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# Operator approach to isospin violation in pion photoproduction

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## Abstract

Unambiguous isospin violation in the strong interaction sector is a key issue in low energy hadronic physics, both experimentally and theoretically. Bernstein has employed the Fermi–Watson theorem to demonstrate that pion photoproduction is a process where isospin violation in the  $\pi N$  system can be revealed, an approach we review here. Here we propose a general operator approach to the phenomenon in pion photoproduction, thereby providing an analogue for the framework that was proposed for  $\pi N$  scattering by Kaufmann and Gibbs. The resulting set of amplitudes could form the basis for determining the multipole amplitudes for photoproduction. Thus, the so resulting phase shift determination from photoproduction can then be used via the Fermi–Watson theorem to resolve discrepancies in  $\pi N$  phase shift analyses. We point out that casting effective Lagrangian results in terms of our framework would be beneficial. The upcoming polarization experiments are an ideal setting to test our approach, and also to constrain better the isotensor currents which strictly are not forbidden.

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## 1. Introduction

Of the key issues in hadronic physics (for a eponymous white paper on the subject, see Ref. [1]), the issue of isospin violation in the low energy strong interaction sector is an important one (see Section 5.1 of [1]), both theoretically and experimentally. Charge symmetry breaking in hadronic reactions (for a review see, Refs. [2,3]) for which there have been remarkable new experimental signatures in the reactions  $dd \rightarrow \alpha\pi^0$  [4] and in the reaction  $np \rightarrow d\pi^0$  [5]. Related investigations in the theoretical front have also been presented, see, Ref. [6]. In general the issue of charge symmetry breaking and isospin conservation (also known as charge independence in the hadronic sector) and violation have consequences in the nucleon–nucleon sector, for a sample of reviews see, e.g., Refs. [7–9].

The situation is less clear in scattering processes: there have been several analyses of  $\pi N$  scattering data with the aim of establishing isospin violation in the system [10,11], which all see evidence for isospin violation, although the numerical size of the violation remains uncertain. This matter is of great importance to low energy strong interaction dynamics as pointed out by Weinberg [12]. Indeed, isospin violation due to the quark mass difference  $m_d - m_u$ , where  $m_d$  and  $m_u$  are fundamental parameters of the standard model [13], will lead to pronounced effects in the  $\pi N$  scattering lengths at leading order in chiral perturbation theory, recalling here that chiral perturbation theory is the effective low energy theory of the strong interactions (for a few excellent reviews, see, e.g. [14]). Meißner and co-workers have worked out the consequences of the quark mass difference to higher order in a series of investigations [15].

A completely general approach to isospin violation  $\pi N$  scattering has been presented in Ref. [16]. In this framework, which is based on the treatment of all possible operators that may arise due to isospin violation are considered and classified thoroughly in terms of operators, denoted by  $\theta_i$ ,  $i = 1, \dots, 10$ . It thus provides a general platform for the analysis of the  $\pi N$  system.

In QCD isospin violation is due to the non-vanishing of  $(m_d - m_u)$ , and is introduced at the level of the microscopic Lagrangian as an isovector. Isospin violation would arise in all hadronic processes involving the strong interactions, as well as the electro-

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magnetic interactions. This could, in principle, at higher orders generate all possible isospin violating operators in each process of interest, that could connect the particles in the initial state to those in the final state subject to the constraints of isospin addition and charge conservation, and the strengths of which ought to be computable in QCD in principle. Indeed the  $\theta_i$  of Kaufmann and Gibbs is precisely this set for  $\pi N$  scattering.<sup>1</sup>

In the effective low-energy theory, it should also be possible to recast all the operators arising in chiral perturbation theory into combinations of the  $\theta_i$  of Kaufmann and Gibbs with calculable coefficients in terms of the low-energy constants and quark masses, for partial results, see Ref. [17].

