_2 and $v_j' \in \tau^n(v_j)$ for $j = 0, 1, \dots, k$. □

Nondeterministic self-productive letters satisfy the next proposition, whose assertion is the same as Theorem 3.1 in [2].

Proposition 4. *Let $G = \langle \Sigma, \tau, \omega \rangle$ be a slender 0L system. If $a \in \Sigma$ is a nondeterministic self-productive letter in G, then for every $n \in \mathbb{N}_+$ there exist $z, z' \in \Sigma^*$ and a finite set of integers I_n such that $\tau^n(a) = \{(zz')^i z \mid i \in I_n\}$.*

A detailed observation of prime 2-factors will be needed in the proof of this proposition.

Lemma 5. *Let $z, z' \in \Sigma^*$ be prime 2-factors of some words x and y with $x = (zz')^{i_1} z$ and $y = (zz')^{i_2} z$ and $i_1 < i_2$. If*

$$xuy = yux$$

for some $u \in \Sigma^*$, then $u = (z'z)^j z'$ for some $j \in \mathbb{N}$.

Proof. By the assumptions we have

$$u(zz')^{i_2 - i_1} = (z'z)^{i_2 - i_1} u. \quad (1)$$

Because u begins with $z'z$, u has the factorization $u = (z'z)^j r$ where $j \geqslant 0$, $|r| < |z'z|$, and r is a prefix of $z'z$ if $|u| \leqslant |(z'z)^{i_2 - i_1}|$. If $|u| > |(z'z)^{i_2 - i_1}|$, u has the factorization $u = (z'z)^{i_2 - i_1} u'$ such that $u'(zz')^{i_2 - i_1} = (z'z)^{i_2 - i_1} u'$. Repeating this process, we have $u = (z'z)^{n(i_2 - i_1)} u' = u'(zz')^{n(i_2 - i_1)}$ with $|u'| \leqslant |(z'z)^{i_2 - i_1}|$. Now u has the factorization $u = (z'z)^j r$ for some prefix r of $z'z$. Then Eq. (1) implies

$$r(zz')^{i_2 - i_1} = (z'z)^{i_2 - i_1} r. \quad (2)$$

The initial segment of length $|rzz'|$ of (2) shows that $rzz' = z'zr$ so that

$$(rz)(z'z) = (z'z)(rz).$$

Now by Lemma 1.2 in [2] we have $z'z = s^l$, $z = vs^p$, $z' = s^{l-p-1} v'$, and $r = s^{m-p-1} v'$ for some $v'v = s$. Since z and z' are prime 2-factors of x and y, the following equations hold:

$$x = (zz')^{i_1} z = z(z'z)^{i_1} = vs^{p + i_1 l} = (vv')^{p + i_1 l} v.$$

The definition of prime 2-factors implies that $p=0$ and $l=1$, i.e., $z=v$, $z'=v'$, and $r=(z'z)^{m-1}z'$. Since $|r| < |z'z|$, we have $r=z'$. □

Proof of Proposition 4. By Lemma 2, for every $n \in \mathbb{N}_+$ and $x_1, x_2, \in \tau^n(a)$, x_1 and x_2 have prime 2-factors $z, z' \in \Sigma^*$ with $x_1=(zz')^{i_1}z$ and $x_2=(zz')^{i_2}z$ for some $i_1 < i_2$. Let x_3 be a word in $\tau^n(a)$. We may assume that $|x_2| < |x_3|$, for otherwise let the shorter two words have prime 2-factors z and z'.

Since a is self-productive, there is a word $v_0av_1av_2av_3 \in \tau^l(a)$ for some $l \in \mathbb{N}_+$ in which v_0, v_1, and v_2 have no occurrences of a and v_3 may have some occurrences of a. Let $v'_i \in \tau^n(v_i)$ $i=0,1,2,3$ be fixed words. Since

$$w_1=v'_0x_1v'_1x_2v'_2x_2v'_3, \quad w_2=v'_0x_2v'_1x_1v'_2x_2v'_3, \quad w_3=v'_0x_2v'_1x_2v'_2x_1v'_3 \in \tau^{l+n}(a)$$

