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## Apparent ghosts and spurious degrees of freedom in non-local theories



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## ABSTRACT

Recent work has shown that non-local modifications of the Einstein equations can have interesting cosmological consequences and can provide a dynamical origin for dark energy, consistent with existing data. At first sight these theories are plagued by ghosts. We show that these apparent ghost-like instabilities do not describe actual propagating degrees of freedom, and there is no issue of ghost-induced quantum vacuum decay.

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## 1. Introduction

Non-local classical equations and non-local field theories have been sporadically studied since the early days of field theory [1]. In general, they present a number of subtle issues concerning ghosts and unitarity, and often it is not obvious even how many degrees of freedom they describe, see e.g. [2–6]. In recent years there has been a renewed interest for non-local models in connection with cosmology. Non-local effective equations of motion can play an important role in explaining the “old” cosmological constant problem, through a degravitation mechanism that promotes Newton’s constant to a non-local operator [7,8] (see also [9–12]), and non-local cosmological models have interesting observational consequences and can play a role in explaining the origin of dark energy [13–26]. Non-local gravity models have also been studied as UV modifications of GR, see e.g. [27–31].

In two recent papers [32,33] we have proposed a non-local approach that allows us to introduce a mass term in the Einstein equations, in such a way that the invariance under diffeomorphisms is not spoiled, and we do not need to introduce an external reference metric (contrary to what happens in the conventional local approach to massive gravity [34–40]). In particular, in [33] a classical model based on the non-local equation has been proposed:

$$G_{\mu\nu} - \frac{d-1}{2d} m^2 (g_{\mu\nu} \square_{\text{ret}}^{-1} R)^{\text{T}} = 8\pi G T_{\mu\nu}. \quad (1.1)$$

Here  $d$  is the number of spatial dimensions (the factor  $(d-1)/2d$  is a convenient normalization of the parameter  $m^2$ ),  $\square = g^{\mu\nu} \nabla_{\mu} \nabla_{\nu}$  is the d’Alembertian operator with respect to the metric  $g_{\mu\nu}$  and,

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quite crucially,  $\square_{\text{ret}}^{-1}$  is its inverse computed with the retarded Green’s function. The superscript T denotes the extraction of the transverse part of the tensor, which exploits the fact that, in a generic Riemannian manifold, any symmetric tensor  $S_{\mu\nu}$  can be decomposed as  $S_{\mu\nu} = S_{\mu\nu}^{\text{T}} + \frac{1}{2}(\nabla_{\mu} S_{\nu} + \nabla_{\nu} S_{\mu})$ , with  $\nabla^{\mu} S_{\mu\nu}^{\text{T}} = 0$  [41,42]. The extraction of the transverse part of a tensor is itself a non-local operation, which involves further  $\square^{-1}$  operators. For instance in flat space, where  $\nabla_{\mu} \rightarrow \partial_{\mu}$ , it is easy to show that

$$S_{\mu\nu}^{\text{T}} = S_{\mu\nu} - \frac{1}{\square} (\partial_{\mu} \partial^{\rho} S_{\rho\nu} + \partial_{\nu} \partial^{\rho} S_{\rho\mu}) + \frac{1}{\square^2} \partial_{\mu} \partial_{\nu} \partial^{\rho} \partial^{\sigma} S_{\rho\sigma}. \quad (1.2)$$

Again, in Eq. (1.1) all  $\square^{-1}$  factors coming from the extraction of the transverse part are defined by the retarded Green’s function, so that Eq. (1.1) satisfies causality. Furthermore, since the left-hand side of Eq. (1.1) is transverse by construction, the energy-momentum tensor is automatically conserved,  $\nabla^{\mu} T_{\mu\nu} = 0$ . Both causality and energy-momentum conservation were lost in the original degravitation proposal [7], and in this sense Eq. (1.1) can be seen as a refinement of the original idea. However, the explicit appearance of retarded Green’s function in the equations of motion has important consequences for the conceptual meaning of an equation such as (1.1), as we will discuss below.

As shown in [33,43], Eq. (1.1) has very interesting cosmological properties, and in particular it generates a dynamical dark energy. Since during radiation dominance (RD) the Ricci scalar  $R$  vanishes, the term  $\square^{-1} R$  starts to grow only during matter dominance (MD), thereby providing in a natural way a delayed onset of the accelerated expansion (similarly to what happens in the model proposed in [13]). Furthermore, this model is highly predictive since it only introduces a single parameter  $m$ , that replaces the cosmological constant in  $\Lambda$ CDM. In contrast, models based on quintessence,  $f(R)$ -gravity, or the non-local model of [13] in which

a term  $Rf(\square^{-1}R)$  is added to the Einstein action, all introduce at least one arbitrary function, which is typically tuned so to get the desired cosmological behavior. In our case, we can fix the value of  $m$  so to reproduce the observed value  $\Omega_{DE} \simeq 0.68$ . This gives  $m \simeq 0.67H_0$ , and leaves us with no free parameter. We then get a pure prediction for the EOS parameter of dark energy. Quite remarkably, writing  $w_{DE}(a) = w_0 + (1-a)w_a$ , in [33] we found  $w_0 \simeq -1.04$  and  $w_a \simeq -0.02$ , consistent with the Planck data, and on the phantom side.

These cosmological features make Eq. (1.1) a potentially very attractive dark energy model. The presence of the  $\square^{-1}$  operator raises however a number of potential problems of theoretical consistency, and the purpose of this paper is to investigate them in some detail. The crucial problem can already be seen linearizing Eq. (1.1) over flat space. Writing  $g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$ , where  $\kappa = (32\pi G)^{1/2}$  and  $\eta_{\mu\nu} = (-, +, \dots, +)$ , the equation of motion of this theory takes the form

$$\mathcal{E}^{\mu\nu,\rho\sigma} h_{\rho\sigma} - \frac{d-1}{d} m^2 P_{\text{ret}}^{\mu\nu} P_{\text{ret}}^{\rho\sigma} h_{\rho\sigma} = -16\pi G T^{\mu\nu}, \quad (1.3)$$

where  $\mathcal{E}^{\mu\nu,\rho\sigma}$  is the Lichnerowicz operator, while

$$P_{\text{ret}}^{\mu\nu} = \eta^{\mu\nu} - \frac{\partial^\mu \partial^\nu}{\square_{\text{ret}}}, \quad (1.4)$$

and  $1/\square_{\text{ret}}$  is the retarded inverse of the flat-space d'Alembertian. Apparently, the corresponding quadratic Lagrangian is

$$\mathcal{L}_2 = \frac{1}{2} h_{\mu\nu} \mathcal{E}^{\mu\nu,\rho\sigma} h_{\rho\sigma} - \frac{d-1}{2d} m^2 h_{\mu\nu} P^{\mu\nu} P^{\rho\sigma} h_{\rho\sigma}. \quad (1.5)$$

