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Constraining a fourth generation of quarks: Non-perturbative Higgs boson mass bounds

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ABSTRACT

We present a non-perturbative determination of the upper and lower Higgs boson mass bounds with a heavy fourth generation of quarks from numerical lattice computations in a chirally symmetric Higgs–Yukawa model. We find that the upper bound only moderately rises with the quark mass while the lower bound increases significantly, providing additional constraints on the existence of a straight-forward fourth quark generation. We examine the stability of the lower bound under the addition of a higher dimensional operator to the scalar field potential using perturbation theory, demonstrating that it is not significantly altered for small values of the coupling of this operator. For a Higgs boson mass of ~ 125 GeV we find that the maximum value of the fourth generation quark mass is ~ 300 GeV, which is already in conflict with bounds from direct searches.

1. Introduction

A heavy fourth generation of quarks is an attractive and simple extension (denoted SM4) of the three-generation Standard Model (SM3) [1,2]. However, heavy fermion effects are expected to significantly contribute to the properties of the Higgs boson, leading to measurable deviations in Higgs production cross sections and branching ratios. The recent discovery of a scalar boson seemingly consistent with the Standard Model expectation therefore casts serious doubt on straight-forward fourth generation scenarios [3].

Another property of the Higgs boson which is sensitive to a possible fourth generation is its mass. Invoking arguments from perturbation theory, the Higgs boson mass is bounded from above by the triviality bound, which reflects the Gaussian nature of the UV fixed point, and from below by the vacuum instability bound, which ensures that the theory has a stable vacuum state.

In perturbation theory, the lower bound can be obtained by examining the effective potential and demanding that it remains bounded from below. As the fermion fields contribute negatively to the effective potential, they have a destabilizing effect which leads (by demanding the stability of the theory) to a lower Higgs boson mass bound. However it is expected that the perturbative expansion breaks down for Yukawa couplings near or less than the perturbative unitarity bound [4], which allows maximal fermion masses of roughly $m_f \sim 500$–600 GeV [5,6].

Due to these considerations, study of heavy fourth generation extensions to the Standard Model necessitates non-perturbative methods. To this end, we employ lattice field theory techniques which allow us to compute lower and upper Higgs boson mass bounds as well as resonance parameters of the Higgs boson non-perturbatively. Such a strategy has already been applied in Refs. [7–10] for the non-perturbative determination of the upper and lower Higgs boson mass bounds and the Higgs boson resonance parameters in the SM3. First results for a fourth quark generation have been presented in Ref. [11]. Investigations of a bulk-phase transition at very large values of the Yukawa coupling have been initiated in Ref. [12] while first studies of the system at non-zero temperatures can be found in Ref. [13]. For a summary of recent work on this model see Ref. [14].

The lattice calculations are possible due to a lattice discretization of the fermion action which respects an exact chiral symmetry at finite lattice spacing a [15], thus ensuring that the chiral character of the Higgs-fermion couplings is respected by the lattice regulator in a conceptually clean manner. This advance has triggered a number of lattice investigations of Higgs–Yukawa like models [7–9, 16–21].

In this Letter we report on a systematic investigation of the lower and upper Higgs boson mass bounds for quark masses ranging from the Standard Model top quark mass of $m_f = m_t \approx 175$ GeV up to masses of $m_f \approx 700$ GeV, which is above the

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perturbative unitarity bound. Our calculations are performed at a fixed lattice cutoff of $\Lambda = \frac{1}{a} \approx 1.5$ TeV which ensures that the fermion and Higgs boson masses are sufficiently far from the cut-off scale. We also rely heavily on the techniques and simulation strategies of Refs. [7–10].

Our results indicate that with increasing quark mass there is a substantial upward shift of the lower Higgs boson mass bound leading to severe constraints on a straight-forward fourth quark generation in light of the recently discovered scalar boson when interpreted as the Standard Model Higgs boson. In order to examine the stability of our results, we analyze the effect of including a higher dimensional operator in the scalar field potential using perturbation theory. For small values of the coupling of this term we confirm that the lower bound is not significantly altered.

