A Novel Bi-level Continuous Formulation for the Cellular Manufacturing System Facility Layout Problem

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Abstract

Cellular Manufacturing Systems (CMSs) play an important role in today’s small- to mid-size production enterprises. The facility layout problem for such manufacturing system is a group-one that includes both intercellular and intracellular relationships. A novel continuous formulation has been developed for the problem to model manufacturing shops with vertical and horizontal aisles and to eliminate any possible overlap between machine tools as well that between cells. The overall approach adopted is a bi-level one; initially an upper-level leader facility layout problem is being solved for each cell at a time; then, a lower-level follower facility layout problem (FLP) is being solved after at the shop level determining the overall layout of the shop, where the position of the different cells are to be determined. A case study from the local machining industry has been utilized to verify the model.

Keywords: Cellular Manufacturing Systems; Unequal Facility Layout Problem; Mathematical Modelling; Overlap Elimination; Aisle Modelling

1. Introduction

The facility layout problem can be defined as an optimization problem that tries to make layouts more efficient by taking into account various interactions between facilities and material handling systems while designing layouts [1]. There is a lot of interest in optimizing FLP (Facility Layout Problem); however, the majority of which have attempted to solve FLP by using a discrete approach [2,3,4]. This approach attempts to assigning \( n \) facilities with the same shape and size to \( n \) distinct predetermined locations. The main critique this widely used modeling scheme has received is the lack of geometric constraints to account for the possible unequal size of the different facilities and the lack of knowledge of the possible locations of the different facilities (cells and machines). Discrete representations are not suited to represent the exact position of facilities in the plant site, the orientation of facilities, pick-up and drop-off points or clearance between facilities [5]. A continuous mathematical representation has been introduced, where the problem becomes the optimal allocation of the two-dimensional coordinates of facilities within a shop. This has been often addressed as Mixed Integer Programming Problems [6]. The continuous approach tries to find optimal arrangement of facilities within the planar site, and yet not overlap each other.

Cellular manufacturing system (CMS) layout has recently begun to receive heightened attention worldwide. An effective CMS implementation help any company improve machine utilization and quality; it also makes reduction in setup time, work-in-process inventory, material handling cost, part makespan, and expediting costs [7,8,9]. In this paper, the layout problem of CMS is being tackled. A Bilevel Mathematical Programming approach has been adopted, in which two-tier mathematical formulations have been developed to solve the layout problem at two different hierarchical levels. First, the layout of the different cells of the manufacturing system is to be determined. Second, and once the coordinate of each cell is determined, the recursive layout of each cell, in which a cell’s machine tools’ layout are to be determined, is performed. Grouping of each manufacturing
cell machine tools is to be done in a prior step using a metaheuristic; however, this part of the overall methodology is not reported in this paper.

2. Critical Literature Review

The block facility layout problem that was originally formulated by Armour and Buffa (1963) [10] is concerned with finding the most efficient arrangement of indivisible departments with unequal area requirements within a facility [11]. As defined in the literature, the objective of the block layout design problem is to minimize the material handling costs inside a facility by considering the following two sets of constraints: (a) department and floor area requirements and (b) department locational restrictions; i.e., departments cannot overlap, must be placed within the facility, and some must be fixed to a location or cannot be placed in specific regions [12, 13, 14, 15, 16].

Cellular layout has been categorized as one of the special cases in their layout problem classification. There are increasing interests in solving the block layout problem by taking continuous approach. Heragu and Kusiak (1991) [17] presented two new models of the facility layout problem, namely linear continuous with absolute values in the objective function and constraints and linear mixed integer. A co-evolutionary approach to the numerical optimization of large facility layouts is introduced by Dunker et al., (2003) [18]. Their work was based on a mixed integer model for the layout constraints and objectives.

It could be concluded, a few publications seem to deal with adopting the continuous approach for CMS. Wu et al., [19, 20] developed a hierarchical genetic algorithm (HGA) to concurrently solve the CF (Cellular Facility) and GL (Group Layout) problems. However, they did not consider the overlap-elimination. There are recent studies that have adopted a continuous approach [21, 22]. There are some common drawbacks to the constraints such as those ones used as non-overlapping constraints for machines; and also the constraints used to force machines to stay within shop floor boundaries. Finally, Arkat et al., assumed that the machines have equal square area and cells are rectangle. However, in the real world these are poor assumption.