Adding the usual gauge fixing term of linearized massless gravity,  $\mathcal{L}_{\text{gf}} = -(\partial^\nu \bar{h}_{\mu\nu})(\partial_\rho \bar{h}^{\rho\mu})$ , and inverting the resulting quadratic form we get the propagator

$$\begin{aligned} \tilde{D}^{\mu\nu\rho\sigma}(k) &= \frac{-i}{2k^2} \left( \eta^{\mu\rho} \eta^{\nu\sigma} + \eta^{\mu\sigma} \eta^{\nu\rho} - \frac{2}{d-1} \eta^{\mu\nu} \eta^{\rho\sigma} \right) \\ &\quad - \frac{1}{d(d-1)} \frac{im^2}{k^2(-k^2+m^2)} \eta^{\mu\nu} \eta^{\rho\sigma}, \end{aligned} \quad (1.6)$$

plus terms proportional to  $k^\mu k^\nu$ ,  $k^\rho k^\sigma$  and  $k^\mu k^\nu k^\rho k^\sigma$ , that give zero when contracted with a conserved energy-momentum tensor. The first term is the usual propagator of a massless graviton, for  $d$  generic. The term proportional to  $m^2$  gives an extra contribution to the saturated propagator  $\tilde{T}_{\mu\nu}(-k) \tilde{D}^{\mu\nu\rho\sigma}(k) \tilde{T}_{\rho\sigma}(k)$ , equal to

$$\frac{1}{d(d-1)} \tilde{T}(-k) \left[ -\frac{i}{k^2} - \frac{i}{(-k^2+m^2)} \right] \tilde{T}(k). \quad (1.7)$$

This term apparently describes the exchange of a healthy massless scalar plus a ghostlike massive scalar. In general, a ghost has two quite distinct effects: at the classical level, it can give rise to runaway solutions. In our cosmological context, rather than a problem this can actually be a virtue, because a phase of accelerated expansion is in a sense an instability of the classical evolution. Indeed, ghosts have been suggested as models of phantom dark energy [44, 45]. The real trouble is that, at the quantum level, a ghost corresponds to a particle with negative energy and induces a decay of the vacuum, through processes in which the vacuum decays into ghosts plus normal particles, and renders the theory inconsistent.

The main purpose of this paper is to discuss and clarify some subtle conceptual issues related to this apparent ghost-like degree of freedom and to show that, in fact, in this theory there is no propagating ghost-like degree of freedom. The paper is organized as follows. In Section 2 we show that the status of a non-local equation such as Eq. (1.1) is that of an effective classical equation, derived from some classical or quantum averaging in

a more fundamental theory. In Section 3 we show that similar apparent ghosts even appear in massless GR when one decomposes the metric perturbation into a transverse-traceless part  $h_{\mu\nu}^{\text{TT}}$  and a trace part  $\eta_{\mu\nu}s$ . This is due to the fact that the relation between  $\{h_{\mu\nu}^{\text{TT}}, s\}$  and the original metric perturbations  $h_{\mu\nu}$  is non-local. We show that (contrary to some statements in the literature) the apparent ghost field  $s$  is not neutralized by the helicity-0 component of  $h_{\mu\nu}^{\text{TT}}$ . Rather, what saves the vacuum stability of GR, in these variables, is that  $s$  (as well as the helicity-0,  $\pm 1$  components of  $h_{\mu\nu}^{\text{TT}}$ ), is a non-propagating field and cannot be put on the external lines, nor in loops. Beside having an intrinsic conceptual interest, this analysis will also show that the same considerations extend straightforwardly to the non-local modification of GR that we are studying. Finally, in Section 4 we will work out the explicit relation between the fake ghost that is suggested by Eq. (1.7), and the spurious degrees of freedom that are known to emerge when a non-local theory is written in local form by introducing auxiliary fields. Section 5 contains our conclusions.

## 2. Non-local QFT or classical effective equations?

A crucial point of Eq. (1.1), or of its linearization (1.3), is that they contain explicitly a retarded propagator. This retarded prescription is forced by causality, which we do not want to give up. We are used, of course, to the appearance of retarded propagators in the solutions of classical equations. Here however the retarded propagator already appears in the equation itself, and not only in its solution. Is it possible to obtain such an equation from a variational principle? The answer, quite crucially, is no. As already observed by various authors [13,18,32], a retarded inverse d'Alembertian cannot be obtained from the variation of a non-local action. Consider for illustration a non-local term in an action of the form  $\int dx \phi \square^{-1} \phi$ , where  $\phi$  is some scalar field, and  $\square^{-1}$  is defined with respect to some Green's function  $G(x; x')$ . Taking the variation with respect to  $\phi(x)$  we get

$$\begin{aligned} \frac{\delta}{\delta\phi(x)} \int dx' \phi(x') (\square^{-1} \phi)(x) \\ &= \frac{\delta}{\delta\phi(x)} \int dx' dx'' \phi(x') G(x'; x'') \phi(x'') \\ &= \int dx' [G(x; x') + G(x'; x)] \phi(x'). \end{aligned} \quad (2.1)$$

We see that the variation of the action automatically symmetrizes the Green's function. It is therefore impossible to obtain in this way a retarded Green's function in the equations of motion, since  $G_{\text{ret}}(x; x')$  is not symmetric under  $x \leftrightarrow x'$ ; rather  $G_{\text{ret}}(x'; x) = G_{\text{adv}}(x; x')$ . The same happens if we take the variation of the Lagrangian (1.5). Writing explicitly the convolution with the Green's function as we did in Eq. (2.1) we find that it is not possible to get the term  $P_{\text{ret}}^{\mu\nu} P_{\text{ret}}^{\rho\sigma}$  in Eq. (1.3). If in the action the term  $\square^{-1}$  that appears in  $P^{\mu\nu}$  is defined by a symmetric Green's function, so that  $G(x; x') = G(x'; x)$ , we find the same Green's function in the equation of motion. If, in contrast, we use  $h_{\mu\nu} P_{\text{ret}}^{\mu\nu} P_{\text{ret}}^{\rho\sigma} h_{\rho\sigma}$  in the action, in the equation of motions we get  $(P_{\text{ret}}^{\mu\nu} P_{\text{ret}}^{\rho\sigma} + P_{\text{adv}}^{\mu\nu} P_{\text{adv}}^{\rho\sigma}) h_{\rho\sigma}$ .

Of course, one can take the point of view that the classical theory is defined by its equations of motion, while the action is simply a convenient "device" that, through a set of well defined rules, allows us to compactly summarize the equations of motion. We can then take the formal variation of the action and at the end replace by hand all factors  $\square^{-1}$  by  $\square_{\text{ret}}^{-1}$  in the equation of motion. This is indeed the procedure used in [13,46], in the context of non-local gravity theories with a Lagrangian of the form  $Rf(\square^{-1}R)$ . As long as we see the Lagrangian as a "device" that, through a well defined procedure, gives a classical equation of motion, this

prescription is certainly legitimate. However, any connection between these classical causal equations of motion and the quantum field theory described by such a Lagrangian is now lost. In particular, the terms in Eq. (1.6) or in Eq. (1.7) that apparently describe the exchange of a healthy massless scalar plus a ghostlike massive scalar are just the propagators that, to reproduce Eq. (1.3), after the variation must be set equal to retarded propagators. Taking them as Feynman propagators in a QFT gives a quantum theory that has nothing to do with our initial classical equation (1.3) and that has dynamical degrees of freedom that, with respect to our original problem, are spurious.

Thus, Eq. (1.1) is not the classical equation of motion of a non-local quantum field theory. To understand its conceptual meaning, we observe that non-local equations involving the retarded propagator appear in various situation in physics, but are never fundamental. They rather typically emerge after performing some form of averaging, either purely classical or at the quantum level. In particular, non-local field equations govern the effective dynamics of the vacuum expectation values of quantum fields, which include the quantum corrections to the effective action. The standard path integral approach provides the dynamics for the in-out matrix element of a quantum field, e.g.  $\langle 0_{\text{out}} | \hat{\phi} | 0_{\text{in}} \rangle$  or, in a semiclassical approach to gravity,  $\langle 0_{\text{out}} | \hat{g}_{\mu\nu} | 0_{\text{in}} \rangle$ . The classical equations for these quantities are however determined by the Feynman propagator, so they are not causal, since they contain both the retarded and the advanced Green's function. This is not surprising, since the in-out matrix element are not directly measurable quantities, but only provide intermediate steps in the QFT computations. Furthermore, even if  $\hat{\phi}$  is a Hermitian operator, its in-out matrix element are complex. In particular, this makes it impossible to interpret  $\langle 0_{\text{out}} | \hat{g}_{\mu\nu} | 0_{\text{in}} \rangle$  as an effective metric. In contrast, the in-in matrix elements are real, and satisfy non-local but causal equations [47, 48], involving only retarded propagators (that can be computed using the Schwinger–Keldysh formalism).