Bernstein [18–21] in a series of publications has pointed out that pion photoproduction is an ideal setting for probing isospin violation in the  $\pi N$  sector, in the reaction  $\gamma p \rightarrow \pi N$ . [For recent experimental information on pion photoproduction, see, e.g. [22].] It may be recalled that in the isospin symmetric limit it is described in terms of two isospin amplitudes corresponding to  $I = 1/2, 3/2$ . A celebrated result associated with pion photoproduction is that the pion photoproduction phases are indeed the  $\pi N$  phase shifts in this limit, which goes under the name of the Fermi–Watson theorem [23], when the electromagnetic interaction is retained to leading order. Bernstein has generalized the theorem in the presence of isospin violation when there are three open channels, and on treating quantities proportional to the isospin violating quantity  $m_d - m_u$  on par with quantities of  $O(e)$ , where  $e$  is the electronic charge. The isospin violation due to the charged and neutral pion mass difference, a quantity of  $O(e^2)$  is put in by hand, as is the elastic phase shift  $\delta_\gamma$ .

One of the important objectives of the present work is to provide an operator framework for the analysis of hadronic isospin violation that feeds into pion photoproduction. Stated differently, we provide a framework for pion photoproduction, analogous to that of Kaufmann and Gibbs for  $\pi N$  scattering. In practice this turns out to be straightforward, and requires an extension due to inclusion of neutron targets. There is considerable amount of data available for this case as well, see, e.g. [24] and references therein.

We note here that some authors have reported many results on the subject of (neutral) pion photoproduction in the isospin conserving case [25]. It is likely that isospin violation is also worked out in chiral perturbation theory for pion photoproduction, and it would be beneficial to cast the results of those computations in terms of the operators presented in this work.

In Section 2 we review the proposal of Bernstein by providing a setting for his version of the Fermi–Watson theorem from a general approach to the unitarity conditions found in the literature [26,27]. This provides a unified framework for inequivalent representations that Bernstein has considered in his treatment of the problem. This would also benefit us, for we shall propose that a determination of photoproduction multipole amplitudes from our operator approach can then be fed back via the Fermi–Watson theorem to resolve the discrepancies in the  $\pi N$  phase shift analyses.

We will then proceed to describe the construction of the operators that enter the photoproduction process and classify the terms according to their tensorial properties in Section 3. We will present expressions for the transition amplitudes which will explicitly demonstrate the isospin violation. We will propose that these ought to be the basis for the analysis of the multipoles of pion photoproduction amplitudes. We shall then compare the determination of isospin violation in such an analysis, with that from the Fermi–Watson approach. We shall finally point out that a phase shift analysis that results from our approach could be fed back into the  $\pi N$  system via the Fermi–Watson theorem to resolve the discrepancy in those analyses.

Of special interest is the possibility of carrying out polarization measurements. In particular, there is the Jefferson Laboratory Letter of Intent [28] (LOI) which proposes to carry out photoproduction experiments at high precision to determine better resonance parameters. We provide a brief discussion on the significance of these measurements in constraining isospin violation considered here in Section 4.

We recall that an analogous situation arose in the past when it was suggested that one may be able to observe an isotensor contribution to the electromagnetic current. This possibility is not disallowed in the standard model at higher order, and was considered seriously in [29,30], while non-vanishing evidence for its effect on photoproduction was also reported in the past [31]. However, later experiments provided null results, for a review, see, e.g. Ref. [32]. We conclude by pointing out that the new polarization measurements that are planned may be used to better constrain the isotensor contributions.

A summary is provided in Section 5. Finally in Appendix A we recall the main features of the original formalism of Kaufmann and Gibbs for  $\pi N$  scattering as this process is so closely allied to pion photoproduction, and in Appendix B we present the contributions of the amplitudes including the possible isotensor contributions to the electromagnetic currents to the reactions of interest and briefly describe the special role that is played by the  $\Delta(1232)$  resonance in constraining the isotensor amplitude.