To conclude, very few researches seem to deal with UA-FLPs with a continuous representation for CMS. Moreover, the majority of the models have been too complex to be attempted using mathematical programming and Operations Research solution methods, and hence are solved using heuristics and metaheuristics, which defeat the purpose of developing a mathematical model in the first place, one could argue. In this paper, a Bilevel a novel continuous mathematical formulation has been developed by taking into account the possibility of having both vertical and horizontal aisles in the shop, as well as constraints to eliminate overlap between machines, as well as that between manufacturing cells. The model has been later on being solved using commercial OR algebraic modeling language and solvers. A case study finally has been employed to demonstrate the working of the model and for validation.

3. Methodology

A two-tier mixed integer non-linear programming model has been developed to solve the intra-cell and inter-cell layout sequentially at two different hierarchical levels, namely at the cellular and shop floor levels. Firstly, the intracell layout is being solved; secondly, and after the layout for all manufacturing cells have been finalized, the overall approach for the whole is being solved. Hence, the leader problem is the layout at the cell level (i.e. intracellular), while follower is the layout for the whole shop (i.e. intercellular). It is important to note that initially when we run the FLP for each cell (leader problem), we define an upper limit for the length and width of each cell using constraints named as within-cell constraint. At leader level, the traffic at intra-level is the material flow among the machines (operations already assigned to machines) located in cell, which is still to be laid out and constructed. Moreover, the material flows in the follower level are inter-travel between cells. Since the Group Formation is done in advance, it is already known which machines are assigned to which cell; i.e., operations of part j processed in cell k are known ahead of time. Therefore, material flows between cells are actually the flows among the operations of parts on machines done in each cell.

From computational and optimization points of view, it is important to note that dividing and conquering the FLP for CMS does not produce an optimal solution; i.e., the solution obtained would not really be the same exact global optimum solving the problem combined in one math model for the two different levels (that is if we are to assume the nonlinear model to be presented in this section is linearized). However, it is important to pinpoint that such models in the literature were complex enough that they were not really being attempted and solved for optimality using OR (Operations Research) exact methods and commercial OR software. Moreover, few of these models carried constraints that were formulated in a way that hindered the ability to solve them using these tools. To elaborate, one of the models had conditions on the decision variables associated with the overlap elimination constraints. Finally, some models even went further and overcomplicated the problem by introducing other elements such as the grouping and clustering that is needed ahead of time for cell formation, as well as the production scheduling of each cell. In our case, we find it far more efficient to go about the clustering problem beforehand using a robust developed simulated annealing metaheuristic.

4. Problem Statement

The problem is to arrange facilities that are cells in follower problem and machine tools in leader problem in the continual planar site. The site has rectangle shape with specified length (L) and width (W). Moreover, there is an aisle in the site by the same length as of site, however with two different vertical dimensions Y_{AisleUpperCoordinate} and Y_{AisleLowerCoordinate}. Aisle divides the site to two sections, upper and lower. No facilities could be arranged in this area. The objective is minimizing total travel-flow cost by considering shape, size and geometric characteristics.
constraints. Each facility has rectangle shape denotes its position by the coordinates of its centre and its predetermined length and width. Hence, the facilities consider as rigid blocks. Facilities are not allowed to overlap each other and have to be assigned in their related boundaries area, which is the site’s boundaries for lower level problem and of machines tools is the boundaries of their related cell. The traditional Cartesian Coordinate System used shown in Figure 1, represents the scheme used in this paper. The distance measurement used in this paper is Manhattan distance.

The problem is formulated under the following assumptions:

1. GF is known in advanced.
2. Machines are not in the same size.
3. Machines must be located within a given area.
4. Machines must not overlap with each other.
5. Cell’s dimensions and orientation are predetermined.
6. Each part type has a number of operations that must be processed based on its operation sequence readily available from the route sheet of parts.
7. The demand for each part type in known and is constant
8. Material handling devices moving the part between machines are assumed to carry only one part at a time
9. Inter and intra-cell movements related to the part types have different costs is related to the distance travelled.

We assume that the rectangular distance between each pair of machines’ centroid.

In determining machine size and dimensions, the workspace required for operator usage and that needed to enforce between the different machines have been taken into account.