2. The model

Here we briefly review the definition of our model. For a more complete treatment, see Ref. [9]. We employ a lattice discretization of the Standard Model Higgs-fermion sector which exactly respects a chiral symmetry at finite lattice spacing. As the dynamics of the complex scalar (Higgs) doublet are expected to be dominated by its interactions with the heaviest fermions, we consider a single degenerate quark doublet only. By the same reasoning, as well as for computational simplicity, we also neglect gauge fields.

One of the defining features of the Standard Model is the chiral structure of the scalar-fermion interactions. In order to reproduce this structure in a lattice discretization it is crucial to maintain a chiral symmetry at finite lattice spacing. This has been a long-standing obstacle to the lattice regularization of Higgs–Yukawa models and was finally overcome by employing the Neuberger 'overlap' [15,22,23] discretization of the fermion action.

Following the proposition in Ref. [15] we can therefore construct a lattice Higgs–Yukawa model with a global $SU(2)_L \times U(1)_Y$ symmetry. Specifically, the fields included in our model are a scalar doublet φ and two fermions, the left-handed components of which are paired into an $SU(2)$ doublet. The lattice action can thus be written as

$$\begin{aligned}
&S = S_F + S_\Phi, \qquad S_F = \sum_{xy} \bar{\psi}_x \mathcal{M}_{xy} \psi_y, \\
&S_\Phi = -\kappa \sum_{x,\mu} \Phi_x^T [\Phi_{x+\mu} + \Phi_{x-\mu}] + \sum_x \Phi_x^T \Phi_x \\
&\qquad + \hat{\lambda} \sum_x \left(\Phi_x^T \Phi_x - 1\right)^2, \\
&\mathcal{M}_{xy} = \mathcal{D}^{(ov)}_{xy} \mathbb{1}_{2\times 2} + y\left(P_+ \phi_x^\dagger \hat{P}_+ + P_- \phi_x \hat{P}_-\right)\delta_{xy},
\end{aligned} \tag{1}$$

where ψ is a doublet of four-component spinor fields, $\mathcal{D}^{(ov)}$ is the free overlap Dirac operator with a Wilson kernel, $\Phi_x^\mu \in \mathbb{R}^4$, and $\phi_x = \Phi_x^\mu \tau_\mu$, with $\tau_\mu = (1, -i\vec{\sigma})$. The left- and right-handed projection operators $P_\pm$ and the modified projectors $\hat{P}_\pm$ are given by

$$\begin{aligned}
&P_\pm = \frac{1 \pm \gamma_5}{2}, \qquad \hat{P}_\pm = \frac{1 \pm \hat{\gamma}_5}{2}, \\
&\hat{\gamma}_5 = \gamma_5\left(\mathbb{1} - \frac{1}{\rho}\mathcal{D}^{(ov)}\right).
\end{aligned} \tag{2}$$

The action introduced above obeys an exact global $SU(2)_L \times U(1)_Y$ lattice chiral symmetry. For $\Omega_L \in SU(2)$ and $\theta \in [0, 2\pi]$ the action is invariant under the transformation

$$\begin{aligned}
&\psi \to U_Y \hat{P}_+ \psi + U_Y \Omega_L \hat{P}_- \psi, \\
&\bar{\psi} \to \bar{\psi} P_+ \Omega_L^\dagger U_Y^\dagger + \bar{\psi} P_- U_Y^\dagger, \\
&\phi \to U_Y \phi \Omega_L^\dagger, \phi^\dagger \to \Omega_L \phi^\dagger U_Y^\dagger
\end{aligned} \tag{3}$$

with $U_Y \equiv \exp(i\theta Y)$, where Y labels the representation of the global hypercharge symmetry group $U(1)_Y$. It should be noted that in the continuum limit the (global) continuum $SU(2)_L \times \mathrm{U}(1)_Y$ chiral symmetry is recovered.