## 2. Fermi–Watson theorem approach to isospin violation

In this section we review the Fermi–Watson theorem approach to isospin violation. Our treatment is based on the approach of Oka [26] and that of Henley [27]. We present expressions presented by the latter, in a notation suited to our needs. For a three

<sup>1</sup> In the following, we shall refer to the framework in which we determine the relevant set of operators for pion photoproduction, as the analogue of the framework of Kaufmann and Gibbs for  $\pi N$  scattering, in recognition of their pioneering approach.

coupled channel S-matrix, in which we have weak coupling between one channel denoted here by  $\gamma$ , with the other two denoted by  $a, b$ , where the channels are mutually absorptive, the following representation holds for the partial waves of the scattering:

$$S^H = \begin{pmatrix} \eta_\gamma e^{2i\delta_\gamma} & i\sqrt{\eta_a\eta_\gamma} S_{a\gamma}^H e^{i(\delta_a+\delta_\gamma)} & i\sqrt{\eta_b\eta_\gamma} S_{b\gamma}^H e^{i(\delta_b+\delta_\gamma)} \\ i\sqrt{\eta_a\eta_\gamma} S_{a\gamma}^H e^{i(\delta_a+\delta_\gamma)} & \sqrt{\eta_a^2 - \eta_a\eta_b\rho^2} e^{2i\delta_a} & i\sqrt{\eta_a\eta_b\rho} e^{i(\delta_a+\delta_b)} \\ i\sqrt{\eta_b\eta_\gamma} S_{b\gamma}^H e^{i(\delta_b+\delta_\gamma)} & i\sqrt{\eta_a\eta_b\rho} e^{i(\delta_a+\delta_b)} & \sqrt{\eta_b^2 - \eta_a\eta_b\rho^2} e^{2i\delta_b} \end{pmatrix},$$

provided:

$$S_{a\gamma}^H = \epsilon_{a\gamma}^H + i\epsilon_{b\gamma}^H \frac{\eta_a\rho}{\eta_\gamma + \sqrt{\eta_b^2 - \eta_a\eta_b\rho^2}}, \tag{1}$$

and

$$S_{b\gamma}^H = \epsilon_{b\gamma}^H + i\epsilon_{a\gamma}^H \frac{\eta_b\rho}{\eta_\gamma + \sqrt{\eta_a^2 - \eta_a\eta_b\rho^2}}. \tag{2}$$

In the above,  $\epsilon_{a\gamma}^H, \epsilon_{b\gamma}^H$  represent the matrix elements for the transitions,  $\rho$  is the absorption parameter in the  $2 \times 2$  subsector spanned by  $a, b$ .

Bernstein has presented expressions for two cases, in the limit when  $\eta_i, i = \gamma, a, b$  are all equal to unity. These correspond to the cases when

- (A)  $a = 0, b = c$ , for the case of elastic and charge exchange scattering in which the three channels of interest are  $\gamma p, \pi^0 p, \pi^+ n$  [19], and
- (B)  $a = 1, b = 3$  which represent the value  $2I$ , where  $I$  is the definite isospin in the  $\pi N$  system, where the three channels of interest are  $\gamma N, (\pi N)^{2I=1}, (\pi N)^{2I=3}$  [20].

In case (A), and in the limit of unit elasticities,  $\rho$  is identified with  $\sin\phi$  in Ref. [19] and the transition matrix elements are the corresponding multipole amplitudes for pion photoproduction. For completeness we reproduce the S-matrix given therein, for the three channels  $\gamma p, \pi^0 p, \pi^+ n$ :

$$\begin{pmatrix} e^{2i\delta_\gamma} & iM'_0 & iM'_c \\ iM'_0 & \cos\phi e^{2i\delta_0} & i\sin\phi e^{i(\delta_0+\delta_c)} \\ iM'_c & i\sin\phi e^{i(\delta_0+\delta_c)} & \cos\phi e^{2i\delta_c} \end{pmatrix}.$$