Similar non-local but causal equations can also emerge from a purely classical averaging procedure, when one separates the dynamics of a system into a long-wavelength and a short-wavelength part. One can then obtain an effective non-local but causal equation for the long-wavelength modes by integrating out the short-wavelength modes, see e.g. [49] for a recent example in the context of cosmological perturbation theory. Another purely classical example comes from the standard post-Newtonian/post-Minkowskian formalisms for GW production [50,51]. In linearized theory the gravitational wave (GW) amplitude  $h_{\mu\nu}$  is determined by  $\square h_{\mu\nu} = -16\pi G T_{\mu\nu}$ , where  $\bar{h}_{\mu\nu} = h_{\mu\nu} - (1/2)h\eta_{\mu\nu}$ . In such a radiation problem this equation is solved with the retarded Green's function,  $\bar{h}_{\mu\nu} = -16\pi G \square_{\text{ret}}^{-1} T_{\mu\nu}$ . When the non-linearities of GR are included, the GWs generated at some perturbative order become themselves sources for the GW generation at the next order. In the far-wave zone, this iteration gives rise to effective equations for  $\bar{h}_{\mu\nu}$  involving  $\square_{\text{ret}}^{-1}$ .

In summary, non-local equations involving  $\square_{\text{ret}}^{-1}$  are *not* the classical equation of motion a non-local QFT (a point already made e.g. in [13,18,24,25,52]). Even if we can find a classical Lagrangian whose variation reproduces them (once supplemented with the  $\square^{-1} \rightarrow \square_{\text{ret}}^{-1}$  prescription after having performed the variation), the quantum field theory described by this Lagrangian has a priori nothing to do with the problem at hand. Issues of quantum consistencies (such as the possibility of a vacuum decay amplitude induced by ghosts) can only be addressed in the fundamental theory that, upon classical or quantum smoothing, produces these non-local (but causal) classical equations.

So, there is no sense, and no domain of validity, in which the Lagrangian (1.5) can be used to define a QFT associated to our theory. To investigate whether the classical equation (1.1) derives from

a QFT with a stable quantum vacuum we should identify the fundamental theory and the smoothing procedure that give rise to it, and only in this framework we can pose the question.

### 3. Vacuum stability in massless and massive gravity

#### 3.1. A fake ghost in massless GR

It is instructive to see more generally how spurious degrees of freedom, and in particular spurious ghosts, can appear when one uses non-local variables. A simple and quite revealing example is provided by GR itself. We have already discussed this example in Appendix B of [53], but it is useful to re-examine and expand it in this context. We consider GR linearized over flat space. The quadratic Einstein–Hilbert action is

$$S_{\text{EH}}^{(2)} = \frac{1}{2} \int d^{d+1}x h_{\mu\nu} \mathcal{E}^{\mu\nu,\rho\sigma} h_{\rho\sigma}. \quad (3.1)$$

We decompose the metric as

$$h_{\mu\nu} = h_{\mu\nu}^{\text{TT}} + \frac{1}{2}(\partial_\mu \epsilon_\nu + \partial_\nu \epsilon_\mu) + \frac{1}{d} \eta_{\mu\nu} s, \quad (3.2)$$

where  $h_{\mu\nu}^{\text{TT}}$  is transverse and traceless,  $\partial^\mu h_{\mu\nu}^{\text{TT}} = 0$ ,  $\eta^{\mu\nu} h_{\mu\nu}^{\text{TT}} = 0$ . The vector  $\epsilon_\mu$  could be further decomposed as  $\epsilon_\mu = \epsilon_\mu^{\text{T}} + \partial_\mu \alpha$ , where  $\partial^\mu \epsilon_\mu^{\text{T}} = 0$ . Under a linearized diffeomorphism  $h_{\mu\nu} \rightarrow h_{\mu\nu} - (\partial_\mu \xi_\nu + \partial_\nu \xi_\mu)$  we have  $\epsilon_\mu \rightarrow \epsilon_\mu - \xi_\mu$  while the tensor  $h_{\mu\nu}^{\text{TT}}$  and the scalar  $s$  are gauge invariant. Plugging Eq. (3.2) into Eq. (3.1) we find that  $\epsilon_\mu$  cancels (as it is obvious from the fact that Eq. (3.1) is invariant under linearized diffeomorphisms and  $\epsilon_\mu$  is a pure gauge mode), and

$$S_{\text{EH}}^{(2)} = \frac{1}{2} \int d^{d+1}x \left[ h_{\mu\nu}^{\text{TT}} \square (h^{\mu\nu})^{\text{TT}} - \frac{d-1}{d} s \square s \right]. \quad (3.3)$$

Performing the same decomposition in the energy–momentum tensor, the interaction term can be written as

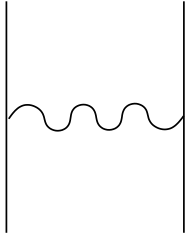
$$S_{\text{int}} = \frac{\kappa}{2} \int d^{d+1}x h_{\mu\nu} T^{\mu\nu} = \frac{\kappa}{2} \int d^{d+1}x \left[ h_{\mu\nu}^{\text{TT}} (T^{\mu\nu})^{\text{TT}} + \frac{1}{d} s T \right], \quad (3.4)$$

so the equations of motion derived from  $S_{\text{EH}}^{(2)} + S_{\text{int}}$  are

$$\square h_{\mu\nu}^{\text{TT}} = -\frac{\kappa}{2} T_{\mu\nu}^{\text{TT}}, \quad (3.5)$$

$$\square s = \frac{\kappa}{2(d-1)} T. \quad (3.6)$$

This result can be surprising, because it seems to suggest that in ordinary massless GR we have many more propagating degrees of freedom than expected: the components of the transverse–traceless tensor  $h_{\mu\nu}^{\text{TT}}$  (i.e. 5 components in  $d=3$ ) plus the scalar  $s$ . Note that these degrees of freedom are gauge invariant, so they cannot be gauged away. Furthermore, from Eq. (3.3) the scalar  $s$  seems a ghost! Of course these conclusions are wrong, and for any  $d$  linearized GR is a ghost-free theory; in particular, in  $d=3$  it only has two radiative degrees of freedom, corresponding to the  $\pm 2$  helicities of the graviton (and, in generic  $d$ ,  $(d+1)(d-2)/2$  radiative degrees of freedom, corresponding to the fact that the little group is  $SO(d-1)$ ). We know very well that the remaining degrees of freedom of GR are physical (i.e. gauge-invariant) but non-radiative. How is this consistent with the fact that  $s$ , as well as all the extra components of  $h_{\mu\nu}^{\text{TT}}$ , satisfy a Klein–Gordon rather than a Poisson equation?



**Fig. 1.** The tree-level Feynman graph describing the exchange of a graviton between two matter lines. In this graph, the contribution due to the exchange of the scalar  $s$  is canceled by the exchange of the helicity-0 component of  $h_{\mu\nu}^{\text{TT}}$ .