This formulation enables a numerical study of the limit $\hat{\lambda} = \infty$ on the lattice by simply enforcing the constraint $\Phi_x^T \Phi_x = 1$, $\forall x$. Also, we can relate the parameters and fields appearing in Eqs. (1) to those appearing in the standard scalar complex doublet continuum Lagrangian ($\mathcal{L} = |\partial_\mu \varphi|^2 + \frac{1}{2} m_0^2 |\varphi|^2 + \lambda |\varphi|^4$) by

$$\begin{aligned}
&\varphi_x = \sqrt{2\kappa} \begin{pmatrix} \Phi_x^2 + i\Phi_x^1 \\ \Phi_x^0 - i\Phi_x^3 \end{pmatrix}, \\
&\lambda = \frac{\hat{\lambda}}{4\kappa^2}, \qquad m_0^2 = \frac{1 - 2\hat{\lambda} - 8\kappa}{\kappa}.
\end{aligned}$$

3. Computational strategy

The cutoff in our model is provided by the inverse lattice spacing ($\Lambda = 1/a$) and we set the physical value of Λ using the phenomenological Higgs field vacuum expectation value (v_R),

$$(\sqrt{2} G_F)^{-\frac{1}{2}} \sim 246\ \text{GeV} = \frac{v_R}{a} \equiv \frac{v}{\sqrt{Z_G} \cdot a}, \tag{4}$$

where Z_G denotes the Goldstone boson field renormalization constant and v the bare scalar field vacuum expectation value. We target a mass range for the degenerate fermion doublet of 175 GeV $\lesssim m_f \lesssim 700$ GeV, while fixing the cutoff at $\Lambda \approx 1.5$ TeV. Although the masses of the fermion doublet are degenerate in this work, we plan to assess the effect of a mass splitting in the near future. At $m_f = m_t = 175$ GeV the splitting $m_b - m_t$ has been taken into account and found to have a small effect on the lower Higgs boson mass bound which, moreover, can be taken into account by the effective potential evaluated in lattice perturbation theory [7].

We now briefly discuss the simulation algorithm. More details can be found in Ref. [9]. The Monte-Carlo simulations are carried out using a variant [24] of the Hybrid Monte Carlo (HMC) algorithm [25] in which the integration over the Grassman fields is done analytically at the expense of introducing the determinant of the fermion action in the updating. Therefore, our remaining degrees of freedom in the Monte Carlo integration are the scalar fields.

Due to the absence of gauge fields, the overlap operator of Eqs. (1) can be constructed exactly in momentum space. However as the Yukawa coupling is local in position space, Fast-Fourier-Transforms (FFT's) are employed to efficiently apply this term. Finally, Fourier acceleration [26] is used to propagate the low momentum modes using coarser time-steps along the HMC evolution, which effectively reduces the autocorrelation between successive scalar field configurations.

It has been demonstrated [7] that the Higgs boson mass is a monotonically increasing function of the bare quartic coupling $\hat{\lambda}$. Therefore, the lower bound for the Higgs boson mass at fixed cutoff and m_f is obtained at $\hat{\lambda} = 0$, while the upper bound is obtained at $\hat{\lambda} = \infty$ [8].

Since we work in a finite volume with no external symmetry breaking source, the naively defined vacuum expectation value is zero in an ensemble average. We therefore resort to a special technique, pioneered in Ref. [27] and employed in Refs. [28–30], of rotating a given scalar field configuration to a preferred direction. It can be shown that in infinite volume this leads to the same vacuum expectation value as obtained in the standard procedure involving the limit of vanishing external source [30].

The determination of the Goldstone boson renormalization constant Z_G, the Higgs boson mass m_H, and the fermion mass m_f

have been discussed in detail in Ref. [8] and are briefly reviewed here. The renormalization constant Z_G is computed from the slope of the inverse Goldstone boson propagator at vanishing Euclidean four-momentum transfer, in the standard on-shell scheme. Operationally, this constant is obtained from fits to the propagator at small momenta.

Due to the existence of massless Goldstone modes in the spontaneously broken phase of our theory, the Higgs boson is unstable and can decay to final states containing an even number of Goldstone bosons. We employ two definitions of the Higgs boson mass which both ignore this finite decay width. The Higgs boson propagator mass m_H^P is derived from fits to the momentum space Higgs propagator defined as

$$\tilde{G}_H(p) = \langle \tilde{h}_p \tilde{h}_{-p} \rangle, \qquad \tilde{h}_p = \frac{1}{\sqrt{L_s^3 \cdot L_t}} \sum_x e^{-ipx} h_x$$

while the correlator mass m_H^C is derived from fits to the Higgs temporal correlation function. Although finite decay width corrections to these formulae are quite different, the mass extracted from both of these procedures typically differs by less than 10%, lending credence to our approximation of a stable Higgs boson. Furthermore, a rigorous study of the Higgs boson decay width at non-vanishing external source has been performed at $m_f = m_t \sim 175$ GeV in Ref. [10] which obtained a narrow decay width for all values of the bare quartic coupling, further supporting the validity of the stable Higgs boson approximation. For this work we quote m_H^P as the central value as it is typically the lowest estimate of m_H.