In this limit the multipoles for pion photoproduction read:

$$M'_0 = e^{i(\delta_\gamma+\delta_c)} [A'_0 \cos(\phi/2) + iA'_c \sin(\phi/2)],$$

$$M'_c = e^{i(\delta_\gamma+\delta_c)} [A'_c \cos(\phi/2) + iA'_0 \sin(\phi/2)].$$

In the above  $A'_0, A'_c$  are quantities proportional to the multipole matrix elements for the charge non-exchange and charge exchange scattering respectively. Bernstein proceeds to relate the quantities above to multipole amplitudes of pion photoproduction. In the near threshold region, we have  $\cos(\phi/2) \rightarrow 1$  and here it is now possible define a quantity

$$\beta \simeq E_{0+}(\gamma p \rightarrow \pi^+ n) a_{cex}(\pi^+ n \rightarrow \pi^0 p),$$

where  $E_{0+}$  is the multipole moment and  $a_{cex}$  is a  $\pi N$  scattering length. This quantity is used to demonstrate the unitarity cusp associated with the two-step process  $\gamma p \rightarrow \pi^+ n \rightarrow \pi^0 p$ , in the limit of isospin conservation, barring the pion mass difference. Furthermore, in this limit Bernstein points out that the presence of isospin violation denoted by  $\delta a_{cex}$  may be detected.

It must be pointed out that away from the threshold region, the limit  $\cos(\phi/2) \rightarrow 1$  no longer holds. Our operator construction of the next section may be used to demonstrate isospin violation away from threshold as well.

In case (B), the result is presented for the case where  $\rho = \sin\psi$ , where  $\psi$  is a small quantity, and for unit elasticities. This corresponds to the S-matrix for the channels  $\gamma N, (\pi N)^{2I=1}, (\pi N)^{2I=3}$ :

$$\begin{pmatrix} e^{2i\delta_\gamma} & iM_1 & iM_3 \\ iM_1 & \cos\psi e^{2i\delta_1} & i\sin\psi e^{i(\delta_1+\delta_3)} \\ iM_2 & i\sin\psi e^{i(\delta_1+\delta_3)} & \cos\psi e^{2i\delta_3} \end{pmatrix}.$$

The unitarity condition then yields:

$$M_1 = e^{i(\delta_\gamma + \delta_1)} [A_1 \cos(\psi/2) + i A_3 \sin(\psi/2)],$$

$$M_3 = e^{i(\delta_\gamma + \delta_3)} [A_3 \cos(\psi/2) + i A_1 \sin(\psi/2)].$$

In the above  $A_1$ ,  $A_3$  are quantities proportional to the multipole matrix elements for the amplitudes of definite isospin in the absence of final state interactions and isospin violation. Bernstein also sets for this case,  $\delta_\gamma = 0$ . Using experimental information based on two independent analyses of  $\pi N$  scattering, Bernstein concludes that at a pion kinetic energy of about 40 MeV,  $\psi \simeq 0.010 \pm 0.004$  [20]. In contrast, it is hoped that the operator approach which is to be described in the next section can assist in unambiguously demonstrating isospin violation, without taking recourse to any information from the  $\pi N$  sector. Furthermore, this coupled channel analysis is valid only below the  $2\pi$  threshold.

### 3. Operator approach to isospin violation

The traditional analysis of pion photoproduction, see Ref. [33], relies on two assumptions:

(a) that the electromagnetic current transforms as

$$\frac{1}{2}(f^s + f^v \tau_0),$$

*viz.* an isoscalar and an isovector part, and, and that;

(b) there is no isospin violation in the hadronic system, due to which the interaction in isospin is proportional to an operator that transforms an isoscalar:

$$O_S \equiv \tau \cdot \Phi. \quad (3)$$

In the above,  $\tau \equiv (\tau_0, \tau_1, \tau_2)$  is an isovector containing the Pauli matrices and  $\Phi$  is an isovector containing the pions.

