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PHYSICS LETTERS B

Physics Letters B 659 (2008) 754–760

www.elsevier.com/locate/physletb

Spontaneous Lorentz violation: Non-Abelian gauge fields as pseudo-Goldstone vector bosons

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Received 22 March 2007; received in revised form 25 October 2007; accepted 14 November 2007

Available online 4 December 2007

Editor: T. Yanagida

Abstract

We argue that non-Abelian gauge fields can be treated as the pseudo-Goldstone vector bosons caused by spontaneous Lorentz invariance violation (SLIV). To this end, the SLIV which evolves in a general Yang–Mills type theory with the nonlinear vector field constraint $\mathrm{Tr}(\boldsymbol{A}_\mu \boldsymbol{A}^\mu) = \pm M^2$ (M is a proposed SLIV scale) imposed is considered in detail. Specifically, we show that in a theory with an internal symmetry group G having D generators not only the pure Lorentz symmetry $SO(1,3)$, but the larger accidental symmetry $SO(D,3D)$ of the SLIV constraint in itself appears to be spontaneously broken as well. As a result, although the pure Lorentz violation on its own still generates only one genuine Goldstone vector boson, the accompanying pseudo-Goldstone vector bosons related to the $SO(D,3D)$ breaking also come into play properly completing the whole gauge multiplet of the internal symmetry group G taken. Remarkably, they appear to be strictly massless as well, being protected by the starting non-Abelian gauge invariance of the Yang–Mills theory involved. When expressed in terms of the pure Goldstone vector modes, this theory look essentially nonlinear and contains a plethora of Lorentz and CPT violating couplings. However, they do not lead to physical SLIV effects which turn out to be strictly cancelled in all the lowest order processes considered.

1. Introduction

The old idea [1] that spontaneous Lorentz invariance violation (SLIV) may lead to an alternative theory of quantum electrodynamics still remains extremely attractive in numerous theoretical contexts [2] (for some later developments, see the papers [3]). The SLIV could generally cause the appearance of massless vector Nambu–Goldstone modes which are identified with photons and other gauge fields underlying the modern particle physics framework like as Standard Model and Grand Unified Theory. At the same time, the Lorentz violation on its own has attracted a considerable attention in recent years as an interesting phenomenological possibility appearing in various quantum field and string theories [4–9].

The first models realizing the SLIV conjecture were based on the four fermion (current–current) interaction, where the gauge field appears as a fermion–antifermion pair composite state [1], in complete analogy with a massless composite scalar field in the original Nambu–Jona-Lasinio model [10]. Unfortunately, owing to the lack of a starting gauge invariance in such models and the composite nature of the Goldstone modes which appear, it is hard to explicitly demonstrate that these modes really form together a massless vector boson as a gauge field candidate. Actually, one must make a precise tuning of parameters, including a cancellation between terms of different orders in the $1/N$ expansion (where N is the number of fermion species involved), in order to achieve the massless photon case [11]. Rather, there are in general three separate massless Goldstone modes, two of which may mimic the transverse photon polarizations, while the third one must be appropriately suppressed.

In this connection, the more instructive laboratory for SLIV consideration proves to be some simple class of the QED type models having from the outset a gauge invariant form, whereas the Lorentz violation is realized through the nonlinear dynami-

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0370-2693/$ – see front matter
doi:10.1016/j.physletb.2007.11.073

cal constraint imposed on the starting vector field A_μ

$$A_\mu^2 = n_\mu^2 M^2, \tag{1}$$

where n_μ is an properly oriented unit Lorentz vector, while M is a proposed SLIV scale (hereafter, as usual, we sum over repeated indices). This constraint means in essence that the vector field A_μ develops the vacuum expectation value $\langle A_\mu(x)\rangle = n_\mu M$ and Lorentz symmetry $SO(1,3)$ breaks down to $SO(3)$ or $SO(1,2)$ depending on the time-like ($n_\mu^2 = +1$) or space-like ($n_\mu^2 = -1$) SLIV. Such QED model was first studied by Nambu a long time ago [12], but only for the time-like SLIV case and in the tree approximation. For this purpose he applied the technique of nonlinear symmetry realizations which appeared to be successful in the handling of the spontaneous breakdown of chiral symmetry in the nonlinear σ model [13] and beyond.[1]