and $|w_1|=|w_2|=|w_3|$, we have $w_1=w_2=w_3$ by Property 3. Then by Lemma 5 $v'_1=(z'z)^{j_1}z'$ and $v'_2=(z'z)^{j_2}z'$. Under the same circumstances $\tau^{l+n}(a)$ contains

$$w_4=v'_0x_1v'_1x_3v'_2x_2v'_3 \quad \text{and} \quad w_5=v'_0x_2v'_1x_3v'_2x_1v'_3$$

with $|w_4|=|w_5|$. Then by Property 3 $w_4=w_5$ and by Lemma 5

$$v'_1x_3v'_2=(z'z)^jz'=(z'z)^{j_1}z'(zz')^{j_3}z(z'z)^{j_2}z'$$

where $j=j_1+j_2+j_3+1$. Hence every word in $\tau^n(a)$ has the form $(zz')^iz$. □

If a persistent unbounded letter is not self-productive, then the letter makes at most two descendants in every step.

Lemma 6. *Let $G=\langle \Sigma, \tau, \omega \rangle$ be a slender 0L system. If a nondeterministic persistent unbounded letter a in G is not self-productive, then for every $n \in \mathbb{N}$* $\mathrm{card}(\tau^n(a)) \leqslant 2$.

Proof. Since a is persistent and not self-productive, there is a sequence of letters $a=a_0, a_1, \ldots, a_n=a$ such that $s_ia_it_i \in \tau(a_{i-1})$ for some $s_it_i \in \Sigma^*$ and $a \notin \mathrm{alph}(\tau^*(s_it_i))$ for every $i=1,2,\ldots,n$.

We first claim that for every $i=1,2,\ldots,n$ $\tau(a_{i-1})$ does not contain a word $s'_ibt'_i$ where b is a persistent letter and $b \neq a_i$. Let us assume the converse, that is, $u, v \in \tau^i(a)$ where u has an occurrence of a_i and v has an occurrence of b. Then $u=u_1a_iu_2$ and we have

$$z_1=u'_1w'_1u_1a_iu_2w'_2u'_2, \quad z_2=u'_1w'_1vw'_2u'_2 \in \tau^{n+i}(a),$$

where $w_1aw_2 \in \tau^{n-i}(a_i)$, and $w'_j \in \tau^i(w_j)$ and $u'_j \in \tau^n(u_j)$ $j=1,2$. Since persistent letters have derivations in which they do not disappear, we can assume $|u_1u_2|_P \leqslant |u'_1u'_2|_P$ where P is the set of persistent letters. Lemma 2 says that $z_1=(xy)^{i_1}x$ and $z_2=(xy)^{i_2}x$ for some $x, y \in \Sigma^*$ and $i_1, i_2 \in \mathbb{N}$. Because a_i occurs once in z_1, $z_1=xyx$ or $z_1=x$ but the latter case makes z_2 have more than one occurrences of a_i. So we have $z_1=xyx$ and $z_2=x$. Hence

$$u'_1w'_1u_1a_iu_2w'_2u'_2=u'_1w'_1vw'_2u'_2yu'_1w'_1vw'_2u'_2.$$

Then y is a subword of $u_1 a_i u_2$ and the inequality

$$|u_1 u_2|_P \geqslant |vw_2' u_2' u_1' w_1' v|_P$$

holds. But this inequality contradicts with the following

$$|u_1 u_2|_P \leqslant |u_1' u_2'|_P < |vw_2' u_2' u_1' w_1' v|_P.$$

Therefore, we have shown that a_i is the only persistent letter derived by a in i steps.

