Boundary-layer transition prediction using a simplified correlation-based model

Xia Chenchao, Chen Weifang *

School of Aeronautics and Astronautics, Zhejiang University, Hangzhou 310027, China

Received 27 January 2015; revised 19 May 2015; accepted 21 June 2015
Available online 23 December 2015

KEYWORDS
Boundary-layer transition; Computational fluid dynamics; Correlation; Transition model; Turbulence model

Abstract This paper describes a simplified transition model based on the recently developed correlation-based \( \gamma - R_{e_k} \) transition model. The transport equation of transition momentum thickness Reynolds number is eliminated for simplicity, and new transition length function and critical Reynolds number correlation are proposed. The new model is implemented into an in-house computational fluid dynamics (CFD) code and validated for low and high-speed flow cases, including the zero pressure flat plate, airfoils, hypersonic flat plate and double wedge. Comparisons between the simulation results and experimental data show that the boundary-layer transition phenomena can be reasonably illustrated by the new model, which gives rise to significant improvements over the fully laminar and fully turbulent results. Moreover, the new model has comparable features of accuracy and applicability when compared with the original \( \gamma - R_{e_k} \) model. In the meantime, the newly proposed model takes only one transport equation of intermittency factor and requires fewer correlations, which simplifies the original model greatly. Further studies, especially on separation-induced transition flows, are required for the improvement of the new model.

1. Introduction

Boundary-layer transitions arise in the majority of applications of aeronautics and astronautics, such as the airfoil of a subsonic passenger plane, the turbofan engine and the hypersonic reentry vehicle. Usually, skin friction and heat transfer rate increase significantly when transition occurs, which could lead to the increase of drag or the aggravation of aerodynamic heating. These troublesome uncertainties on aerodynamic or aerothermodynamic characteristics of aircraft necessitate the accurate prediction of boundary-layer transition, not only from the view point of economy, but also safety. Complex though the physics of transition is, study of boundary-layer transition has been the topic of great importance and urgency over the past several decades, and substantial numbers of satisfactory findings have been achieved.

As an effective means of exploring the mechanism of fluid, experiment plays an important role in revealing instability phenomena of boundary layer and finding new flow scenarios.1 Typical works can be traced from Lee and Wu, who reviewed plenty of experimental results for wall-bounded flows.2 Besides
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In this study, new correlations for $F_{\text{simpl}}$ and $Re_t$ are introduced to control the length of transition region and transition onset respectively, and the single intermittency factor transport equation is coupled with the two-equation shear stress transportation (SST) turbulence model, resulting in a three-equation transition model. The new model is implemented into an in-house CFD solver, followed by several simulations and analyses for the evaluation of its performance. Finally, conclusions and recommendations are made at the end of this paper.

2. Turbulence and transition modeling

2.1. Two-equation SST turbulence model

The two-equation SST turbulence model, originally developed by Menter,\cite{22} is the combination of $k-\omega$ and $k-\epsilon$ turbulence models. It can be switched from $k-\omega$ model near the wall to $k-\epsilon$ model away from the wall through well-designed blending functions, which will be defined in the following text. The SST model has been applied to large quantities of flows and shows excellent performances of accuracy and robustness. Overall, it has been regarded as one of the most successful turbulence models, not only in the area of aeronautics, but also in the industrial community. Hence, all the fully turbulent simulations in this work are performed by SST model. The transport equations for turbulent kinetic energy and specific dissipation rate are as follows:

\[
\frac{\partial (\rho k)}{\partial t} + \frac{\partial (\rho u_i k)}{\partial x_i} = \frac{\partial}{\partial x_j} \left[ \nu_t \frac{\partial k}{\partial x_j} \right] + P_k - \beta' \rho \omega k - \frac{\rho \sigma_{\omega,k}}{\omega} \frac{\partial \omega}{\partial x_i} \frac{\partial k}{\partial x_j} + \frac{1}{\omega} \frac{\partial (\rho \sigma_{\omega,k} \omega)}{\partial x_i} \frac{\partial k}{\partial x_j}
\]

\[
\frac{\partial (\rho \omega)}{\partial t} + \frac{\partial (\rho \omega u_i)}{\partial x_i} = \frac{\partial}{\partial x_j} \left[ \nu_t \frac{\partial \omega}{\partial x_j} \right] + \frac{\rho \sigma_{\omega,k}}{\omega} \frac{\partial k}{\partial x_i} + \frac{1}{\omega} \frac{\partial (\rho \sigma_{\omega,k} \omega)}{\partial x_i} \frac{\partial \omega}{\partial x_j} + \frac{\omega}{\rho} \left( \frac{\partial}{\partial x_j} \left[ \sigma_{\omega,k} \frac{\partial \omega}{\partial x_j} \right] + \frac{\partial}{\partial x_i} \left[ \sigma_{\omega,k} \frac{\partial \omega}{\partial x_i} \right] \right)
\]

where $\rho$ is density, $t$ is time, $k$ is turbulent kinetic energy, $u_i$ is velocity component, $x_j$ is coordinate component, $\omega$ is specific dissipation rate, $\mu$ and $\nu_t$ are laminar and turbulent eddy viscosity respectively, $v_t$ is kinematic eddy viscosity, $\beta'$, $\beta$, $\zeta$, $\sigma_{\omega,k}$, and $\sigma_{\omega,l}$ are model constants. $F_1$ is the blending function and defined as:

\[
F_1 = \max \left( \arg(\gamma') \right)
\]

\[
\arg_{\iota} = \min \left( \frac{\sqrt{\frac{2}{0.09\alpha y^{\prime}}} \left( \frac{500\mu}{\nu^{\prime}y^{\prime}} \right) \frac{4\rho \sigma_{\omega,k}}{\sigma_{\omega,l} \mathrm{CD}_{\text{lim}}}}{\iota} \right)
\]

\[
\mathrm{CD}_{\text{lim}} = \max \left( \frac{2\rho \sigma_{\omega,k} \frac{1}{\omega} \frac{\partial k}{\partial x_i} \frac{\partial \omega}{\partial x_j}}{\mathrm{CD}_{\omega,l} \left( 10^{-10} \right)} \right)
\]