In the present Letter, we mainly address ourselves to the Yang–Mills gauge fields as the possible vector Goldstone modes (Section 3) as soon as some basic ingredients of the Goldstonic QED model are established in a general SLIV case (Section 2). This problem has been discussed many times in the literature within quite different contexts, such as the Yang–Mills gauge fields as the Goldstone modes for the spontaneous breaking of general covariance in a higher-dimensional space [17] or for the nonlinear realization of some special infinite parameter gauge group [18]. However, all these considerations look rather speculative and optional. Specifically, they do not give a correlation between the SLIV induced photon case, from the one hand, and the Yang–Mills gauge field case, from the other. In contrast, our approach is solely based on the spontaneous Lorentz violation thus properly generalizing the Goldstonic QED model [12] to the non-Abelian internal symmetry case. Just in this approach the interrelation between both of cases appears to be most transparent. We will see that in the Yang–Mills theory case with an internal symmetry group G having D generators not only the pure Lorentz symmetry part $SO(1,3)$ in the symmetry $SO(1,3)\times G$ of the Lagrangian, but a much higher accidental symmetry $SO(D,3D)$ of the SLIV constraint $\mathrm{Tr}(\boldsymbol{A}_\mu \boldsymbol{A}^\mu) = \pm M^2$ in itself also happens to spontaneously broken. As a result, many extra massless modes, the pseudo-Goldstone vector bosons (PGB), have to arise. Actually, while the spontaneous Lorentz violation on its own still generates only one genuine Goldstone vector boson, the accompanying vector PGBs related to the $SO(D,3D)$ breaking also come into play properly completing the whole gauge multiplet of the internal symmetry group G taken. Remarkably, in contrast to the familiar scalar PGB case [13] the vector PGBs remain strictly massless being protected by the starting non-Abelian gauge invariance of the Yang–Mills theory involved. Then in Section 4 we show by some examples of the lowest order SLIV processes that, while the Goldstonic non-Abelian theory contains a rich variety of Lorentz and CPT violating couplings, it proves to be physically indistinguishable from the Yang–Mills theory. Actually, one of the goals of the present work is to explicitly demonstrate that a conventional Yang–Mills theory (as well as QED) is in fact a spontaneously broken theory. The Lorentz violation, due to the quadratic field constraint of the type (1), renders this theory highly nonlinear in the Goldstone vector modes, though physically equivalent to the usual one. So, as well as in the pure QED case, the SLIV only means the noncovariant gauge choice in the otherwise gauge invariant and Lorentz invariant Yang–Mills theory. However, even a tiny breaking of the starting gauge invariance at very small distances influenced by gravity would render the SLIV physically significant. For the SLIV scale comparable with the Planck one, the spontaneous Lorentz violation could become directly observable at low energies. We summarize the results obtained in the final Section 5.

2. Goldstonic quantum electrodynamics

The simplest SLIV model is given by a conventional QED Lagrangian for the charged fermion field ψ

$$L(A,\psi) = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\gamma\cdot\partial - m)\psi - eA_\mu\bar{\psi}\gamma^\mu\psi, \tag{2}$$

where the nonlinear vector field constraint (1) is imposed [12]. We can rewrite the Lagrangian $L(A,\psi)$ in terms of the physical photons now identified as being the SLIV generated vector Goldstone bosons. For this purpose one can use the following handy parametrization for the vector potential A_μ

$$A_\mu = a_\mu + \frac{n_\mu}{n^2}(n\cdot A) \quad \left(n^2 \equiv n_\mu^2\right), \tag{3}$$

where the a_μ is the pure Goldstonic mode satisfying

$$n\cdot a = 0, \tag{4}$$

while the effective Higgs mode (or the A_μ component in the vacuum direction) is given according to the above nonlinear constraint (1) by

$$n\cdot A = \left(M^2 - n^2 a_\nu^2\right)^{\frac{1}{2}} = M - \frac{n^2 a_\nu^2}{2M} + O\left(1/M^2\right) \tag{5}$$

taking, for definiteness, the positive sign for the above square root and expanding it in powers of a_ν^2/M^2. Putting then the parametrization (3) with the SLIV constraint (5) into our basic gauge invariant Lagrangian (2) one comes to the truly Goldstonic model for QED. This model might seem unacceptable since it contains, among other terms, the inappropriately large

[1] Actually, the simplest possible way to obtain the above supplementary condition (1) could be an inclusion the "standard" quartic vector field potential $V(A) = -\frac{m_A^2}{2}A_\mu^2 + \frac{\lambda_A}{4}(A_\mu^2)^2$ into the QED type Lagrangian, as can be motivated to an extent from some models in superstring theory [14]. This potential inevitably causes the spontaneous violation of Lorentz symmetry in a conventional way, much as an internal symmetry violation is caused in a linear σ model for pions [13]. As a result, one has a massive Higgs mode (with mass $\sqrt{2}m_A$) together with a massless Goldstone mode associated with photon. Furthermore, just as in the pion model one can go from the linear model for the SLIV to the nonlinear one taking a limit $\lambda_A \to \infty$, $m_A^2 \to \infty$ (while keeping the ratio m_A^2/λ_A to be finite). This immediately leads to the constraint (1) for vector potential A_μ with $n_\mu^2 M^2 = m_A^2/\lambda_A$, as it appears from a validity of its equation of motion. Another motivation for the nonlinear vector field constraint (1) might be an attempt to avoid the infinite self-energy of the electron in a classical electrodynamics, as was originally indicated by Dirac [15] and extended later to various vector field theory cases [16].