The answer, as discussed in [53], is related to the fact that  $h_{\mu\nu}^{\text{TT}}$  and  $s$  are non-local functions of the original metric perturbation  $h_{\mu\nu}$ . In particular, inverting Eq. (3.2) one finds that

$$s = \left( \eta^{\mu\nu} - \frac{1}{\square} \partial^\mu \partial^\nu \right) h_{\mu\nu} = P^{\mu\nu} h_{\mu\nu}. \tag{3.7}$$

The fact that  $s$ , as a function of  $h_{\mu\nu}$ , is non-local in time means that the initial data assigned on  $h_{\mu\nu}$  on a given time slice are not sufficient to evolve  $s$ , so a naive counting of degrees of freedom goes wrong. A simple but instructive example of what exactly goes wrong is provided by a scalar field  $\phi$  that satisfies a Poisson equation  $\nabla^2 \phi = \rho$ . If we define a new field  $\tilde{\phi}$  from  $\tilde{\phi} = \square^{-1} \phi$ , the original Poisson equation can be rewritten as

$$\square \tilde{\phi} = \nabla^{-2} \rho \equiv \tilde{\rho}, \tag{3.8}$$

so now  $\tilde{\phi}$  looks like a propagating degree of freedom. However, for  $\rho = 0$  our original equation  $\nabla^2 \phi = \rho$  only has the solution  $\phi = 0$ . If we want to rewrite it in terms of  $\tilde{\phi}$  without introducing spurious degrees of freedom we must therefore supplement Eq. (3.8) with the condition that, when  $\rho = 0$ ,  $\tilde{\phi} = 0$ . In other words, the homogeneous plane wave solutions of Eq. (3.8),

$$\tilde{\phi}_{\text{hom}}(x) = \int \frac{d^3 \mathbf{k}}{(2\pi)^3} [a_{\mathbf{k}} e^{i\mathbf{k}x} + a_{\mathbf{k}}^* e^{-i\mathbf{k}x}] \tag{3.9}$$

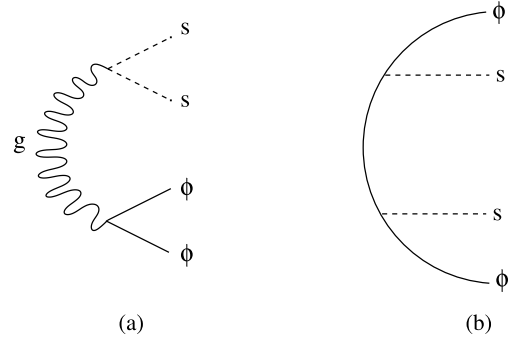
is fixed uniquely by the original equation, and  $a_{\mathbf{k}}, a_{\mathbf{k}}^*$  cannot be considered as free parameters that, upon quantization, give rise to the creation and annihilation operators of the quantum theory.

Exactly the same situation takes place in GR, for the field  $s$  and for the extra components in  $h_{\mu\nu}^{\text{TT}}$ . For instance, writing  $s$  in terms of the variables entering the (3 + 1) decomposition (and specializing to  $d = 3$ ) one finds

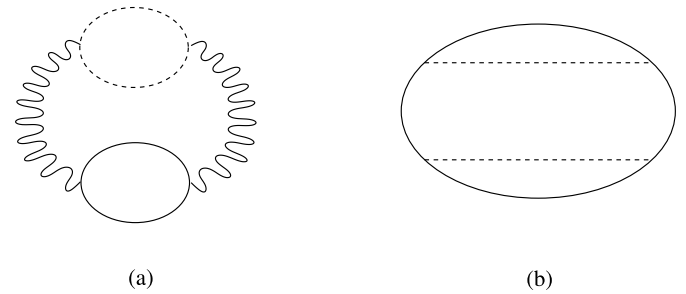
$$s = 6\Phi - 2\square^{-1} \nabla^2 (\Phi + \Psi), \tag{3.10}$$

where  $\Phi$  and  $\Psi$  are the scalar Bardeen’s variable defined in flat space (see [53]). Since  $\Phi$  and  $\Psi$  are non-radiative and satisfy Poisson equations,  $s$  is non-radiative too, and it is just the  $\square^{-1}$  factor in Eq. (3.10) that (much as the  $\square^{-1}$  in the definition of  $\tilde{\phi}$ ), potentially introduces a fake propagating degree of freedom. In order to eliminate such a spurious degree of freedom, we must supplement Eq. (3.6) with the condition that  $s = 0$  when  $T = 0$ , i.e. we must discard again the homogeneous solution of Eq. (3.6) (and similarly for Eq. (3.5)). This implies that, at the quantum level, there are no creation and annihilation operators associated to  $s$ . Therefore  $s$  cannot appear on the external legs of a Feynman diagram, and there is no Feynman propagator associated to it, so it cannot circulate in the loops.

One might also observe that the contribution to the propagator of  $s$  is canceled by an equal and opposite contribution due to the helicity-0 component of  $h_{\mu\nu}^{\text{TT}}$ , see Appendix A1 of [38]. However this only shows that, in the classical matter–matter interaction



**Fig. 2.** Examples of vacuum decay graphs that would be induced by  $s$  if we were allowed to put it on the external legs. The wavy lines denotes gravitons, the solid line a generic matter field  $\phi$  (or a graviton itself) and the dashed line the would-be ghost field  $s$ .



**Fig. 3.** The vacuum-to-vacuum diagrams corresponding to the processes shown in Fig. 2.

described by tree level diagrams such as that in Fig. 1, these contributions cancel and we remain, as expected, with the contribution from the exchange of the helicity  $\pm 2$ . This cancellation has nothing to do with the vacuum stability of GR. Consider in fact the graphs shown in Fig. 2. If  $s$  were a dynamical ghost field that can be put on the external line, these graphs would describe a vacuum decay process. Such a process is kinematically allowed because the ghost  $s$  carries a negative energy that compensates the positive energies of the other final particles. There is no corresponding vacuum decay graph in which we replace  $s$  by the helicity-0 component of  $h_{\mu\nu}^{\text{TT}}$ , since the latter is not a ghost, and the process is no longer kinematically allowed. In any case, these processes have different final states, so the positive probability for, e.g., the decay  $\text{vac} \rightarrow ss\phi\phi$  shown in Fig. 2 (where  $\phi$  is any normal matter field, or a graviton) cannot be canceled by anything.

It is interesting, and somewhat subtle, to understand the same point in terms of the imaginary part of vacuum-to-vacuum diagrams. The vacuum-to-vacuum diagrams corresponding to the processes of Fig. 2 are shown in Fig. 3. Here, whenever we have a dashed line corresponding to  $s$ , we indeed have a corresponding graph where this line is replaced by the propagation of the helicity-0 component of  $h_{\mu\nu}^{\text{TT}}$ , and one might believe that these graphs cancel. In fact this is not true, due to a subtlety in the  $i\epsilon$  prescriptions of the propagators. For a normal particle the usual scalar propagator is  $-i/(k^2 + m^2 - i\epsilon)$  (with our  $(-, +, +, +)$  signature). For a propagating ghost the correct prescription is instead  $i/(k^2 - m^2 + i\epsilon)$ . As discussed in [54], this  $+i\epsilon$  choice propagates negative energies forward in time but preserves the unitarity of the theory and the optical theorem. With a  $-i\epsilon$  choice, in contrast, ghosts carry positive energy but negative norm, and the probabilistic interpretation of QFT is lost. This latter choice is therefore unacceptable. In our case  $m = 0$  and the sum of the contributions to each internal line of the “healthy” helicity-0 component of  $h_{\mu\nu}^{\text{TT}}$  and the ghost  $s$  is

$$-\frac{i}{k^2 - i\epsilon} + \frac{i}{k^2 + i\epsilon}. \quad (3.11)$$

We see that, because of the different  $i\epsilon$  prescriptions, these two terms do not cancel. Indeed, the ghost contribution to the diagrams of Fig. 3 generates an imaginary part that corresponds to the modulus square of the corresponding diagrams in Fig. 2, as required by unitarity. In contrast, the contribution to the diagram of Fig. 3 from the helicity-0 component of  $h_{\mu\nu}^{\text{TT}}$  has no imaginary part, again in agreement with unitarity, since the processes corresponding to Fig. 2, with  $s$  replaced by the helicity-0 component of  $h_{\mu\nu}^{\text{TT}}$ , are not kinematically allowed.