Finally, we compute the quark mass from the exponential decay of the temporal correlation function $C_f(t)$ at large Euclidean time separations t, defined as

$$C_f(t) = \frac{1}{L_s^6} \sum_{\vec{x},\vec{y}} \operatorname{Re}\operatorname{Tr}(\psi_{L,(0,\vec{x})} \cdot \bar{\psi}_{R,(t,\vec{y})}), \tag{5}$$

where the left- and right-handed spinors are defined using the projection operators in Eqs. (2).

4. Numerical results

As mentioned above, massless Goldstone excitations are present in the spontaneously broken phase of our model. Since we work at zero external symmetry breaking source, finite size effects are not exponentially suppressed but rather algebraic in nature and proportional to inverse even powers of the linear extent of the lattice, i.e. $\mathcal{O}(1/L_s^2)$ at leading order [28–30]. These finite size effects can be quite substantial and an infinite volume extrapolation of our results for the renormalized vacuum expectation value and all masses is required.

Finite volume data for the Higgs boson propagator mass (m_H^P) are shown in Fig. 1 as an example of our infinite volume extrapolations. The lattice data are plotted versus $1/L_s^2$ and extrapolated to the infinite volume limit by means of a linear fit ansatz according to the aforementioned leading order behaviour. Due to the observed curvature arising from the non-leading finite volume corrections, only those volumes with $L_s \geqslant 16$ have been included. One also finds that the infinite volume extrapolation can be reliably performed at ranges of lattice volumes from $12^3 \times 32$ to $24^3 \times 32$ if subleading corrections are considered.

The results of the Higgs boson masses for the lower and upper Higgs boson mass bounds as a function of the quark mass at a fixed cutoff of $\Lambda = 1.5$ TeV are finally presented in Fig. 2. All data for the upper bound have been extrapolated to infinite volume, as have the lower Higgs boson mass bounds at $m_f \approx 160, 192, 420$ GeV Only lower Higgs boson mass bounds at

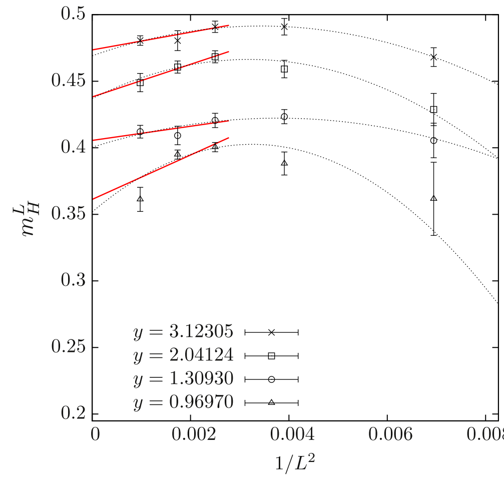

Fig. 1. Infinite volume extrapolation of the Higgs boson mass extracted from fits to the momentum space Higgs propagator (m_H^P). The data are for the Higgs boson mass relevant for the upper bound ($\hat{\lambda} = \infty$) and a range of bare Yukawa couplings corresponding to fermion masses from the physical top quark mass to ~ 700 GeV. The infinite volume extrapolation is performed by fitting the data to a linear function in $1/L_s^2$ taking only data with $L_s \geqslant 16$ into account. Such fits are shown as solid lines, while quadratic fits which extend over the entire range of volumes are shown as dotted lines.