Sets:
- \( P = \{1,2,3,\ldots,P\} \) index set of part types
- \( M = \{1,2,3,\ldots,M\} \) index set of machine types
- \( C = \{1,2,3,\ldots,C\} \) index set of cell types
- \( \mathcal{O}_p = \{1,2,3,\ldots,\mathcal{O}_p\} \) index set of operations indices for part \( p \)

Parameters
- \( L \): Horizontal dimension of shop floor
- \( W \): vertical dimension of shop floor
- \( Y_{\text{AisleUpperCoordinate}} \): Vertical dimension of upper side of aisle
- \( Y_{\text{AisleLowerCoordinate}} \): Vertical dimension of lower side of aisle
- \( X_{\text{AisleLeftCoordinate}} \): horizontal dimension of left side of aisle
- \( X_{\text{AisleRightCoordinate}} \): horizontal dimension of right side of aisle
- \( l_i \): length of machine \( i \)
- \( w_i \): width of machine \( i \)
- \( l_c \): length of cell \( c \)
- \( w_c \): width of cell \( c \)
- \( C_{A_j} \): Intracell transfer unit cost for part \( j \)
- \( C_{E_j} \): Inter-cell transfer unit cost for part \( j \)
- \( D_j \): demand quantity for part \( j \)
- \( U_{ja} \): 1, if operation \( o \) of part \( p \) can be done by machine \( i \), otherwise 0
- \( U_{ja} \): 1, if operation \( o \) of part \( j \) can be done by machine which is located in cell \( c \), otherwise 0
- \( Q_{ie} \): 1, if machine \( i \) is assigned in cell \( c \)

Decision variable
- \( x_{ij} \): X coordinate of machine \( i \) (Machine is denoted by its centroid)
- \( y_{ij} \): Y coordinate of machine \( i \) (Machine is denoted by its centroid)
- \( x_{ic} \): X coordinate of cell \( c \) (Cell is denoted by its centroid)
- \( y_{ic} \): Y coordinate of cell \( c \) (Cell is denoted by its centroid)
- \( Z_{iu} \): 1, if machine \( i \) is arranged in the same horizontal level as machine \( i \), and 0 otherwise
- \( W_{ic} \): 1, if cell \( c \) is arranged in the same horizontal level as cell \( c \) and 0 otherwise

\( Z_c \): 1, if cell \( c \) is arranged in out of aisle horizontal boundaries and 0 otherwise

\( W_c \): 1, if cell \( c \) is arranged in out of aisle vertical boundaries and 0 otherwise

The continuous bi-level programming problem is defined as:

\[
\min \sum_{j=1}^{p} \sum_{o=1}^{q} p_{o-1} Y_{M} U_{jo} U_{jo+1}(|x_{ij} - x_{il}| + |y_{ij} - y_{il}|) C_{A_j} D_j
\]

s.t.

\[
x_{ij} - x_{il} \geq Z_{iu}(l_i + l_o)/2 \quad i, u = 1, \ldots, M
\]

\[
y_{ij} - y_{il} \geq (1 - Z_{iu})(w_i + w_o)/2 \quad i, u = 1, \ldots, M
\]

\[
x_{ij} + \frac{L}{2} \geq 0 \quad i = 1, \ldots, M
\]

\[
y_{ij} + \frac{W}{2} \geq 0 \quad i = 1, \ldots, M
\]

\[
x_{ij} + \frac{L}{2} \leq W_c(w_c + w_o)/2 \quad c = 1, \ldots, C
\]

\[
y_{ij} - y_{il} \geq (1 - W_c)(w_o + w_c)/2 \quad c = 1, \ldots, C
\]

Aisle Constraints:

Horizontal Aisles:

\[
(Y_{ic} + w_c/2) - Y_{\text{AisleLowerCoordinate}} \leq M Z_c
\]

\[
(Y_{ic} - W_c/2) \leq M (1 - Z_c)
\]

Vertical Aisles:

\[
x_{ic} + l_c/2) - X_{\text{AisleLeftCoordinate}} \leq M W_c
\]

\[
(x_{ic} - l_c/2) \leq M (1 - W_c)
\]

Equations 4 and 9 are the objective function of upper-level and lower level program respectively. The first objective function minimizes the total intracell transportation cost of parts and the second one minimize the intercell transportation cost of parts. Equations 4 to 7 and set of equations 10 to 13 are within-site constraints that ensure each machine tool and
cell are assigned within the boundaries of its corresponding cell and shop floor respectively. Equations 2 and 3; and 14 and 15 forces overlap elimination for machine tools and cell respectively. Equations 16 to 19 in leader problem ensure that no cells would be assigned in aisle boundaries. Equations 8 and 20 specify that the decision variables are binary and integer variables.

Figure 1: The scheme of shop floor

5. Case Study:

The case study is a Carbide Tool Inc manufactures and distributes metalworking tools. The company is dedicated to develop specialized Carbide, PCD (Polycrystalline diamond) and CBN (Cubic Boron Nitride) inserts, as well as multitask tooling for the aerospace, automotive and mold-die industries. The current layout of the company has is far from efficient and optimal. The facility at the company was classified as a CMS. Group formation was performed, and machine tools were assigned to their respective cells, which followed a product layout. There are five different kinds of family of cutting insert tools produced. Each part has specific monthly demand. Since some of machine tools have more than one unit, the demand is being shared among the units of those machines. The main operation done on inserts is grinding done by different grinding machine tools. Some of the machine tools have identical copies on the shop floor to increase productivity. Moreover, there are three workstations such as inspection, wash, and packaging. The operations sequence for each cutting insert tool is different from others. In other words, all the operations are not being done for each part. The list of operations of each inserts and the machine tools using for those operation are shown in table 1.