In the past, assumption (a) has been questioned (and it has been shown that even in the standard model, at higher order in the electromagnetic field, this assumption is violated). This is the basis of the isotensor contribution to the electromagnetic current, see Appendix B,<sup>2</sup> which transforms as

$$\frac{f^t}{2\sqrt{15}}(\tau_1 + \tau_2 - 2\tau_0).$$

There has been no treatment of a departure from assumption (b) in general in the literature. In fact, by providing all possible isospin violating terms in this context, here we are providing the general operator framework accounting for strong isospin violation in the process. This amounts to providing the counterpart for pion photoproduction, of the framework of Kaufmann and Gibbs for  $\pi N$  scattering.

Isospin violation in the hadronic system can arise from the most general term of the type  $\tau_i \Phi_j$ ,  $i, j = 0, 1, 2$ . The nine possible combinations can be organized into a scalar  $O_S$ , a vector whose components are given by

$$-i\epsilon^{ijk} \tau_j \Phi_k,$$

and a traceless symmetric tensor whose components are

$$(\tau_i \Phi_j + \tau_j \Phi_i)(1 - \delta^{ij}), \quad \tau_1 \Phi_1 - \tau_2 \Phi_2, \quad \tau_1 \Phi_1 + \tau_2 \Phi_2 - 2\tau_0 \Phi_0.$$

Of the operators listed above, the  $i = 0$  component of the vector operator alone, and the last of the tensor components listed above alone conserve electric charge. Therefore we can introduce 2 operators:

$$O_V \equiv -i(\tau_1 \Phi_2 - \tau_2 \Phi_1), \quad (4)$$

$$O_T \equiv \tau_1 \Phi_1 + \tau_2 \Phi_2 - 2\tau_0 \Phi_0. \quad (5)$$

The set  $O_S$ ,  $O_V$ ,  $O_T$  for pion photoproduction, is the counterpart of the set  $\theta_i$ ,  $i = 1, \dots, 10$  of  $\pi N$  scattering in the Kaufmann and Gibbs framework (see Appendix A). It may be reiterated that  $O_S$  is isospin conserving while the other two,  $O_V$ ,  $O_T$  are isospin violating.

We begin by recalling that the overall matrix element for the scalar case involves the amplitudes that we shall denote by  $A_S^{(-)}$ ,  $A_S^{(+)}$  and  $A_S^{(0)}$  when the Pauli matrices appearing in the interaction of the nucleon with the photon and pion are arranged as

$$\left( \frac{1}{2} A_S^{(-)} [\tau_i, \tau_0] + \frac{1}{2} A_S^{(+)} \{\tau_i, \tau_0\} + A_S^{(0)} \right) \Phi_i. \quad (6)$$

<sup>2</sup> The determination of this contribution to the amplitude is a tremendous experimental challenge. We shall discuss this further in Section 4. In Appendix B, we also provide a short discussion on the mechanism for the relevant isotensor contributions.

In analogy therefore, the new operators contribute to the matrix element for the cases of vector and tensor through amplitudes denoted by  $A_R^{(-)}$ ,  $A_R^{(+)}$ ,  $R = V, T$  associated with the commutator, anti-commutator accompanying  $f^v$  and  $A_R^{(0)}$ ,  $R = V, T$  accompanying  $f^s$ .

The contributions of these amplitudes to the physical reactions may now be evaluated in a straightforward manner which then reads for the reactions denoted by  $R_a$ ,  $a = 1, 2, 3, 4$ , and defined below. They read:

$$R_1: T(\gamma n \rightarrow \pi^0 n) = -A_S^{(0)} + A_S^{(+)} + 2A_T^{(0)} - 2A_T^{(+)}, \quad (7)$$

$$R_2: T(\gamma p \rightarrow \pi^0 p) = A_S^{(0)} + A_S^{(+)} - 2A_T^{(0)} - 2A_T^{(+)}, \quad (8)$$