Lorentz violating fermion bilinear $eM\bar{\psi}(\gamma\cdot n)\psi$ which appears when the expansion (5) is applied to the fermion current interaction term $eA_\mu\bar{\psi}\gamma^\mu\psi$ in the Lagrangian L (2). However, due to local invariance of the Lagrangian L this term can be gauged away by making an appropriate redefinition of the fermion field according to

$$\psi \to e^{ieM(n\cdot x)}\psi, \tag{6}$$

through which the above fermion bilinear is exactly cancelled by an analogous term stemming from the fermion kinetic term. So, one eventually comes to the essentially nonlinear SLIV Lagrangian for the Goldstonic a_μ field of the type (taken to the first order in a_ν^2/M^2)

$$\begin{aligned} L(a,\psi) = {} & -\frac{1}{4}f_{\mu\nu}f^{\mu\nu} - \frac{1}{2}\delta(n\cdot a)^2 - \frac{1}{4}f_{\mu\nu}h^{\mu\nu}\frac{n^2a_\rho^2}{M} \\ & + \bar{\psi}(i\gamma\cdot\partial + m)\psi - ea_\mu\bar{\psi}\gamma^\mu\psi \\ & + \frac{en^2a_\rho^2}{2M}\bar{\psi}(\gamma\cdot n)\psi. \end{aligned} \tag{7}$$

We have denoted its strength tensor by $f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$, while $h^{\mu\nu} = n^\mu\partial^\nu - n^\nu\partial^\mu$ is a new SLIV oriented differential tensor acting on the infinite series in a_ρ^2 coming from the expansion of the effective Higgs mode (5) from which we have only included the first order term $-n^2a_\nu^2/2M$ throughout the Lagrangian $L(a,\psi)$. We have also explicitly introduced the orthogonality condition $n\cdot a = 0$ into the Lagrangian through the second term which can be treated as the gauge fixing term (taking the limit $\delta\to\infty$), and, furthermore, we have retained the notation ψ for the redefined fermion field.

The Lagrangian (7) completes the Goldstonic QED construction for the charged fermion field ψ. The model, as one can see, contains the massless Goldstone modes given by the three broken generators of the Lorentz group (while keeping the massive Higgs mode frozen). These modes, when lumped together, constitute a single Goldstone vector boson associated with photon.[2] In the limit $M\to\infty$ the model is indistinguishable from a conventional QED taken in the general axial (temporal or pure axial) gauge. So, for this part of the Lagrangian $L(a,\psi)$ given by the zero-order terms in $1/M$ the spontaneous Lorentz violation only means the noncovariant gauge choice in otherwise the gauge invariant (and Lorentz invariant) theory. Remarkably, furthermore, also all the other (first and higher order in $1/M$) terms in the $L(a,\psi)$ (7), though being by themselves the Lorentz and CPT violating ones, appear not to cause the physical SLIV effects due to strict cancellations in the physical processes involved. So, the nonlinear constraint (1) imposed on the standard QED Lagrangian (2) appears in fact to be a possible gauge choice, while the S-matrix remains unaltered under such a gauge convention. This conclusion was first confirmed at the tree level [12] and recently extended to the one-loop approximation [19]. All the one-loop contributions to the photon–photon, photon–fermion and fermion–fermion interactions violating the physical Lorentz invariance were shown to be exactly cancelled with each other, in the manner observed earlier for the simplest tree-order diagrams. This suggests that the vector field constraint $A_\mu^2 = n_\mu^2M^2$ having been treated as a nonlinear gauge choice at the tree (classical) level, remains as just a pure gauge condition when quantum effects are also taken into account. Remarkably, this conclusion appears to work also for a general Abelian theory case [20], particularly, when the internal $U(1)$ charge symmetry is spontaneously broken hand in hand with the Lorentz one. As a result, the massless photon being first generated by the Lorentz violation become then massive due to the standard Higgs mechanism, while the SLIV condition in itself remains to be a gauge choice.[3]

3. Goldstonic Yang–Mills theory

In this section, we extend our discussion to the non-Abelian internal symmetry case given by a general group G with generators t^i ($[t^i,t^j] = ic^{ijk}t^k$ and $\mathrm{Tr}(t^it^j) = \delta^{ij}$ where c^{ijk} are structure constants and $i,j,k = 0,1,\ldots,D-1$). The corresponding vector fields which transform according to its adjoint representation are given in the proper matrix form $\boldsymbol{A}_\mu = A_\mu^it^i$, while the matter fields (fermions, for definiteness) are presented in the fundamental representation column ψ^r ($r = 0,1,\ldots,d-1$) of G. By analogy with the above Goldstonic QED case, we take for them a conventional Yang–Mills type Lagrangian

$$\begin{aligned} \mathfrak{L}(\boldsymbol{A},\psi) = {} & -\frac{1}{4}\mathrm{Tr}\big(\boldsymbol{F}_{\mu\nu}\boldsymbol{F}^{\mu\nu}\big) + \bar{\psi}(i\gamma\cdot\partial - m)\psi \\ & + g\bar{\psi}\boldsymbol{A}_\mu\gamma^\mu\psi \end{aligned} \tag{8}$$

(where $\boldsymbol{F}_{\mu\nu} = \partial_\mu\boldsymbol{A}_\nu - \partial_\nu\boldsymbol{A}_\mu - ig[\boldsymbol{A}_\mu,\boldsymbol{A}_\nu]$ and g stands for the universal coupling constant in the theory) with the nonlinear SLIV constraint

$$\mathrm{Tr}\big(\boldsymbol{A}_\mu\boldsymbol{A}^\mu\big) = \mathfrak{n}_\mu^2M^2, \quad \mathfrak{n}_\mu^2 = \pm 1 \tag{9}$$

imposed.[4] One can easily see that, although we propose only the $SO(1,3)\times G$ invariance in the theory, the SLIV constraint taken (9) possesses, in fact, a much higher accidental symmetry

[2] Strictly speaking one can no longer use the standard definition of photon as a state being the spin-1 representation of the (now spontaneously broken) Poincare group. However, due to gauge symmetry of the starting QED Lagrangian (2) the separate SLIV Goldstone modes appear combined in such a way that a standard photon (taken in an axial gauge (4)) emerges.