To sum up, what saves the vacuum stability in GR is not a cancellation between the contributions of the ghost  $s$  and that of the helicity-0 component of  $h_{\mu\nu}^{\text{TT}}$ . If one treats them as propagating degrees of freedom there is no such cancellation, and one reaches the (wrong) conclusion that in GR the vacuum is unstable. Rather, vacuum stability is preserved by the fact that the field  $s$ , as well as the extra components of  $h_{\mu\nu}^{\text{TT}}$ , are non-radiative. There are no destruction nor creation operators associated to them, and we are not allowed to put these fields on the external lines or in loops.

In other words, the theory defined by Eq. (3.3) is not equivalent to that defined by the quadratic Einstein–Hilbert action (3.1), because the non-local transformation between  $h_{\mu\nu}$  and  $\{h_{\mu\nu}^{\text{TT}}, s\}$  introduces spurious propagating modes. We can still describe GR using the formulation in terms of  $\{h_{\mu\nu}^{\text{TT}}, s\}$ , but in this case we must impose on Eq. (3.6) the boundary condition that  $s = 0$  when  $T = 0$  (and similarly for the extra components of  $h_{\mu\nu}^{\text{TT}}$  in Eq. (3.5)), in order to eliminate these spurious modes.

### 3.2. The apparent ghost in the non-local massive theory

It is now straightforward to make contact between linearized GR and its non-local massive deformation given by Eq. (1.3). Integrating by parts the operator  $P^{\mu\nu}$  and using Eqs. (3.3) and (3.7), the Lagrangian (1.5) can be written as

$$\begin{aligned} \mathcal{L}_2 &= \frac{1}{2} h_{\mu\nu} \mathcal{E}^{\mu\nu, \rho\sigma} h_{\rho\sigma} - \frac{d-1}{2d} m^2 (P^{\mu\nu} h_{\mu\nu})^2 \\ &= \frac{1}{2} \left[ h_{\mu\nu}^{\text{TT}} \square (h^{\mu\nu})^{\text{TT}} - \frac{d-1}{d} s (\square + m^2) s \right]. \end{aligned} \quad (3.12)$$

Thus, the non-local term in Eq. (1.5) is simply a mass term for the field  $s$ . However, in the original equation of motion (1.3) that this action is supposed to reproduce, the non-local term was defined by the retarded Green's function. Thus, in order not to introduce spurious propagating degrees of freedom, we must simply continue to impose the condition that  $s$  is a non-radiative field, just as we did in GR. In other words, now the equation of motion (3.6) is replaced by

$$(\square + m^2)s = \frac{\kappa}{2(d-1)} T, \quad (3.13)$$

and, just as in Eq. (3.9), we must refrain from interpreting the coefficients of the plane waves  $e^{\pm ikx}$  with  $k^2 = m^2$  as free parameters that, upon quantization, give rise to creation and annihilation operators. Thus, again, there are no creation and annihilation operators associated to  $s$ , which therefore cannot appear on the external lines of graphs such as those in Fig. 2, nor in the internal lines of graphs such as those in Fig. 3, and there is no vacuum decay. Observe that, since the pole of  $s$  is now massive while that of the helicity-0 mode of  $h_{\mu\nu}^{\text{TT}}$  remain massless, there is no cancellation among them in the tree graphs that describe the classical matter–matter interaction, which is therefore modified at cosmological distances, compared to GR. This is just as we want, since our aim is to modify classical gravity in the IR. In contrast, the lack

of cancellation between  $s$  and the helicity-0 mode has nothing to do with unitarity and vacuum decay. As discussed above, this cancellation does not take place even in the  $m = 0$  case. Graphs such as those in Fig. 2 could not be canceled by anything, and the reason why the vacuum decay amplitude in GR is zero is that these graphs simply do not exist, because we cannot put  $s$  on the external lines.<sup>1</sup>

It is instructive to compare the situation with the usual local theory of linearized massive gravity. With a generic local mass term, the quadratic Lagrangian reads

$$\mathcal{L}_{\text{FP}} = \frac{1}{2} h_{\mu\nu} \mathcal{E}^{\mu\nu, \rho\sigma} h_{\rho\sigma} - \frac{m^2}{2} (b_1 h_{\mu\nu} h^{\mu\nu} + b_2 h^2). \quad (3.14)$$

Using again the decomposition (3.2), now the action depends also on  $\epsilon^\mu$ , since the invariance under linearized diffeomorphisms is broken. Writing  $\epsilon_\mu = \epsilon_\mu^T + \partial_\mu \alpha$ , the scalar sector now depends both on  $s$  and  $\alpha$ , with  $h_{\mu\nu}^{\text{scalar}} = \partial_\mu \partial_\nu \alpha + (1/d)\eta_{\mu\nu} s$ . In particular, the mass term in Eq. (3.14) produces a term proportional to  $(b_1 + b_2)(\square\alpha)^2$ . For  $b_1, b_2$  generic, this higher-derivative term gives rise to a ghost, and the Fierz–Pauli tuning  $b_1 + b_2 = 0$  is designed so to get rid of it. Indeed, this longitudinal mode of the metric of the form  $\partial_\mu \partial_\nu \alpha$  is nothing but the mode that is isolated using the Stückelberg formalism, and the dRGT theory [34,35] is constructed just to ensure that its equations of motion remains of second order, even at the non-linear level. The situation is quite different from that in Eq. (3.12), where no higher-derivative term is generated, but we just added a mass term to an already non-radiative field.

### 4. Spurious degrees of freedom from auxiliary fields

An alternative way of studying the degrees of freedom of a non-local theory is to transform it into a local theory by introducing auxiliary fields, see e.g. [55,56]. As it has been recognized in various recent papers [18,24,57,58], such “localization” procedure introduces however spurious solutions, and in particular spurious ghosts. This is in fact an equivalent way of understanding that the apparent ghosts of these theories do not necessarily correspond to propagating degrees of freedom. An example that has been much studied is the non-local model originally proposed by Deser and Woodard [13], which is based on the action

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} R [1 + f(\square^{-1}R)], \quad (4.1)$$

for some function  $f$ . This can be formally rewritten in local form introducing two fields  $\xi(x)$  and  $\phi(x)$  and writing

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \{ R [1 + f(\phi)] + \xi (\square\phi - R) \}. \quad (4.2)$$

Thus,  $\xi$  is a Lagrange multiplier that enforces the equation  $\square\phi = R$ , so that formally  $\phi = \square^{-1}R$ . The kinetic term  $\xi\square\phi = -\partial_\mu \xi \partial^\mu \phi$  can be diagonalized writing  $\xi = \varphi_1 + \varphi_2$ ,  $\phi = \varphi_1 - \varphi_2$ , and then

$$\begin{aligned} S &= \frac{1}{16\pi G} \int d^4x \sqrt{-g} \{ R [1 + f(\varphi_1 - \varphi_2) - \varphi_1 - \varphi_2] \\ &\quad - \partial_\mu \varphi_1 \partial^\mu \varphi_1 + \partial_\mu \varphi_2 \partial^\mu \varphi_2 \}, \end{aligned} \quad (4.3)$$