$$R_3: T(\gamma n \rightarrow \pi^- p) = \sqrt{2}A_S^{(0)} - \sqrt{2}A_S^{(-)} - \sqrt{2}A_V^{(0)} + \sqrt{2}A_V^{(-)} + \sqrt{2}A_T^{(0)} - \sqrt{2}A_T^{(-)}, \quad (9)$$

$$R_4: T(\gamma p \rightarrow \pi^+ n) = \sqrt{2}A_S^{(0)} + \sqrt{2}A_S^{(-)} + \sqrt{2}A_V^{(0)} + \sqrt{2}A_V^{(-)} + \sqrt{2}A_T^{(0)} + \sqrt{2}A_T^{(-)}. \quad (10)$$

We take this opportunity to suggest that this set of amplitudes be the basis for the analysis of pion photoproduction multipole analysis. In this manner, isospin violation in the hadronic sector could be probed with no recourse to the Fermi–Watson theorem. The amplitudes  $A_R^{(\pm)}$ ,  $A_R^{(0)}$ ,  $R = V, T$  get contributions due to  $(m_d - m_u) \neq 0$ . The vector amplitudes receive contributions at leading order in this quantity, while the tensor will receive contributions only at higher order.

We may infer from this that the analogue of the triangle relation of Kaufmann and Gibbs (see Appendix A) for pion photoproduction reads:

$$T(\gamma n \rightarrow \pi^- p) + T(\gamma p \rightarrow \pi^+ n) = -\sqrt{2}(T(\gamma n \rightarrow \pi^0 n) - T(\gamma p \rightarrow \pi^0 p)) \quad (11)$$

in the absence of isospin violation, *viz.*, when all the amplitudes due to  $O_V, O_T$  are set to zero.

In light of the expressions above, it may be seen that indeed one cannot probe the vector like isospin violating interactions without a charge exchange reaction involving the nucleons. This is in accordance with the observations of Weinberg [12] and those of Bernstein [18–20]. On the other hand, it is possible to observe isospin violating interactions of the tensor type in neutral pion production. Such an interaction is not likely to be important in the low-energy regime where the isospin violating contribution can be coupled at leading order only in a vectorlike manner. However, our framework opens up the possibility of probing isospin violation at higher energies by considering the reactions above.

#### 4. Polarization experiments

Bernstein [18–20] has pointed out repeatedly the availability of polarized targets/beams would significantly enhance the capacity of experiments to probe isospin violation. The main reason for this is that polarization affords the possibility of measuring the multipole amplitude  $\text{Im}(E_{0+})$  (for notation see Ref. [33]) in the near threshold region. In this regard, we draw attention to the Jefferson Laboratory LOI [28] where it has been proposed to carry out photo production experiments using target/beam polarization. It is expected that there will be high statistics experiments, including also neutron targets (the proposal here involves both deuteron as well as carbon targets). Indeed, the measurements of the low multipoles of photoproduction amplitudes could be used to study the deviations from the isospin conserving relation given in Eq. (11). It should also be possible to determine, process by process, the contributions of the isospin violating amplitudes to the phases of the multipole amplitudes. Indeed, there is already data from the experiment E94-104 [34] at high energies, which could possibly be analyzed for isospin violating effects at these energies.

Another of the important objectives set out in the Jefferson Laboratory LOI [28] is to determine better the parameters of the resonance denoted by  $P_{33}(1232)$ . In this regard, we now turn to the issue of the determination of the isotensor contribution to the electromagnetic current. (Note also that our amplitude  $A_T^{(0)}$ , upto a numerical factor is not distinguishable from the isotensor contribution to the electromagnetic current.) The determination of this amplitude is a very challenging one from an experimental point of view due to the contributions from  $A_S^{(0)}$  in the non-resonant region. However, one place where a clear signature can be seen is at an energy corresponding to the production of an  $I = 3/2$  resonance, the  $\Delta(1232)$ , where the isoscalar part of the current makes no contribution, and the production amplitude would involve only the isovector and isotensor parts, resulting in an interference between the two contributions. Here, Sanda and Shaw [31] find evidence for an isotensor contribution to the electromagnetic current from data obtained with polarized photons. However, from analysis of later experimental data there have been null results presented in the literature. In Ref. [35] an experiment with tagged photons found no evidence for isotensor component to the crosssection, while Ref. [36] reports results from an experiment that observes the differential crosssection at several different angles, which also found no evidence. Null results are also reported in Ref. [37], based on experiments with polarized photons. Here it has been pointed out that target asymmetries could play a role in resolving ambiguities. In the light of the proposals presented in the LOI, and