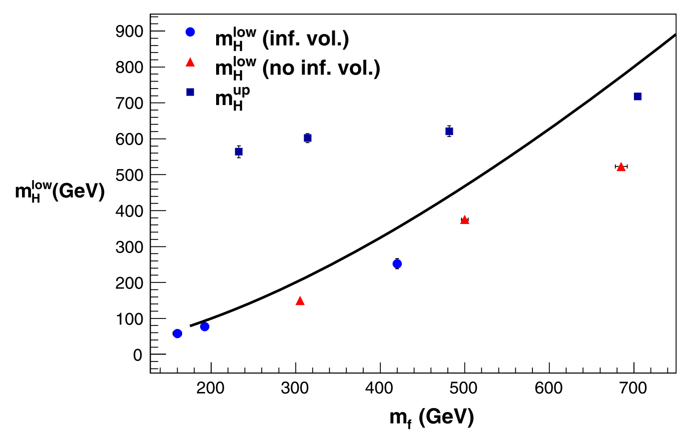

Fig. 2. The lower and upper Higgs boson mass bounds as a function of the fermion mass m_f for a fixed cutoff of $\Lambda = 1.5$ TeV. The solid line represents a perturbative lattice effective potential calculation for the lower Higgs boson mass bound.

$m_f \approx 305, 500, 685$ GeV are taken on our presently largest lattice of size $24^3 \times 32$. However, the finite size corrections of these points will only provide a small change and the behaviour of the lower Higgs boson mass bound as a function of the fermion mass will not be significantly altered.

We find that the upper Higgs boson mass bound is only moderately shifted by about 20% when the fermion mass is increased from 175 GeV to 700 GeV. On the contrary, the lower Higgs boson mass bound increases drastically with increasing quark mass. Such a significant shift of the lower bound has already been observed in [7,8] for a quark mass of $m_f \approx 700$ GeV. We also show in Fig. 2 a lattice perturbative computation of the lower Higgs boson mass bound using the effective potential. The result of this calculation, which is discussed in the next section, is represented by the solid line in Fig. 2. Even up to fermion masses of $m_f \approx 700$ GeV the lattice perturbative calculation qualitatively describes the simulation data rather well. This observation allows us to test the stability of the lower bound against the addition of a higher dimensional operator within the framework of the perturbative lattice effective potential. Moreover, direct Monte Carlo simulations with such a term were found to be well described by perturbation theory [9] within a small range of couplings of this higher dimensional operator.

5. Effective potential

In addition to non-perturbative numerical data, it is instructive to calculate the lower Higgs boson mass bound perturbatively using the effective potential. To this end we follow Ref. [9] and calculate the effective potential to 1-loop in the large-N_f expansion.

It should be noted that these calculations are performed with the same lattice regularization used in the simulations maintaining finite spatial and temporal extents. Thus, the loop corrections are calculated numerically, by explicitly performing sums over the lattice momenta. An infinite volume extrapolation of these perturbative results is also performed.

The vacuum expectation value and lower Higgs boson mass bound are obtained from this effective potential in the usual way, namely

$$\frac{d}{d\bar{\phi}}V(\bar{\phi})|_{\bar{\phi}=v} = 0,$$
$$\frac{d^2}{d\bar{\phi}^2}V(\bar{\phi})|_{\bar{\phi}=v} = m_H^2, \tag{6}$$

where $\bar{\phi}$ denotes the average value of the scalar field. The results of such a determination are shown in Fig. 2. Although quantitatively different, the numerical data agree with the qualitative behaviour of the perturbative result even for rather large fermion masses.

Based on this agreement, we estimate the effect of an additional higher dimensional operator in the scalar field potential using the perturbative expansion discussed above. To this end we add to Eqs. (1) the term $\lambda_6\phi^6$. It has been demonstrated [9] that perturbation theory successfully describes non-perturbative numerical results for the range $\lambda_6 = [0, 0.01]$. Therefore, at this preliminary stage we examine couplings in this range only. Furthermore, in the presence of such a term we must modify the stability criterion of the effective potential.

Due to the triviality of the theory, the cutoff Λ cannot be completely removed while maintaining non-zero values of the renormalized quartic and Yukawa couplings. However, the existence of a scaling regime suggests that predictions of the model are affected only mildly by small cutoff effects provided that the cutoff is larger than the relevant scales in the theory, namely v_R, m_f, m_H. It has been determined in the pure ϕ^4 theory [31] that a suitable definition of the scaling regime is the situation where $m/\Lambda < 0.5$, where $m = v_R, m_H$. We adopt here this observation also for m_f, keeping $m_f/\Lambda < 0.5$.