Next let $\{u, v_1, v_2\} \subseteq \tau^n(a)$ for some n. Then v_1 and v_2 do not contain any persistent letters nor any ancestors of persistent letters, in other words, they are words over mortal letters. Since a is unbounded, for arbitrarily large $k \in \mathbb{N}$ there is a word $w = w_1 w_2 \cdots w_k \in L(G)$ where $w_i = w_{i0} a w_{i1} a w_{i2} a$ for $i = 1, 2, \ldots, k-1$. Then $\tau^n(w_i)$ has the words of the same length,

$$w_{i0}' v_1 w_{i1}' u w_{i2}' v_2 \quad \text{and} \quad w_{i0}' v_2 w_{i1}' u w_{i2}' v_1,$$

where $w_{ij}' \in \tau^n(w_{ij})$ $j = 0, 1, 2$. Because $L(G)$ is slender, $v_1 w_{i1}' u w_{i2}' v_2 = v_2 w_{i1}' u w_{i2}' v_1$ holds for all but fixed number of i's. By Lemmas 2 and 5, $v_1 = (xy)^{i_1} x$, $v_2 = (xy)^{i_2} x$ and u is a subword of $(xy)^j$ for sufficiently large j. This means that u is a word over mortal letters and that a is also mortal. This is a contradiction. Thus $\mathrm{card}\,\tau^n(a) \leqslant 2$ for every n. □

Finally, we consider nonpersistent letters. A nonpersistent unbounded letter is a descendant of a persistent unbounded letter. So the next lemma exhausts all cases.

Lemma 7. *Let $G = \langle \Sigma, \tau, \omega \rangle$ be a slender 0L system and let a be a persistent unbounded letter in G. If b is a descendant of a, then b derives a or b is deterministic.*

Proof. Let $sbt \in \tau^n(a)$ for some $n > 0$ and $s, t \in \Sigma^*$. If b is nondeterministic, i.e., $\{u, v\} \in \tau^k(b)$ for some $k > 0$, then we have $|u| \neq |v|$ and we can assume $|u| < |v|$. The set $\tau^{n+k}(a)$ has a subset $\{s'ut', s'vt'\}$ where $s' \in \tau^k(s)$ and $t' \in \tau^k(t)$. Since b is unbounded, there exist some words $x, y \in \Sigma^*$ and some integers $0 \leqslant i_1 < i_2$ such that $u = (xy)^{i_1} x$ and $v = (xy)^{i_2} x$. Since a is persistent, there is a word $z \in \tau^{n+k}(a)$ such that z derives a. Because G is slender and a is unbounded in G, the same argument of the proof of Lemma 6 shows that a subword $aw_1 aw_2 a$ of a word in $L(G)$ implies the equality

$$(xy)^{i_1} xt' w_1' z w_2' s' (xy)^{i_2} x = (xy)^{i_2} xt' w_1' z w_2' s' (xy)^{i_1} x \in \tau^{n+k}(aw_1 aw_2 a)$$

in which $w_j' \in \tau^{n+k}(w_j)$ $j = 1, 2$. Then z is a subword of $(xy)^j$ for sufficiently large j. This implies that xy derives a and b also derives a. □

Proposition 4 and Lemmas 6 and 7 completes the proof of Theorem 3.1 in [2]. In this note the theorem has a different number.

Theorem 8. *If a 0L system $G = \langle \Sigma, \tau, \omega \rangle$ is slender, then G satisfies the following condition:*

(8.1) *For every unbounded letter a in* Σ *and for every* $n\in\mathbb{N}_+$ *there exist* $zz'\in\Sigma^+$ *and a finite set* $I\subset\mathbb{N}$ *such that* $\tau^n(a)=\{(zz')^iz \mid i\in I\}$.

Proof. An unbounded letter is persistent or a descendant of a persistent letter. If a is self-productive, Proposition 4 shows the condition. If a is persistent and not self-productive, then Lemmas 2 and 6 verify condition (8.1). If a is a descendant of a persistent letter and a is not persistent, then a is deterministic by Lemma 7. Finally every deterministic letter obviously satisfies condition (8.1). □

Now we modify the discussion after Theorem 3.1 in [2]. The following arguments work instead of Lemmas 3.5, 3.6, and 4.5 in [2], whose proofs depend on the insufficient proof of Theorem 3.1 in [2]. Lemma 3.5 is used in the proof of Lemma 3.6 only and we prove in this note a lemma corresponding to Lemma 3.6 without Lemma 3.5. So we omit Lemma 3.5 in [2]. We give, in the next section, new versions of Lemmas 3.6 and 4.5 as well as pathological exceptions of the main theorem (Theorem 4.3) of [2] and a new condition to avoid them.