due to the likelihood of the availability of polarization and other facilities at Jefferson Laboratory detailed therein, the upcoming experiments there could play a crucial role in settling this question.<sup>3</sup>

## 5. Discussion and summary

In this work, we have revisited the issue of observing isospin violation in the hadronic sector from pion photoproduction. We have pointed out that the application of the Fermi–Watson theorem is one approach that has been considered in the past. We have presented a comparison of different techniques used to arrive at the pertinent expressions by appealing to general treatments. We have pointed out that there is a general operator approach also to the phenomenon, which we have worked out here. In essence, it is the counterpart of the Kauffman and Gibbs construction for  $\pi N$  scattering.

The operator approach described here, yields a set of amplitudes for pion photoproduction which should be the basis for the determination of the multipole amplitudes for the processes of interest. The resulting phase shifts can then be inserted into a Fermi–Watson like system to provide a set of constraints for  $\pi N$  phase shift analyses which are in mutual disagreement at the moment. The Fermi–Watson approach of Bernstein uses well known  $\pi N$  phase shift analyses to establish isospin violation in photoproduction. This latter is also constrained to be valid only below the  $2\pi$  threshold, and is to leading order in the electric charge. The treatment presented in case (A) of Bernstein requires the  $\pi N$  scattering length as an input to demonstrate isospin violation at the photoproduction threshold, while the treatment in case (B) requires  $\pi N$  phase shift analyses as an input. Our treatment does not require these inputs.

We have considered the virtues of polarization experiments and have pointed out that at the upcoming facilities, one may constrain isotensor contributions to the electromagnetic current, expected to arise at higher orders better. Our work is likely to be a useful platform for the construction of results from effective Lagrangians and a basis for analysis of crosssections which can be used to constrain isospin violation in the hadronic sector. Finally, we point out here that a determination of photoproduction amplitudes including the effects due to the isospin violating operators presented here, could then be used via the Fermi–Watson theorem to resolve the discrepancies in the  $\pi N$  phase shift analyses.

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## Appendix A. Formalism of Kaufmann and Gibbs for $\pi N$ scattering

For the ten reactions (listed in Table 1 of Ref. [16]) of interest, a general analysis of isospin violation in terms of a set of 10 standard operators designated  $\theta_i$ ,  $i = 1, \dots, 10$  (listed in Table 2 of Ref. [16]) is presented. The matrix elements for all these operators are listed (see Table 3 of Ref. [16]) and the result is also presented in the isospin basis (see Table 4 of Ref. [16]<sup>4</sup>). We note here some of the features of the work:

- (1)  $\theta_{1,5}$  are isospin conserving,  $\theta_{4,6,(10)}$  violate isospin but are invariant under charge reflection,  $\theta_{3,7,(9)}$  conserve neither isospin nor charge reflection,  $\theta_8$  besides not conserving isospin and charge reflection, only connects  $I = 3/2$  states;
- (2) In the elementary examples of isospin violation, the combination

$$\theta_3 - \sqrt{\frac{1}{3}}\theta_5 + \sqrt{\frac{2}{3}}\theta_6 \tag{A.1}$$

represents the Coulomb interaction and that this combination gives the product of the nucleon and pion charge;