It is in this scaling regime only that results which are universal up to small cutoff effects can be obtained from our model. Therefore, the vacuum stability criterion should ensure that a stable vacuum exists throughout the scaling regime. Indeed, the running of the renormalized quartic and Yukawa couplings (and thus vacuum stability considerations) have been demonstrated to be severely regularization dependent outside of the scaling regime defined above [21]. Therefore, we choose as our stability criterion

$$\frac{d^2}{d\bar{\phi}^2}V(\bar{\phi}) > 0, \quad \bar{\phi} < 0.5. \tag{7}$$

In the $\lambda_6 = 0$ case, this results in the well-known stability criterion $\hat{\lambda} \geqslant 0$. We have examined the effect of a finite λ_6 in the range $\lambda_6 = [0.0, 0.01]$ which results in a less than 1% difference in the lower bound. However for larger values of $\lambda_6 \sim 0.1$, deviations as large as 15% have been observed in the lower Higgs boson mass bound. As we are unsure of the applicability of perturbation theory in this region, further non-perturbative numerical simulations must be performed to properly assess the effect of this higher dimensional operator for larger values of λ_6.

6. Outlook and conclusions

In this Letter we have performed a non-perturbative lattice investigation of the lower and upper Higgs boson mass bounds in an extension of the Standard Model with a heavy fourth quark generation. Since the heavy quark masses lead to large values of the Yukawa coupling, such a non-perturbative computation is necessary in order to have reliable results for the Higgs boson mass bounds at large m_f. We include only the dominant interactions, which are expected to be the Higgs–Yukawa interactions of the heavy fermions. In particular, we neglect all gauge fields.

Our chirally invariant lattice regularization of the Higgs–Yukawa sector of the Standard Model (as proposed in Ref. [15]) obeys a global $SU(2)_\mathrm{L} \times U(1)_\mathrm{Y}$ symmetry at finite lattice spacing, providing a formulation that preserves the chiral nature of the Higgs-fermion couplings. In this setup, for a fixed cutoff of $\Lambda = 1.5$ TeV, we have studied a range of quark masses 175 GeV $\lesssim m_f \lesssim$ 700 GeV. We find that the upper Higgs boson mass bound is only moderately increased by about 20% for larger quark masses. The lower Higgs boson mass bound however, changes quite substantially as is summarized in Fig. 2 and assumes for e.g. $m_f \approx 500$ GeV a value of $m_H^{\mathrm{low}} \approx 500$ GeV. Assuming $m_H \sim 125$ GeV, this puts severe constraints on the existence of a fourth generation of quarks given the current status of direct searches [32,33], which typically quote constraints $m_{t'}, m_{b'} \gtrsim$ 400–500 GeV which are fully consistent with the analysis of Ref. [3].

The cutoff dependence of this lower bound has been investigated extensively in previous work both at $m_f \sim 175$ GeV and $m_f \sim 700$ GeV in Refs. [7,11], and found to be weakly dependent on the value of the cutoff. In particular, at $m_f \sim 700$ GeV, the change in the lower bound was not statistically significant as the cutoff was varied over the range 1.5–3.5 TeV. This is consistent with the weak cutoff dependence of the lower Higgs boson mass bound found in a lattice perturbative effective potential calculation [7,11,14].

Confronting the non-perturbatively computed lower Higgs boson mass bounds with a lattice perturbative calculation of the effective potential, we found that the effective potential describes the simulation data rather well, even for $m_f \approx 700$ GeV. Based on this fact, as well as previous numerical work, we also used the perturbative effective potential to test the stability of the lower Higgs boson mass bound against addition of a higher dimensional operator for a small range of couplings. We found that the lower Higgs boson mass bound is quite insensitive against such an addition, although more non-perturbative numerical simulations are required to examine the effect of the higher dimension operator for larger couplings. Nonetheless, this preliminary work provides some confidence that the results for the lower bound are robust. Furthermore, although we work with a mass degenerate quark doublet in this work, the dependence of the lower bound on the mass splitting was found to be small at $m_f = 175$ GeV, but will investigated at larger quark masses in the near future.

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