[3] Note in this connection that there was discussed [12] a possibility of an explicit construction of the gauge function corresponding to the nonlinear gauge constraint (1) that would eliminate the need for all the kinds of checks of gauge invariance mentioned above. Remarkably, the equation for this gauge function appears to be mathematically equivalent to the classical Hamilton–Jacobi equation of motion for a charged particle. Thus, this gauge function should in principle exist because there is a solution to the classical problem. However, this formal analogy only works for the time-like SLIV ($n_\mu^2 = +1$) in the pure QED leaving aside a general Abelian theory when the gauge invariance can spontaneously be broken. Apart from that, it cannot be generally extended to the non-Abelian case (see Section 3).

[4] As in the Abelian case, the existence of such a constraint could be related with some nonlinear σ type SLIV model proposed for the vector field multiplet A_μ^i in the Yang–Mills theory (8). Note in this connection that, due to its generic antisymmetry, the familiar quadrilinear terms $-\frac{1}{4}g^2\,\mathrm{Tr}([\boldsymbol{A}_\mu,\boldsymbol{A}_\nu])^2$ in the Lagrangian (8) do not contribute into the SLIV since they identically vanish for any single-valued vacuum configuration $\langle A_\mu^i\rangle$.

$SO(D,3D)$ determined by the dimensionality D of the G group adjoint representation to which the vector fields A^i_μ belong. This symmetry is indeed spontaneously broken at a scale M

$$\langle A^i_\mu(x)\rangle = \mathfrak{n}^i_\mu M, \tag{10}$$

with the vacuum direction given now by the 'unit' rectangular matrix $\mathfrak{n}^i_\mu$ describing simultaneously both of the generalized SLIV cases, time-like ($SO(D,3D)\to SO(D-1,3D)$) or space-like ($SO(D,3D)\to SO(D,3D-1)$), respectively, depending on the sign of the $\mathfrak{n}^2_\mu \equiv \mathfrak{n}^i_\mu \mathfrak{n}^{\mu,i} = \pm 1$. This matrix has in fact only one non-zero element for both cases, subject to the appropriate $SO(D,3D)$ rotation. They are, specifically, $\mathfrak{n}^0_0$ or $\mathfrak{n}^0_3$ provided that the vacuum expectation value (10) is developed along the $i=0$ direction in the internal space and along the $\mu=0$ or $\mu=3$ direction, respectively, in the Minkowskian space–time. As we shall soon see, in response to each of these two breakings, side by side with one true vector Goldstone boson corresponding to the spontaneous violation of actual $SO(1,3)\otimes G$ symmetry of the total Lagrangian $\mathfrak{L}$, $D-1$ vector pseudo-Goldstone bosons related to breaking of the accidental $SO(D,3D)$ symmetry of the SLIV constraint taken (9) per se are also produced. Remarkably, in contrast to the familiar scalar PGB case [13] the vector PGBs remain strictly massless being protected by the non-Abelian gauge invariance of the starting Lagrangian (8). Together with the aforementioned true vector Goldstone boson they complete the whole Goldstonic vector field multiplet of the internal symmetry group G.

Actually, as in the above Abelian case, after the explicit use of the corresponding SLIV constraint (9), which is so far the only supplementary condition for vector field multiplet A^i_μ, one can identify the pure Goldstone field modes a^i_μ as follows:

$$A^i_\mu = a^i_\mu + \frac{\mathfrak{n}^i_\mu}{\mathfrak{n}^2}(\mathfrak{n}\cdot A), \quad \mathfrak{n}\cdot a \equiv \mathfrak{n}^i_\mu a^{\mu,i} = 0 \quad \left(\mathfrak{n}^2 \equiv \mathfrak{n}^2_\mu\right). \tag{11}$$

At the same time, an effective Higgs mode (i.e., the A^i_μ component in the vacuum direction $\mathfrak{n}^i_\mu$) is given by the product $\mathfrak{n}\cdot A \equiv \mathfrak{n}^i_\mu A^{\mu,i}$ determined by the SLIV constraint

$$\mathfrak{n}\cdot A = \left[M^2 - \mathfrak{n}^2\left(a^i_\nu\right)^2\right]^{\frac{1}{2}} = M - \frac{\mathfrak{n}^2(a^i_\nu)^2}{2M} + O\left(1/M^2\right). \tag{12}$$