3. Changes in lemmas and the main theorem

First we restate Lemma 4.5 in [2]. Because of a slight modification of Theorem 4.3 in [2], which is mentioned later, the assertion 2 of the next lemma is changed from the corresponding assertion of Lemma 4.5 in [2] (assertion (i)). The next lemma says nothing about the subword which does not derive a; for example, if a is derived by v_1, nothing is stated about v_0, while assertion (i) of Lemma 4.5 in [2] stated $v_0\in M^*$. But a pathological exception shows that $v_0\in M^*$ is not always valid.

Lemma 9. *Let* $L(G)$ *be slender and a be a persistent non-deterministic unbounded letter occurring in* $L(G)$*. If a is non self-productive, then*:
1. $\tau^i(a)=u_ia_iu_i'$ *or* $\tau^i(a)=\{u_ia_iu_i',1\}$ *for every i where* a_i *is an ancestor of a and* u_i *and* u_i' *are words over deterministic mortal letters.*
2. *There is a stem letter b such that* $v_0bv_1\in\tau^k(b)$ *where a is the only persistent letter derived by* v_1 *or a is the only persistent letter derived by* v_0 *and the period of a is a divisor of k.*

Proof. 1. By Lemma 6, $\tau^i(a)=\{x_ix_i'x_i,x_i\}$ for every $i>0$ if it is not a singleton. Let $\tau^l(a)=\{x_lx_{l_1}ax_{l_2}x_l,x_l\}$ where l is the period of a. Then

$$\tau^{l+i}(a)=\{\tau^i(x_lx_{l_1})x_ix_i'x_i\tau^i(x_{l_2}x_l),\tau^i(x_lx_{l_1})x_i\tau^i(x_{l_2}x_l),\tau^i(x_l)\}.$$

But since $\tau^{l+i}(a)$ has at most two elements and $x_ix_i'x_i\neq x_i$, we have

$$\tau^i(x_lx_{l_1})x_i\tau^i(x_{l_2}x_l)=\tau^i(x_l),$$

that is $x_i=1$ and $\tau^i(x_{l_1}x_{l_2})=1$ for all $i\in\mathbb{N}_+$. Note that $\tau^l(a)$ cannot be a singleton because a is nondeterministic.

2. Since a is unbounded and not self-productive, there is a stem letter b which produces a, that is, $v_0bv_1 \in \tau^k(b)$ and v_0 or v_1 generates a. We can assume, without loss of generality, that v_0 generates a. We assume that v_0 generates a persistent letter c which is different from a. Let p be the common multiple of periods of a and c. We note that $\tau^p(a) = \{u, 1\}$ and a is the only occurrence of persistent letter in u. Since a and c are persistent, there is a subword $w = aw_1cw_2a$ of a word in $L(G)$ such that $\tau^p(w)$ has the words

$$uw_1'vw_2' \quad \text{and} \quad w_1'vw_2'u$$

of the same length where $w_i' \in \tau^p(w_i)$ $i = 1, 2$ and $v \in \tau^p(c)$. Since $L(G)$ is slender, they are identical. Then by Lemma 1.2 in [2], u and $w_1'vw_2'$ are powers of a common word. This implies that any persistent letter occurring in v is a and contradicts the fact that $\tau^p(c)$ contains a word which has an occurrence of c. Thus a is the only persistent letter derived by v_0.

Let $a_0 = a, a_1, a_2, \ldots, a_l = a$ be the sequence of words such that a_{i-1} derives a_i $i = 1, 2, \ldots, l$. If l is not a divisor of k, there is a subword $aw_1a_iw_2a$ of a word in $L(G)$ with $0 < i < l$. Then the same argument as above leads to a contradiction. Hence l is a divisor of k. □

The next lemma is the new version of Lemma 3.6 in [2]. The conclusion 1 of the next lemma is added to Lemma 3.6 in [2] because nonself-productive case must be considered separately from self-productive case.