<table>
<thead>
<tr>
<th>ID</th>
<th>Machine</th>
<th>Dimension</th>
<th>X</th>
<th>Y</th>
<th>Dog Bone</th>
<th>S Shape</th>
<th>Triangular</th>
<th>Top Notch</th>
<th>Diamond-type 1</th>
<th>Diamond-type 2</th>
<th>Diamond-type 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>Double disk (1)</td>
<td></td>
<td>12.67</td>
<td>5</td>
<td>O1</td>
<td>O2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M2</td>
<td>Blanchard (2)</td>
<td></td>
<td>6</td>
<td>9.07</td>
<td>O1</td>
<td>O1</td>
<td>O1</td>
<td>O1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M3</td>
<td>Wendt (3)</td>
<td></td>
<td>8.5</td>
<td>6.1</td>
<td>O1</td>
<td>O2</td>
<td>O4</td>
<td>O2</td>
<td></td>
<td></td>
<td></td>
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<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M4</td>
<td>Polish (1)</td>
<td></td>
<td>6</td>
<td>5</td>
<td>O1</td>
<td>O1</td>
<td>O2</td>
<td>O2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M5</td>
<td>EWAG (1)</td>
<td></td>
<td>4.3</td>
<td>7.3</td>
<td>O1</td>
<td>O1</td>
<td>O2</td>
<td>O2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M6</td>
<td>Surface grinding (2)</td>
<td></td>
<td>7</td>
<td>6</td>
<td>O4</td>
<td>O3</td>
<td>O3</td>
<td>O3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M7</td>
<td>Surface grinding (1)</td>
<td></td>
<td>6</td>
<td>7.54</td>
<td>O1</td>
<td>O1</td>
<td>O1</td>
<td>O1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M8</td>
<td>Swing fixture (1)</td>
<td></td>
<td>8</td>
<td>6</td>
<td>O2</td>
<td>O2</td>
<td>O2</td>
<td>O2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M9</td>
<td>V-bottom (1)</td>
<td></td>
<td>7</td>
<td>6</td>
<td>O3</td>
<td>O3</td>
<td>O3</td>
<td>O3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M10</td>
<td>Wire-cutting (2)</td>
<td></td>
<td>7.8</td>
<td>6.7</td>
<td>O4</td>
<td>O4</td>
<td>O4</td>
<td>O4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>7.8</td>
<td>5.7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M11</td>
<td>Laser M/C (1)</td>
<td></td>
<td>7.6</td>
<td>9.74</td>
<td>O5</td>
<td>O5</td>
<td>O5</td>
<td>O5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M12</td>
<td>Brazing (1)</td>
<td></td>
<td>4</td>
<td>1.8</td>
<td>O6</td>
<td>O6</td>
<td>O6</td>
<td>O6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M13</td>
<td>ETCH (1)</td>
<td></td>
<td>3</td>
<td>4</td>
<td>O5</td>
<td>O5</td>
<td>O5</td>
<td>O5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ST1</td>
<td>Inspection (1)</td>
<td></td>
<td>4</td>
<td>3</td>
<td>O6</td>
<td>O6</td>
<td>O7</td>
<td>O7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ST2</td>
<td>Wash (1)</td>
<td></td>
<td>5</td>
<td>3</td>
<td>O7</td>
<td>O7</td>
<td>O8</td>
<td>O8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ST3</td>
<td>Packing (1)</td>
<td></td>
<td>18</td>
<td>9</td>
<td>O8</td>
<td>O8</td>
<td>O9</td>
<td>O9</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*The number of units for each machine tools shown in bracket.