<sup>1</sup> Observe that, because of the lack of cancellation due to the signs of the  $i\epsilon$  factors in Eq. (3.11), the argument proposed in Section 6.1 of [32] (where the contribution to the vacuum-to-vacuum amplitude coming from the massive ghost and the massless scalars in Eq. (1.7) partially canceled, modulo corrections  $\mathcal{O}(m^2/E^2)$ ), is incorrect. However, we now see that the result is even stronger. The vacuum decay amplitude in the theory with finite  $m$  is not suppressed by factors  $m^2/\Lambda^2$  (where  $\Lambda$  is the UV cutoff) but is in fact identically zero, because  $s$  cannot appear on the external lines of a Feynman diagram, neither for  $m = 0$  nor for  $m \neq 0$ .

and we see that one of the two auxiliary fields ( $\varphi_2$ , given our signature) is a ghost. However, this apparent ghost is a spurious degree of freedom, as it is immediately understood observing that the above formal manipulation hold even when the function  $f(x)$  is equal to a constant  $f_0$  [57] (or, in fact, even when  $f = 0$ ). In this case the original action (4.1) is obviously the same as GR with a rescaled Newton constant, and certainly has no ghost (and, in fact, it has no ghost also for a broad class of functions  $f(\square^{-1}R)$  [24]). Once again, the point is that Eq. (4.3) is equivalent to Eq. (4.1) only if we discard the homogeneous solution of  $\square\phi = R$ , and therefore there are no annihilation and creation operators associated to  $\phi$  (nor to  $\xi$ ). A similar example has been given, for a non-local model based on the term  $R^{\mu\nu}\square^{-1}G_{\mu\nu}$ , in [18], where it was also clearly recognized that the auxiliary ghost field that results from the localization procedure never exists as a propagating degree of freedom, and does not appear in the external lines of the Feynman graphs.

Exactly the same happens in our model. To *define* the model we must specify what  $\square^{-1}$  actually means. In general, an equation such as  $\square U = -R$  is solved by

$$U = -\square^{-1}R = U_{\text{hom}}(x) - \int d^{d+1}x' \sqrt{-g(x')} G(x; x') R(x'), \quad (4.4)$$

where  $U_{\text{hom}}(x)$  is any solution of  $\square U_{\text{hom}} = 0$  and  $G(x; x')$  is any a Green's function of the  $\square$  operator. To define our model we must specify what definition of  $\square^{-1}$  we use, i.e. we must specify the Green's function and the solution of the homogeneous equation. In our case we use the retarded Green's function, but still we must complete the definition of  $\square^{-1}$  by specifying  $U_{\text{hom}}(x)$ . A possible choice is  $U_{\text{hom}}(x) = 0$ . Then, in Eq. (1.1),

$$(\square^{-1}R)(x) \equiv \int d^{d+1}x' \sqrt{-g(x')} G_{\text{ret}}(x; x') R(x'). \quad (4.5)$$

A similar choice must be made in the non-local operators which enter in the extraction of the transverse part in Eq. (1.1). Thus, at the linearized level, with this definition of  $\square^{-1}$ , in Eq. (1.3) we have

$$P_{\text{ret}}^{\rho\sigma} h_{\rho\sigma} \equiv h(x) - \int d^{d+1}x' G_{\text{ret}}(x; x') (\partial^\rho \partial^\sigma h_{\rho\sigma})(x'), \quad (4.6)$$

and similarly

$$\begin{aligned} P_{\text{ret}}^{\mu\nu} P_{\text{ret}}^{\rho\sigma} h_{\rho\sigma} & \equiv \eta^{\mu\nu} \left[ h(x) - \int d^{d+1}x' G_{\text{ret}}(x; x') (\partial^\rho \partial^\sigma h_{\rho\sigma})(x') \right] \\ & - \partial^\mu \partial^\nu \int d^d x' G_{\text{ret}}(x; x') \\ & \times \left[ h(x') - \int d^{d+1}x'' G_{\text{ret}}(x'; x'') (\partial^\rho \partial^\sigma h_{\rho\sigma})(x'') \right]. \end{aligned} \quad (4.7)$$

Consider now what happens if we rewrite the theory in a local form, introducing  $U = -\square^{-1}R$  and  $S_{\mu\nu} = -Ug_{\mu\nu}$ . Formally, Eq. (1.1) can be written as

$$G_{\mu\nu} - \frac{d-1}{2d} m^2 S_{\mu\nu}^\top = 8\pi GT_{\mu\nu}, \quad (4.8)$$

where  $S_{\mu\nu} = S_{\mu\nu}^\top + \frac{1}{2}(\nabla_\mu S_\nu + \nabla_\nu S_\mu)$ . To make contact with Eq. (1.3) we linearize over flat space and we use Eq. (1.2). Then Eq. (4.8) can be rewritten as the coupled system

$$\mathcal{E}^{\mu\nu,\rho\sigma} h_{\rho\sigma} - \frac{d-1}{d} m^2 P_{\text{ret}}^{\mu\nu} U = -16\pi GT^{\mu\nu}, \quad (4.9)$$

$$\square U = -R. \quad (4.10)$$

The introduction of a second auxiliary field  $S \equiv -\square^{-1}U$  puts also the term  $P_{\text{ret}}^{\mu\nu}U$  into local form, by introducing a further equation  $\square S = -U$ . Such a local form of the equations can be convenient, particularly for numerical studies, because it transforms the original integro-differential equations into a set of coupled differential equations. However, exactly as in the example discussed above, it introduces spurious solutions. The choice of homogeneous solution, that in the original non-local formulation amounts to a definition of the theory, is now translated into a choice of initial conditions on the fields  $U(x)$ ,  $S(x)$ . There is one, and only one choice, that gives back our original models. For instance, if the original non-local theory is defined through Eq. (4.5), we must choose the initial conditions on  $U$  in Eq. (4.10) such that the solution of the associated homogeneous equation vanishes. In any case, whatever the choices made in the definition of  $\square^{-1}$ , the corresponding homogeneous solution of Eq. (4.10) is fixed, and does not represent a free field that we can take as an extra degree of freedom of the theory. In flat space this homogeneous solution is a superposition of plane waves of the form (3.9), and the coefficients  $a_{\mathbf{k}}$ ,  $a_{\mathbf{k}}^*$  are fixed by the definition of  $\square^{-1}$  (e.g. at the value  $a_{\mathbf{k}} = a_{\mathbf{k}}^* = 0$  if we use the definition (4.5)), and at the quantum level it makes no sense to promote them to annihilation and creation operators. There is no quantum degree of freedom associated to them.

Observe that this argument is not related to a linearization of the equations if motion, say over Minkowski or over any other background. Simply, the definition of the  $\square^{-1}$  operator in the original non-local theory is such that, in the fully nonlinear equation  $\square U = -R$ , we must discard (or anyhow fix) the homogeneous solution. Thus, the fact that  $U$  does not give rise to propagating degrees of freedom is true at the fully nonlinear level.

Comparing Eq. (4.9) with Eq. (1.3) we see that, at the linearized level,  $U = P_{\text{ret}}^{\rho\sigma} h_{\rho\sigma}$ . Therefore at the linearized level  $U$  is the same as the variable  $s$  given in Eq. (3.7). The fact that the homogeneous solutions for  $U$  does not represent a free degree of freedom means that the same holds for  $s$ . We therefore reach the same conclusion of the previous section: the homogeneous solutions for  $s$  do not describe propagating degrees of freedom, and at the quantum level there are no creation and annihilation operators associated to it.