As earlier in the Abelian case, we take the positive sign for the square root and expand it in powers of $(a^i_\nu)^2/M^2$. Note that the general Goldstonic modes a^i_μ, apart from pure vector fields, contain the $D-1$ scalar ones, $a^{i'}_0$ and $a^{i'}_3$ ($i'=1,\dots,D-1$), for the time-like ($\mathfrak{n}^i_\mu = \mathfrak{n}^0_0 g_{\mu 0}\delta^{i0}$) and space-like ($\mathfrak{n}^i_\mu = \mathfrak{n}^0_3 g_{\mu 3}\delta^{i0}$) SLIV, respectively. They can be eliminated from the theory if one puts the appropriate supplementary conditions on the a^i_μ fields which are still constraint free. Using their overall orthogonality (11) to the physical vacuum direction $\mathfrak{n}^i_\mu$ one can formulate these supplementary conditions in terms of a general axial gauge for the entire a^i_μ multiplet

$$n\cdot a^i \equiv n_\mu a^{\mu,i} = 0, \quad i = 0,\dots,D-1, \tag{13}$$

where n_μ is the unit Lorentz vector analogous to that introduced in the Abelian case, which is now oriented in Minkowskian space–time so as to be parallel to the vacuum matrix $\mathfrak{n}^i_\mu$. For such a choice the simple equation holds

$$\mathfrak{n}^i_\mu = s^i n_\mu \quad \left(s^i \equiv \frac{n\cdot \mathfrak{n}^i}{n^2}\right), \tag{14}$$

showing that the rectangular vacuum matrix $\mathfrak{n}^i_\mu$ has the factorized "two-vector" form. As a result, apart from the Higgs mode excluded earlier by the orthogonality condition (11), all the scalar fields are also eliminated, and only pure vector fields, $a^i_{\mu'}$ ($\mu'=1,2,3$) or $a^i_{\mu''}$ ($\mu''=0,1,2$) for time-like or space-like SLIV, respectively, are left in the theory.

We now show that these Goldstonic vector fields, denoted generally as a^i_μ but with the supplementary conditions (13) understood, appears truly massless when the starting non-Abelian Lagrangian $\mathfrak{L}$ (8) is subjected to the SLIV constraint (9) or (12). Actually, putting the parametrization (11) with the SLIV constraint (12) into the Lagrangian (8) one is led to the highly nonlinear Yang–Mills theory in terms of the pure Goldstonic gauge field modes a^i_μ. However, as in the above Abelian case, one should first use the local invariance of the Lagrangian $\mathfrak{L}$ to gauge away the apparently large Lorentz violating terms, which appear in the theory in the form of fermion and vector field bilinears. As one can readily see, they stem from the effective Higgs mode expansion (12) when it is applied to the couplings $g\bar\psi \boldsymbol{A}_\mu\gamma^\mu\psi$ and $-\frac{1}{4}g^2\,\mathrm{Tr}([\boldsymbol{A}_\mu,\boldsymbol{A}_\nu])^2$, respectively, in the Lagrangian (8). Analogously to the Abelian case, we make the appropriate redefinitions of the fermion (ψ) and vector ($\boldsymbol{a}_\mu \equiv a^i_\mu t^i$) field multiplets

$$\psi \to U(\omega)\psi, \qquad \boldsymbol{a}_\mu \to U(\omega)\boldsymbol{a}_\mu U(\omega)^\dagger,$$
$$U(\omega) = e^{igM(\mathfrak{n}^i\cdot x)t^i}. \tag{15}$$

Since the phase of the transformation matrix $U(\omega)$ is linear in the space–time coordinate and, on the other hand, the vacuum matrix $\mathfrak{n}^i_\mu$ has only one nonzero element ($\mathfrak{n}^0_0$ or $\mathfrak{n}^0_3$ for the particular SLIV cases) the following equalities are evidently satisfied:

$$\partial_\mu U(\omega) = ig\mathfrak{n}^i_\mu t^i U(\omega) = igU(\omega)\mathfrak{n}^i_\mu t^i. \tag{16}$$

One can readily confirm that the above-mentioned Lorentz violating terms are thereby cancelled with the analogous bilinears stemming from their kinetic terms. So, the final Lagrangian for the Goldstonic Yang–Mills theory takes the form (to the first order in $(a^i_\nu)^2/M^2$)

$$\begin{aligned}\mathfrak{L}(a,\psi) = {} & -\frac{1}{4}\mathrm{Tr}\left(\boldsymbol{f}_{\mu\nu}\boldsymbol{f}^{\mu\nu}\right) - \frac{1}{2}\delta\left(n\cdot a^i\right)^2 \\ & + \frac{1}{4}\mathrm{Tr}\left(\boldsymbol{f}_{\mu\nu}\boldsymbol{h}^{\mu\nu}\right)\frac{\mathfrak{n}^2(a^i_\nu)^2}{M} + \bar\psi(i\gamma\cdot\partial - m)\psi \\ & + g\bar\psi\boldsymbol{a}_\mu\gamma^\mu\psi - \frac{g\mathfrak{n}^2(a^i_\nu)^2}{2M}\bar\psi\left(\gamma\cdot\mathfrak{n}^k\right)t^k\psi. \end{aligned}\tag{17}$$

Here the tensor $\boldsymbol{f}_{\mu\nu}$ is, as usual, $\boldsymbol{f}_{\mu\nu} = \partial_\mu\boldsymbol{a}_\nu - \partial_\nu\boldsymbol{a}_\mu - ig[\boldsymbol{a}_\mu,\boldsymbol{a}_\nu]$, while $\boldsymbol{h}_{\mu\nu}$ is a new SLIV oriented tensor of the type

$$\boldsymbol{h}_{\mu\nu} = \boldsymbol{n}_\mu\partial_\nu - \boldsymbol{n}_\nu\partial_\mu + ig\left([\boldsymbol{n}_\mu,\boldsymbol{a}_\nu] - [\boldsymbol{n}_\nu,\boldsymbol{a}_\mu]\right),$$
$$\boldsymbol{n}_\mu \equiv \mathfrak{n}^k_\mu t^k, \tag{18}$$

acting on the infinite series in $(a^i_\nu)^2$ coming from the expansion of the effective Higgs mode (12) from which we have only included the first order term $-\mathfrak{n}^2(a^i_\nu)^2/2M$ throughout the Lagrangian $\mathfrak{L}(a,\psi)$. We have explicitly introduced the (axial) gauge fixing term into the Lagrangian, corresponding to the supplementary conditions (13) imposed. We have also retained the original notations for the fermion and vector fields after the transformations (15).