The company’s shop floor is a rectangular shape. The current layout they have is process layout. There is no special material handling device for transforming unfinished products among machine tools. A tray with capacity of 250 pieces arranged for inserts transformation. By considering table 1 would be obvious that the number of operations done on each part inserts tool is large enough, hence the amount of travelling taking place every day on their floor. Additionally, all the raw materials are carried from the back side of the floor and transformed all the way to the front for starting the operations. This makes extra movement which leads to extra material handling cost. The inspection site and shipping department were not properly positioned with the current
layout. It is located at the end of hall and washing at the front. These two operations are the last operations have to be done for all parts. Transforming all finished part to washing site also is time and cost consuming. All the parts need the combination of these three operations: surface grinding, top and bottom grinding and periphery grinding that can be done by Double Disk, Blanchard and Wendt. Since they are using process layout, Wendt machines group are located in upper side of hall and Blanchard and Double Disk machines arranged in the lower side. Therefore, it could be concluded there is much unnecessary traffic that is primarily due to following a non-convenient layout scheme, aside from all other factors listed that were not really optimized. GF done in advance, there are 4 cells with specific types of machine tools. The characterization of GF is as follows:

Table 2. GF results

<table>
<thead>
<tr>
<th>No.</th>
<th>Name</th>
<th>Machine tools / Work Station</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Primary Cell</td>
<td>Double Disc (1) Blanchard (2) Polish (1) Wendt (3)</td>
</tr>
<tr>
<td>2</td>
<td>Grinding Cell</td>
<td>Surface Grinding (2) Swing Fixture (2) V-Bottom (1)</td>
</tr>
<tr>
<td>3</td>
<td>Diamond Cell</td>
<td>Wire-cutting (2) Surface Grinding (1) EWAG (1) Brazing (1) Laser M/c (1)</td>
</tr>
<tr>
<td>4</td>
<td>Final Cell</td>
<td>ETCH (1) Inspection (1) Wash (1) Packing (1)</td>
</tr>
</tbody>
</table>

*The number of units for each machine tools shown in bracket.

FICO Xpress Optimization Suit Software has been used to solve the continuous formulation of this paper. Since the mathematical formulation is nonlinear both Successive Linear Programming (SLP) and Non-linear Programming (NLP) solver have been used.

5.1. Intracellular Layout:

For the leader problem the layout of the different machine tools and work stations in their respective cells are being solved. The intracellular travel cost per unit distance per one unit of each respective part are \( \varepsilon10, \varepsilon20, \varepsilon20, \varepsilon25, \varepsilon30 \) respectively for Dog Bone, S Shape, Triangular, Top Notch and all types of Diamond. The results of intracellular layout represented in table 3 and the sketch of layout plan in each cell shown in figure 2.

Table 3. Intracellular costs

<table>
<thead>
<tr>
<th>Cell’s Name</th>
<th>Dimension</th>
<th>Material Handling Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary Cell</td>
<td>35 x 25</td>
<td>( \varepsilon737,175 )</td>
</tr>
<tr>
<td>Grinding Cell</td>
<td>26 x 30</td>
<td>( \varepsilon151,625 )</td>
</tr>
<tr>
<td>Diamond Cell</td>
<td>30 x 16</td>
<td>( \varepsilon659,100 )</td>
</tr>
<tr>
<td>Final Cell</td>
<td>34 x 18</td>
<td>( \varepsilon862,875 )</td>
</tr>
</tbody>
</table>

5.2. Intercellular Layout:

In the follower problem, the four cells with the exact dimensions have been assigned in the whole shop floor with 97\( \times \)60 feet length and width. The intercellular travel cost per unit distance per one unit of each respective part Dog Bone, S Shape, Triangular, Top Notch, all types of Diamond among cells are \( \varepsilon10, \varepsilon20, \varepsilon40, \varepsilon10, \varepsilon15 \) respectively. The traveling flows costs among cells are \( \varepsilon852617 \). The final sketch which is showing intercellular and intracellular layout has been shown as follows:

Figure 2. Sketch of intercellular and intracellular layout plan

6. Conclusion and Future Work:

CMS is one of the most important facility layout problems, which does include group formation, intracellular layout, and intercellular layout. The focus of this paper is on the latter two aspects of the problem. The bi-level approach taken in this paper is to optimize the intracellular and intercellular facility layout problem in a sequential manner by solving a leader and follower problem at the cell- and shop floor-levels respectively. A continuous mixed-integer nonlinear programming mathematical formulation has been developed for each of the two levels. A novel overlap elimination constraint has been formulated. For the follower problem, vertical and horizontal aisle constraints have been developed.

The develop model has been demonstrated and verified using a realistic industrial case study. The group formation has been done before and the results presented in this paper. Based on this results there are 4 main cells by special kind of machine tool. The results out of the model in this case study is intuitive. Generally speaking, solving both leader and follower problem, the layout generated was optimized in a sense to minimize travel-flow cost. It was found out that that for each of the facilities layout problems being solved at both the leader- and follower-level, the facilities have been lamped
together in a way to minimize the travel in between, and hence the travel-flow cost. For future work, the developed model will be linearized and solved for optimality.

7. References