## 5. Conclusions

Non-local modifications of GR have potentially very interesting cosmological consequences. At the conceptual level, they raise however some issues of principle which must be understood before using them confidently to compare with cosmological observations. In particular, these equations feature the *retarded* inverse of the d'Alembertian. The retarded prescription ensures causality, but at the same time the fact  $\square_{\text{ret}}^{-1}$  appears not only in the solution of such equations, but already in the equations themselves, tells us that such equations cannot be fundamental. Rather, they are effective classical equations.

Such non-local effective equations can emerge in a purely classical context. Typical examples are obtained when integrating out the short-wavelength modes to obtain an effective theory for long-wavelength modes. Another example is given by the formalism for gravitational-wave production, beyond leading order. In both cases one basically re-injects a retarded solution, obtained to lowest order, into the equation governing the next-order corrections. Another way to obtain non-local equations is by performing a quantum averaging, in particular when working with the in-in expectation values of the quantum fields, and deriving these equations from an effective action that takes into account the radiative corrections. In particular, in semiclassical quantum gravity we can write such effective non-local (but causal) equations for an effective metric  $\langle 0_{\text{in}} | \hat{g}_{\mu\nu} | 0_{\text{in}} \rangle$ .

We have seen (in agreement with various recent works, e.g. [18,24,57,58]) that, if one is not careful, it is quite easy to introduce spurious degrees of freedom in these models, which furthermore are ghost-like. Basically, this originates from the fact that the kernel of the  $\square^{-1}$  operator is non-trivial: the equation  $\square^{-1}(0) = f$  does not imply that  $f = 0$  but only that  $f$  satisfies  $\square f = 0$ . The non-local equations that we are considering only involve the retarded solutions of equations of the form  $\square f = j$ , for some source  $j$ , i.e.

$$f(x) = \int dx' G_{\text{ret}}(x; x') j(x'). \quad (5.1)$$

However, any action principle that (with some more or less formal manipulation, as discussed in Section 2) reproduces the equation  $\square f = j$  will automatically carry along the most general solution of this equation, of the form

$$f(x) = f_{\text{hom}} + \int dx' G(x; x') j(x'), \quad (5.2)$$

where  $\square f_{\text{hom}} = 0$  and  $G(x; x')$  is a generic Green's function. In order to recover the solutions that actually pertain to our initial non-local theory we must impose the appropriate boundary conditions, that amount to choosing  $G(x; x') = G_{\text{ret}}(x; x')$  and fixing once and for all the homogeneous solution. In particular, one should be careful not to use the corresponding Lagrangian at the quantum level, and one should not include the corresponding fields in the external lines or in the loops. The corresponding particles, some of which are unavoidably ghost-like, do not correspond to propagating degrees of freedom in our original problem, and the quantization of these spurious solutions does not make sense.

We have seen in particular how the above considerations apply to the model defined by Eq. (1.1). We have found that the apparent ghost signaled by the second term in Eq. (1.7) is actually a non-radiative degree of freedom, and we have also seen that in the  $m \rightarrow 0$  limit it goes smoothly into a non-radiative degree of freedom of GR. Finally, we have shown how the same conclusion emerges from the point of view of the spurious degrees of freedom induced by the localization procedure. The conclusion is that the apparent ghost of Eq. (1.7) by itself is not an indication of any problem of consistency of the theory at the quantum level, and Eq. (1.1), taken as an effective classical equation, defines a consistent classical theory that can be used for cosmological purposes.

The main message of this paper is that non-local theories featuring retarded propagators in the equations of motion can only be understood as effective classical theories derived from some more fundamental QFT through some appropriate smoothing procedure, and the existence of ghosts at the quantum level can only be addressed in such an underlying theory. Non-local terms that, interpreted in momentum space as Feynman propagators, seem to imply the presence of a ghost, cannot be interpreted as such, simply because they derive from classical terms constructed with retarded propagators. The whole issue of quantum consistency can only be addressed in the underlying fundamental theory, and this holds both for non-local terms that appear at the linearized level, as well as for possible “ghost-like” non-local terms in the full non-linear classical equations.

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## References

- [1] A. Pais, G. Uhlenbeck, On field theories with nonlocalized action, *Phys. Rev.* 79 (1950) 145–165.
- [2] D. Eliezer, R. Woodard, The problem of nonlocality in string theory, *Nucl. Phys. B* 325 (1989) 389.
- [3] J.Z. Simon, Higher derivative Lagrangians, nonlocality, problems and solutions, *Phys. Rev. D* 41 (1990) 3720.
- [4] R.P. Woodard, Avoiding dark energy with  $1/R$  modifications of gravity, *Lect. Notes Phys.* 720 (2007) 403–433, arXiv:astro-ph/0601672.
- [5] N. Barnaby, N. Kamran, Dynamics with infinitely many derivatives: the initial value problem, *J. High Energy Phys.* 0802 (2008) 008, arXiv:0709.3968.
- [6] I.B. Ilhan, A. Kovner, Some comments on ghosts and unitarity: the Pais–Uhlenbeck oscillator revisited, *Phys. Rev. D* 88 (2013) 044045, arXiv:1301.4879.
- [7] N. Arkani-Hamed, S. Dimopoulos, G. Dvali, G. Gabadadze, Nonlocal modification of gravity and the cosmological constant problem, arXiv:hep-th/0209227.
- [8] G. Dvali, S. Hofmann, J. Khoury, Degravitation of the cosmological constant and graviton width, *Phys. Rev. D* 76 (2007) 084006, arXiv:hep-th/0703027.
- [9] G. Dvali, G. Gabadadze, Gravity on a brane in infinite volume extra space, *Phys. Rev. D* 63 (2001) 065007, arXiv:hep-th/0008054.
- [10] G. Dvali, G. Gabadadze, M. Shifman, Diluting cosmological constant in infinite volume extra dimensions, *Phys. Rev. D* 67 (2003) 044020, arXiv:hep-th/0202174.
- [11] A. Barvinsky, Nonlocal action for long distance modifications of gravity theory, *Phys. Lett. B* 572 (2003) 109–116, arXiv:hep-th/0304229.
- [12] G. Dvali, Predictive power of strong coupling in theories with large distance modified gravity, *New J. Phys.* 8 (2006) 326, arXiv:hep-th/0610013.
- [13] S. Deser, R. Woodard, Nonlocal cosmology, *Phys. Rev. Lett.* 99 (2007) 111301, arXiv:0706.2151.
- [14] T. Koivisto, Dynamics of nonlocal cosmology, *Phys. Rev. D* 77 (2008) 123513, arXiv:0803.3399.
- [15] T. Koivisto, Newtonian limit of nonlocal cosmology, *Phys. Rev. D* 78 (2008) 123505, arXiv:0807.3778.
- [16] S. Capozziello, E. Elizalde, S. Nojiri, S.D. Odintsov, Accelerating cosmologies from non-local higher-derivative gravity, *Phys. Lett. B* 671 (2009) 193–198, arXiv:0809.1535.
- [17] A. Barvinsky, Dark energy and dark matter from nonlocal ghost-free gravity theory, *Phys. Lett. B* 710 (2012) 12–16, arXiv:1107.1463.
- [18] A.O. Barvinsky, Serendipitous discoveries in nonlocal gravity theory, *Phys. Rev. D* 85 (2012) 104018, arXiv:1112.4340.
- [19] E. Elizalde, E. Pozdeeva, S.Y. Vernov, De Sitter universe in non-local gravity, *Phys. Rev. D* 85 (2012) 044002, arXiv:1110.5806.
- [20] Y. Zhang, M. Sasaki, Screening of cosmological constant in non-local cosmology, *Int. J. Mod. Phys. D* 21 (2012) 1250006, arXiv:1108.2112.
- [21] E. Elizalde, E. Pozdeeva, S.Y. Vernov, Reconstruction procedure in nonlocal models, *Class. Quantum Gravity* 30 (2013) 035002, arXiv:1209.5957.
- [22] S. Park, S. Dodelson, Structure formation in a nonlocally modified gravity model, *Phys. Rev. D* 87 (2013) 024003, arXiv:1209.0836.
- [23] K. Bamba, S. Nojiri, S.D. Odintsov, M. Sasaki, Screening of cosmological constant for De Sitter Universe in non-local gravity, phantom-divide crossing and finite-time future singularities, *Gen. Relativ. Gravit.* 44 (2012) 1321–1356, arXiv:1104.2692.
- [24] S. Deser, R. Woodard, Observational viability and stability of nonlocal cosmology, *J. Cosmol. Astropart. Phys.* 1311 (2013) 036, arXiv:1307.6639.
- [25] P.G. Ferreira, A.L. Maroto, A few cosmological implications of tensor nonlocalities, arXiv:1310.1238.
- [26] S. Dodelson, S. Park, Nonlocal gravity and structure in the universe, arXiv:1310.4329.
- [27] H. Hamber, R.M. Williams, Nonlocal effective gravitational field equations and the running of Newton's  $G$ , *Phys. Rev. D* 72 (2005) 044026, arXiv:hep-th/0507017.
- [28] J. Khoury, Fading gravity and self-inflation, *Phys. Rev. D* 76 (2007) 123513, arXiv:hep-th/0612052.
- [29] T. Biswas, T. Koivisto, A. Mazumdar, Towards a resolution of the cosmological singularity in non-local higher derivative theories of gravity, *J. Cosmol. Astropart. Phys.* 1011 (2010) 008, arXiv:1005.0590.
- [30] L. Modesto, Super-renormalizable quantum gravity, *Phys. Rev. D* 86 (2012) 044005, arXiv:1107.2403.
- [31] F. Briscese, A. Marciano, L. Modesto, E.N. Saridakis, Inflation in (super-)renormalizable gravity, *Phys. Rev. D* 87 (2013) 083507, arXiv:1212.3611.
- [32] M. Jaccard, M. Maggiore, E. Mitsou, A non-local theory of massive gravity, *Phys. Rev. D* 88 (2013) 044033, arXiv:1305.3034.
- [33] M. Maggiore, Phantom dark energy from nonlocal infrared modifications of general relativity, *Phys. Rev. D* 89 (2014) 043008, arXiv:1307.3898.
- [34] C. de Rham, G. Gabadadze, Generalization of the Fierz–Pauli action, *Phys. Rev. D* 82 (2010) 044020, arXiv:1007.0443.
- [35] C. de Rham, G. Gabadadze, A.J. Tolley, Resummation of massive gravity, *Phys. Rev. Lett.* 106 (2011) 231101, arXiv:1011.1232.