The theory we here derived is an essence a generalization of the nonlinear QED model [12] for the non-Abelian case. As one can see, this theory contains the massless vector boson multiplet a^i_μ (consisting of one Goldstone and $D-1$ pseudo-Goldstone vector states) which gauges the starting internal symmetry G. In the limit $M\to\infty$ it is indistinguishable from a conventional Yang–Mills theory taken in the general axial gauge. So, for this part of the Lagrangian $\mathfrak{L}(a,\psi)$ given by the zero-order in $1/M$ terms the spontaneous Lorentz violation only means the noncovariant gauge choice in the otherwise gauge invariant (and Lorentz invariant) theory. Furthermore, one may expect that, just as it appears in the nonlinear QED model, also all the first and higher order in $1/M$ terms in the $\mathfrak{L}$ (17), though being by themselves the Lorentz and CPT violating ones, do not cause the physical SLIV effects due to the mutual cancellation of their contributions to the physical processes involved.

4. The lowest order SLIV processes

Let us now show that simple tree level calculations related to the Lagrangian $\mathfrak{L}(a,\psi)$ confirm in essence this proposition. As an illustration, we consider SLIV processes in the lowest order in g and $1/M$ being the fundamental parameters of the Lagrangian (17). They are, as one can readily see, the vector-fermion and vector–vector elastic scattering going in the order g/M which we are going to consider in some detail as soon as the Feynman rules in the Goldstonic Yang–Mills theory are established.

4.1. Feynman rules

The corresponding Feynman rules, apart from the ordinary Yang–Mills theory rules for

(i) the vector-fermion vertex

$$-ig\gamma_\mu t^i, \tag{19}$$

(ii) the vector field propagator (taken in a general axial gauge $n^\mu a^i_\mu=0$)

$$D^{ij}_{\mu\nu}(k)=-\frac{i\delta^{ij}}{k^2}\left(g_{\mu\nu}-\frac{n_\mu k_\nu+k_\mu n_\nu}{n\cdot k}+\frac{n^2k_\mu k_\nu}{(n\cdot k)^2}\right), \tag{20}$$

which automatically satisfies the orthogonality condition $n^\mu\times D^{ij}_{\mu\nu}(k)=0$ and on-shell transversality $k_\mu D^{ij}_{\mu\nu}(k,k^2=0)=0$ (the latter means that free vector fields with polarization vector $\epsilon^i_\mu(k,k^2=0)$ are always appeared transverse $k^\mu\epsilon^i_\mu(k)=0$);

(iii) the 3-vector vertex (with vector field 4-momenta k_1,k_2 and k_3; all 4-momenta in vertexes are taken ingoing throughout)

$$gc^{ijk}\big[(k_1-k_2)_\gamma g_{\alpha\beta}+(k_2-k_3)_\alpha g_{\beta\gamma}+(k_3-k_1)_\beta g_{\alpha\gamma}\big], \tag{21}$$

include the new ones, violating Lorentz and CPT invariance, for

(iv) the contact 2-vector-fermion vertex

$$i\frac{g\mathfrak{n}^2}{M}\big(\gamma\cdot\mathfrak{n}^k\big)\tau^k g_{\mu\nu}\delta^{ij}, \tag{22}$$

(v) another 3-vector vertex

$$-\frac{i\mathfrak{n}^2}{M}\big[\big(k_1\cdot\mathfrak{n}^i\big)k_{1,\alpha}g_{\beta\gamma}\delta^{jk}+\big(k_2\cdot\mathfrak{n}^j\big)k_{2,\beta}g_{\alpha\gamma}\delta^{ki} + \big(k_3\cdot\mathfrak{n}^k\big)k_{3,\gamma}g_{\alpha\beta}\delta^{ij}\big], \tag{23}$$

where the second index in the vector field 4-momenta k_1,k_2 and k_3 denotes their Lorentz components;

(vi) the extra 4-vector vertex (with the vector field 4-momenta $k_{1,2,3,4}$ and their proper differences $k_{12}\equiv k_1-k_2$, etc.)