- (3)  $\pi^0$ – $\eta$  mixing, a quantity that receives contributions at leading order in  $(m_d - m_u)$  transforms as

$$\sqrt{\frac{8}{9}}\theta_2 + \sqrt{\frac{40}{9}}\theta_7; \tag{A.2}$$

<sup>3</sup> In this regard, it should be noted that pion electroproduction experiments have been consistently seeing evidence for an isotensor amplitude. Data from the recent SLAC experiments NE11 and E133 yield [38], for the ratio  $\sigma_n/\sigma_p$  for the crosssections on neutron and proton targets,  $0.72 \pm 0.09$ , to be contrasted with the value from older data of Köbberling [39], of  $0.91 \pm 0.03$ , at the  $\Delta(1232)$  resonance. This ratio should be unity in the absence of isotensor contributions.

<sup>4</sup> We point out here that all the signs corresponding to the entries of  $\theta_9$  and  $\theta_{10}$  in Table 4 of Ref. [16] need to be consistently reversed.

(4) The triangle identity, which holds in the isospin conserving limit, can be expressed as

$$T(\pi^+ p \rightarrow \pi^+ p) - T(\pi^- p \rightarrow \pi^- p) = \sqrt{2}T(\pi^+ p \rightarrow \pi^0 n). \quad (\text{A.3})$$

In the work of Kaufmann and Gibbs a treatment of a final state theorem is presented, which allows one to transform certain  $I = 1$  operators into other  $I = 1$  operators by left and right multiplication by isospin conserving operators. In particular, it could turn an operator that transforms with the transformation law of  $\rho - \omega$  mixing ( $\theta_3$ ) into one that has the transformation law of  $\pi^0 - \eta$  mixing.

## Appendix B. Isotensor contributions

We begin by recalling that in the limit of isospin conservation in the hadron sector, we have the isospin relations for the amplitudes  $t^{2I}$ ,  $I = 3/2, 1/2$  reading

$$\begin{aligned} A^3 &= A_S^{(+)} - A_S^{(-)}, \\ A^1 &= A_S^{(+)} + 2A_S^{(-)}. \end{aligned}$$

Note that  $A^0 \equiv A_S^{(0)}$  contributes to  $I = 1/2$  amplitude. The admission of an isotensor operator can lead up to  $I = 3/2$  state, when represented by an amplitude  $A^2$ . These together yield for the processes of interest [31]:

$$\begin{aligned} R_1: & -A^0 + A^1/3 + 2A^2/\sqrt{15} + 2A^3/3, \\ R_2: & A^0 + A^1/3 - 2A^2/\sqrt{15} + 2A^3/3, \\ R_3: & \sqrt{2}(A^0 - A^1/3 + A^2/\sqrt{15} + A^3/3), \\ R_4: & \sqrt{2}(A^0 + A^1/3 + A^2/\sqrt{15} - A^3/3). \end{aligned}$$

It may also be noted that the amplitude  $A^2$  contributes to the  $R_i$ ,  $i = 1, 2, 3, 4$  in the same way as  $\sqrt{15}A^2$ . These amplitudes are the basis of the analysis of photoproduction amplitudes in Ref. [31]. In this Letter, the multipole amplitudes  $M_{1+}^i$ ,  $i = 0, 1, 2, 3$  have been studied in detail.

We present here some salient features of the possibility of detecting the signature of the isotensor amplitude from the formation of the resonance  $\Delta(1232)$  [30,31] (see also Refs. [40,41]). At the resonance, the isoscalar amplitude does not contribute to the crosssection except for the nonresonant background. If, for example, the dominant multipoles  $M_{1+}$  are being probed, then the presence of the isotensor would lead to an interference term proportional to  $\text{Re}(M_{1+}^2 M_{1+}^3)$ . The model for the isotensor term which is the basis of the analysis of Sanda and Shaw [31] is written down in the static model of Chew et al. [42], by introducing an isotensor  $\gamma \Delta n$  coupling, and required the resulting multipole moment to verify a fixed- $t$  dispersion relation. This allows for the isotensor interaction to participate in the resonance production.

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