- [36] C. de Rham, G. Gabadadze, A.J. Tolley, Ghost free massive gravity in the Stückelberg language, *Phys. Lett. B* 711 (2012) 190–195, arXiv:1107.3820.
- [37] S. Hassan, R.A. Rosen, Resolving the ghost problem in non-linear massive gravity, *Phys. Rev. Lett.* 108 (2012) 041101, arXiv:1106.3344.
- [38] S. Hassan, R.A. Rosen, A. Schmidt-May, Ghost-free massive gravity with a general reference metric, *J. High Energy Phys.* 1202 (2012) 026, arXiv:1109.3230.
- [39] S. Hassan, R.A. Rosen, Confirmation of the secondary constraint and absence of ghost in massive gravity and bimetric gravity, *J. High Energy Phys.* 1204 (2012) 123, arXiv:1111.2070.
- [40] K. Hinterbichler, Theoretical aspects of massive gravity, *Rev. Mod. Phys.* 84 (2012) 671–710, arXiv:1105.3735.
- [41] S. Deser, Covariant decomposition and the gravitational Cauchy problem, *Ann. Inst. Henri Poincaré* 7 (1967) 149.
- [42] J.J. York, Covariant decompositions of symmetric tensors in the theory of gravitation, *Ann. Inst. Henri Poincaré* 21 (1974) 319.
- [43] S. Foffa, M. Maggiore, E. Mitsou, Cosmological dynamics and dark energy from non-local infrared modifications of gravity, arXiv:1311.3435.
- [44] R. Caldwell, A phantom menace?, *Phys. Lett. B* 545 (2002) 23–29, arXiv:astro-ph/9908168.
- [45] S.M. Carroll, M. Hoffman, M. Trodden, Can the dark energy equation-of-state parameter  $w$  be less than  $-1$ ?, *Phys. Rev. D* 68 (2003) 023509, arXiv:astro-ph/0301273.
- [46] M. Soussa, R.P. Woodard, A nonlocal metric formulation of MOND, *Class. Quantum Gravity* 20 (2003) 2737–2752, arXiv:astro-ph/0302030.
- [47] R. Jordan, Effective field equations for expectation values, *Phys. Rev. D* 33 (1986) 444–454.
- [48] E. Calzetta, B. Hu, Closed time path functional formalism in curved space-time: application to cosmological back reaction problems, *Phys. Rev. D* 35 (1987) 495.
- [49] S.M. Carroll, S. Leichenauer, J. Pollack, A consistent effective theory of long-wavelength cosmological perturbations, arXiv:1310.2920.
- [50] L. Blanchet, Gravitational radiation from post-Newtonian sources and inspiralling compact binaries, *Living Rev. Relativ.* 9 (2006) 4.
- [51] M. Maggiore, *Gravitational Waves*, vol. 1. Theory and Experiments, Oxford University Press, 2007, 574 pp.
- [52] N. Tsamis, R. Woodard, Nonperturbative models for the quantum gravitational back reaction on inflation, *Ann. Phys.* 267 (1998) 145–192, arXiv:hep-ph/9712331.
- [53] M. Jaccard, M. Maggiore, E. Mitsou, Bardeen variables and hidden gauge symmetries in linearized massive gravity, *Phys. Rev. D* 87 (2013) 044017, arXiv:1211.1562.
- [54] J.M. Cline, S. Jeon, G.D. Moore, The phantom menaced: constraints on low-energy effective ghosts, *Phys. Rev. D* 70 (2004) 043543, arXiv:hep-ph/0311312.
- [55] S. Nojiri, S.D. Odintsov, Modified non-local-F(R) gravity as the key for the inflation and dark energy, *Phys. Lett. B* 659 (2008) 821–826, arXiv:0708.0924.
- [56] S. Jhingan, S. Nojiri, S. Odintsov, M. Sami, I. Thongkool, S. Zerbini, Phantom and non-phantom dark energy: the cosmological relevance of non-locally corrected gravity, *Phys. Lett. B* 663 (2008) 424–428, arXiv:0803.2613.
- [57] N. Koshelev, Comments on scalar-tensor representation of nonlocally corrected gravity, *Gravit. Cosmol.* 15 (2009) 220–223, arXiv:0809.4927.
- [58] T.S. Koivisto, Cosmology of modified (but second order) gravity, *AIP Conf. Proc.* 1206 (2010) 79–96, arXiv:0910.4097.