$$\begin{aligned}-\frac{\mathfrak{n}^2g}{M}\big[&c^{ijp}\delta^{kl}g_{\alpha\beta}g_{\gamma\delta}\big(\mathfrak{n}^p\cdot k_{12}\big)+c^{klp}\delta^{ij}g_{\alpha\beta}g_{\gamma\delta}\big(\mathfrak{n}^p\cdot k_{34}\big)\\ &+c^{ikp}\delta^{jl}g_{\alpha\gamma}g_{\beta\delta}\big(\mathfrak{n}^p\cdot k_{13}\big)+c^{jlp}\delta^{ik}g_{\alpha\gamma}g_{\beta\delta}\big(\mathfrak{n}^p\cdot k_{24}\big)\\ &+c^{ilp}\delta^{jk}g_{\alpha\delta}g_{\beta\gamma}\big(\mathfrak{n}^p\cdot k_{14}\big)+c^{jkp}\delta^{il}g_{\alpha\delta}g_{\beta\gamma}\big(\mathfrak{n}^p\cdot k_{23}\big)\big],\end{aligned} \tag{24}$$

where we have not included the terms which might contain contractions of the vacuum matrix $\mathfrak{n}^p_\mu$ with vector field polarization vectors $\epsilon^i_\mu(k)$ in the vector–vector scattering amplitude since these contractions are vanished due to the gauge taken (13), $\mathfrak{n}^p\cdot\epsilon^i=s^p(n\cdot\epsilon^i)=0$ (as follows according to a factorized two-vector form for the matrix $\mathfrak{n}^p_\mu$ (14)).

Just the rules (i)–(vi) are needed to calculate the lowest order processes mentioned in the above.

4.2. Vector boson scattering on fermion

This process is directly related to two SLIV diagrams one of which is given by the contact a^2-fermion vertex (22), while another corresponds to the pole diagram with the longitudinal a-boson exchange between Lorentz violating a^3 vertex (23) and ordinary a-boson–fermion one (19). Since ingoing and outgoing a-bosons appear transverse $(k_1\cdot\epsilon^i(k_1)=0,\ k_2\cdot\epsilon^j(k_2)=0)$ only the third term in this a^3 coupling (23) contributes to the pole diagram so that one comes to a simple matrix element $i\mathcal{M}$ from both of diagrams

$$\begin{aligned}i\mathcal{M}=i\frac{gn^2}{M}\bar{u}(p_2)\tau^l\big[\big(\gamma\cdot\mathfrak{n}^l\big)+i\big(k\cdot\mathfrak{n}^l\big)\gamma^\mu k^\nu D_{\mu\nu}(k)\big]\\ \times u(p_1)\big[\epsilon(k_1)\cdot\epsilon(k_2)\big],\end{aligned} \tag{25}$$

where the spinors $u(p_{1,2})$ and polarization vectors $\epsilon^i_\mu(k_1)$ and $\epsilon^j_\mu(k_2)$ stand for ingoing and outgoing fermions and a-bosons, respectively, while k is the 4-momentum transfer $k=p_2-p_1=k_1-k_2$. Upon further simplifications in the square bracket related to the explicit form of the a boson propagator $D_{\mu\nu}(k)$

(20) and matrix $\mathfrak{n}^i_\mu$ (14), and using the fermion current conservation $\bar{u}(p_2)(\hat{p}_2 - \hat{p}_1)u(p_1) = 0$, one is finally led to the total cancellation of the Lorentz violating contributions to the a-boson–fermion scattering in the g/M approximation.

Note, however, that such a result may be in some sense expected since from the SLIV point of view the lowest order a-boson–fermion scattering discussed here is hardly distinct from the photon–fermion scattering considered in the nonlinear QED case [12]. Actually, the fermion current conservation which happens to be crucial for the above cancellation works in both of cases, whereas the couplings which are peculiar to the Yang–Mills theory have not yet touched on. In this connection the next example seems to be more instructive.

4.3. Vector–vector scattering

The matrix element for this process in the lowest order g/M is given by the contact SLIV a^4 vertex (24) and the pole diagrams with the longitudinal a-boson exchange between the ordinary a^3 vertex (21) and Lorentz violating a^3 one (23), and vice versa. There are six pole diagrams in total describing the elastic a–a scattering in the s- and t-channels, respectively, including also those with an interchange of identical a-bosons. Remarkably, the contribution of each of them is exactly canceled with one of six terms appeared in the contact vertex (24). Actually, writing down the matrix element for one of the pole diagrams with ingoing a-bosons (with momenta k_1 and k_2) interacting through the vertex (21) and outgoing a-bosons (with momenta k_3 and k_4) interacting through the vertex (23) one has

$$\begin{aligned} i\mathcal{M}^{(1)}_{\text{pole}} = -i\frac{gn^2}{M}c^{ijp}\delta^{kl}\big[&(k_1-k_2)_\mu g_{\alpha\beta} \\ &+ (k_2-k)_\alpha g_{\beta\mu} + (k-k_1)_\beta g_{\alpha\mu}\big] \\ &\times D^{pq}_{\mu\nu}(k) g_{\gamma\delta} k_\nu \big(n^q\cdot k\big) \\ &\times \big[\epsilon^{i,\alpha}(k_1)\epsilon^{j,\beta}(k_2)\epsilon^{k,\gamma}(k_3)\epsilon^{l,\delta}(k_4)\big], \end{aligned} \tag{26}$$