Lemma 10. *Let $a, b \in \Sigma$ be nondeterministic unbounded persistent letters which satisfy the condition that, for every nonnegative integer N, there exists a word $w \in L(G)$ such that $|w|_a > N$ and $|w|_b > N$. Then:*
1. *a and b are not self-productive and $a = b$.*
2. *a and b are self-productive and*

$$u_nu_n' = t_nv_nv_n't_n^{-1}$$

for every $n \in \mathbb{N}_+$ where $\tau^n(a) = \{(u_nu_n')^iu_n \mid i \in I_n\}$ and $\tau^n(b) = \{(v_nv_n')^jv_n \mid j \in J_n\}$ are the factorizations given by Theorem 8 and t_n is a suffix of v_nv_n'.

Proof. If a and b are not self-productive, then the previous lemma says that $a = b$. For otherwise, the fact that there is a stem letter c such that $v_0cv_1 \in \tau^k(c)$ and v_0 generates a and v_1 generates b, or vice versa, implies that $L(G)$ is not slender.

Next, consider the case that a is self-productive and b is not self-productive. Let l be the period of b and $\tau^l(b) = \{v, 1\}$. By Theorem 8 we have $\tau^l(a) = \{(u_lu_l')^iu_l \mid i \in I_l\}$. Let $(u_lu_l')^{i_1}u_l$ and $(u_lu_l')^{i_1+c}u_l$ be two words in $\tau^l(a)$, d be the least common multiple of $c|u_lu_l'|$ and $|v|$, and k be $\min(|w|_a, |w|_b)$. Then any word in $\tau^l(w)$ which has $id/|v|$ occurrences of v and $(k-i)d/c|u_lu_l'|$ occurrences of $(u_lu_l')^{i_1+c}u_l$ with the other occurrences of a is replaced for $(u_lu_l')^{i_1}u_l$ has the same length. This contradicts the slenderness of $L(G)$.

If a and b are self-productive, then the same argument as above shows the equation

$$(u_n u_n')^i u_n x = x(v_n v_n')^j v_n,$$

where $\tau^n(b) = \{(v_n v_n')^j v_n \mid j \in J_n\}$. We note that there are arbitrarily large i and j which fulfill the above equation. Then by Lemma 1.1 in [2],

$$(u_n u_n')^i u_n = yz \quad \text{and} \quad v_n(v_n' v_n)^j = zy.$$

Now the lemma follows from Lemma 1.6 in [2]. □

Finally, we change the condition (4) of the assertion (i) of Theorem 4.1 in [2] in order to avoid pathological cases examplifed below.

Example 11. Let $G = \langle \{a,b,c\}, \tau, b\rangle$ where $\tau(a) = \{a, 1\}$, $\tau(b) = \{abc\}$, and $\tau(c) = c^2$ be a $0L$ system. Then

$$\tau^i(b) = a^{[i]} b c^{2^i - 1},$$

where $a^{[i]}$ stands for the set $a^{[i]} = \{1, a, a^2, \ldots, a^i\}$. Since every word in $L(G)$ has different length, $L(G)$ is thin. But $(\tau^i(b))_{i \geqslant 0}$ is not an ultimately extended free generated sequence.

Such pathological cases are excluded by the following slight change.

Condition (4) of the assertion (i) of Theorem 4.3 in [2]. *w has a factorization $w = w_1 w_2 \cdots w_l$ such that,* (a) *the* $0L$ *system* $G_i = \langle \Sigma, \tau, w_i\rangle$ *generates a slender language of type* (1), (2), *or* (3); *or* (b) *there is a stem letter $w_j = a_j$ such that a_j has a production $ua_jv \in \tau^n(a_j)$ where u has, say, an unbounded nondeterministic letter and v has deterministic letters only. In this case if τ is modified to τ' where $ua_j \in \tau'^n(a_j)$, then $\langle \Sigma, \tau', a_j\rangle$ generates a slender language of type* (2) *and if τ is modified to τ'' where $a_jv \in \tau''^n(a_j)$, then $\langle \Sigma, \tau'', a_j\rangle$ generates a slender language of type* (1).

The proof to Theorem 4.3 in [2] is now clear in all cases. If a stem letter which generates a nondeterministic unbounded letter a generates no other persistent letter, we just follow the proof given in [2]. The case in which a stem letter generates deterministic persistent letters falls into the condition (4) above.

A futher characterization of such cases, as well as the general problem about the decidability of slenderness for $0L$ languages, remains open.

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