where polarization vectors $\epsilon^{i,\alpha}(k_1)$, $\epsilon^{j,\beta}(k_2)$, $\epsilon^{k,\gamma}(k_3)$ and $\epsilon^{l,\delta}(k_4)$ belong, respectively, to ingoing and outgoing a-bosons, while $k = -(k_1 + k_2) = k_3 + k_4$ according to the momentum running in the diagrams taken above. Again, as in the previous case of vector–fermion scattering, due to the fact that outgoing a-bosons appear transverse ($k_3 \cdot \epsilon^k(k_3) = 0$ and $k_4 \cdot \epsilon^l(k_4) = 0$), only the third term in the Lorentz violating a^3 coupling (23) contributes to this pole diagram. After evident simplifications related to the a-boson propagator $D_{\mu\nu}(k)$ (20) and matrix $\mathfrak{n}^i_\mu$ (14) one comes to the expression which is exactly cancelled with the first term in the contact SLIV vertex (24) when it is properly contracted with a-boson polarization vectors. Likewise, other terms in this vertex provide the further one-to-one cancellation with the remaining pole matrix elements $i\mathcal{M}^{(2-6)}_{\text{pole}}$. So, again, the Lorentz violating contribution to the vector–vector scattering is absent in Goldstonic Yang–Mills theory in the lowest g/M approximation.

4.4. Other processes

Many other tree level Lorentz violating processes, related to a bosons and fermions, appear in higher orders in the basic SLIV parameter $1/M$. They come from the subsequent expansion of the effective Higgs mode (12) in the Lagrangian (17). Again, their amplitudes are essentially determined by an interrelation between the longitudinal a-boson exchange diagrams and the corresponding contact a-boson interaction diagrams which appear to cancel each other thus eliminating physical Lorentz violation in theory.

Most likely, the same conclusion can be derived for SLIV loop contributions as well. Actually, as in the massless QED case considered earlier [19], the corresponding one-loop matrix elements in Goldstonic Yang–Mills theory either vanish by themselves or amount to the differences between pairs of the similar integrals whose integration variables are shifted relative to each other by some constants (being in general arbitrary functions of external four-momenta of the particles involved) that in the framework of dimensional regularization leads to their total cancellation.

So, the Goldstonic vector field theory (17) for a non-Abelian charge-carrying matter is likely to be physically indistinguishable from a conventional Yang–Mills theory.

5. Conclusion

The spontaneous Lorentz violation in 4-dimensonal flat Minkowskian space–time was shown to generate vector Goldstone bosons both in Abelian and non-Abelian theories with the corresponding nonlinear vector field constraint (1) or (9) imposed. In the Abelian case such a massless vector boson is naturally associated with photon. In non-Abelian case, although the pure Lorentz violation still generates only one genuine Goldstone vector boson, the accompanying vector PGBs related to a violation of the larger accidental symmetry $SO(D, 3D)$ of the SLIV constraint (9) in itself come also into play properly completing the whole gauge multiplet of the internal symmetry group G taken. Remarkably, they remain strictly massless being protected by the starting gauge invariance of the Yang–Mills theory involved. These theories, both Abelian and non-Abelian, though being essentially nonlinear in the Goldstone vector modes, appear to be physically indistinguishable from conventional QED and Yang–Mills theories. One could actually see that just the gauge invariance ensures that our theories do not have unreasonably large, proportional to the SLIV scale, Lorentz violation in the fermion and vector field interaction terms (as those which could otherwise stem from their large bilinears in Sections 2 and 3). Furthermore, it appears also to ensure that all the physical Lorentz violating effects, even those suppressed by this SLIV scale, are non-observable (as was explicitly shown in Section 4). As a result, Abelian and non-Abelian SLIV theory appear, respectively, as standard QED and Yang–Mills theory taken in the nonlinear gauge (to which the vector field constraints (1) and (9) are virtually reduced), while the S-matrix remains unaltered under such a gauge convention.

In conclusion, it seems plausible that gauge fields, both Abelian and non-Abelian, might have a true Goldstonic nature. However, the most fundamental question whether the physical Lorentz violation takes place, that only could uniquely point toward such a possibility, is still an open question. Note that we do not mean here direct (and quite arbitrary in essence) Lorentz non-invariant extensions of QED or Standard Model which were intensively discussed on their own in recent years [6–8]. Rather, our goal is a construction of genuine SLIV models which would generate gauge fields as the proper vector Goldstone bosons, from one hand, and could lead to observed Lorentz violating effects, from the other. In this connection, somewhat natural framework for the physical Lorentz violation to occur would be a model where the internal gauge invariance were slightly broken at very small distances through some high-order operators stemming from the gravity-influenced area. Such physical SLIV effects would be seen in terms of powers of ratio M/M_{Pl} (where M_{Pl} is the Planck mass). So, for the SLIV scale comparable with the Planck one they would become directly observable. Remarkably enough, if one has such internal gauge symmetry breaking in an ordinary Lorentz invariant theory this breaking appears vanishingly small at low energies being properly suppressed by the Planck scale. However, the spontaneous Lorentz violation would render it physically significant: the higher Lorentz scale, the greater SLIV effects observed. If true, it would be of particular interest to have a better understanding of the internal gauge symmetry breaking mechanism that brings the spontaneous Lorentz violation to low energies. We return to this basic question elsewhere.

Acknowledgements

We would like to thank Colin Froggatt, Rabi Mohapatra and Holger Nielsen for useful discussions and comments. One of us (J.L.C.) is grateful for the warm hospitality shown to him during a visit to Center for Particle and String Theory at University of Maryland where part of this work was carried out.

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