The variable $y$ is the distance from the cell to the nearest wall. Instead of computing the production term exactly, the vorticity magnitude is adopted for approximation:

\[
\overline{P}_k = \mu \frac{\Omega^2}{3} \left( \frac{\partial}{\partial x_i} \frac{\partial k}{\partial x_j} \right) \quad P_k = \min \left( \overline{P}_k, 10 \beta' \rho \omega k \right)
\]

where $\Omega$ is the magnitude of vorticity and defined as:

\[
\Omega = \sqrt{2W_y W_y} \quad W_y = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i} \right)
\]

The kinematic eddy viscosity is calculated by

\[
v_t = \frac{a_x k}{\max(a_x, \alpha_{SF})}
\]
where the constant $\alpha_1 = 0.31$, $S$ is the invariant of strain rate and defined as:

$$S = \sqrt{2S_{ij}S_{ij}}, \quad S_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$

The blending function $F_2$ is defined as

$$F_2 = \tanh(\text{arg}_2)$$

$$\text{arg}_2 = \max\left( \frac{\sqrt{\varepsilon}}{0.15\tau \cdot 0.2}, \frac{\eta}{\tau} \right)$$

Coefficients of the model are calculated from the formula

$$\phi = F_1\phi_1 + (1 - F_1)\phi_2$$

The subscript “1” is for the $k-\varepsilon$ model and subscript “2” is for the $k-\omega$ model. Two sets of coefficients used at present are: $\sigma_1 = 0.85$, $\sigma_{ao} = 0.50$, $\beta_1 = 0.075$, $\chi_1 = 5/9$, $\sigma_{e2} = 1.0$, $\sigma_{e1} = 0.856$, $\beta_2 = 0.0828$, $\zeta_2 = 0.44$.

2.2. Description of $\gamma - Re_{ci}$ and simplified transition model

The $\gamma - Re_{ci}$ transition model is a local correlation-based model, in which the localization is achieved by the definition of a vorticity Reynolds number and the formulation of a transport equation for transition momentum thickness Reynolds number. Moreover, the intermittency factor is used for the control of production of turbulent kinetic energy. The transport equation for intermittency factor is listed as

$$\frac{\partial (\gamma \phi)}{\partial t} + \frac{\partial}{\partial x_j} \left[ \mu \frac{\partial \phi}{\partial x_j} \right] = P_\phi + E_\phi$$

(3)

where $\gamma$ is the intermittency factor.

The production and destruction terms of intermittency factor are

$$P_\phi = F_{\text{length}} \frac{\partial \phi}{\partial t} (\gamma F_{\text{onset}})^{0.5} (1 - c_{el1})$$

(4)

$$E_\phi = c_{e2} \Delta \Omega F_{\text{turb}} (1 - c_{el2})$$

(5)

where $F_{\text{length}}$ is the transition length function, $F_{\text{onset}}$ controls the onset of transition and defined as:

$$F_{\text{onset}} = \max(F_{\text{onset},2} - F_{\text{onset},3}, 0)$$

$$F_{\text{onset},1} = \frac{Re}{2.193 Re_{ci}}$$

$$F_{\text{onset},2} = \min(\max(F_{\text{onset},1}, F_{\text{onset},1}^a), 2.0)$$

$$F_{\text{onset},3} = \max\left( 1 - \frac{Re}{2.53}, 0 \right)$$

with

$$Re_{ci} = \frac{\rho k^2}{\mu \phi}, \quad Re_{ci} = \frac{\rho \phi^2 S}{\mu}$$

The transport equations for transition momentum thickness Reynolds number is formulated as

$$\frac{\partial (\rho \dot{Re}_h)}{\partial t} + \frac{\partial}{\partial x_j} \left[ \mu \frac{\partial \dot{Re}_h}{\partial x_j} - \sigma_{h} (\mu + \mu_1) \frac{\partial Re}{\partial x_j} \right] = P_{\dot{Re}_h}$$

(6)

where $\dot{Re}_h$ is the transition momentum thickness Reynolds number, the source term is

$$P_{\dot{Re}_h} = c_{e1} \Delta \Omega (Re - \dot{Re}_h) (1.0 - F_{\dot{Re}_h})$$

(7)

with

$$F_{\dot{Re}_h} = \min\left( \max\left( \frac{F_{\text{wake}} \exp\left( -\frac{(\gamma - 1/2)^2}{1.0 - (1/2) \mu_{\text{lin}}} \right)}{1.0} \right), 1.0 \right)$$

$$t = \frac{500h}{\rho U^2}, \quad F_{\text{wake}} = \exp\left( -\left( \frac{Re_{ci}}{1 \times 10^2} \right)^2 \right), \quad Re_{ci} = \frac{\rho \gamma^2 \mu}{\mu}$$

$$\delta = \frac{500 \Delta \Omega}{U} \delta_{BL}, \quad \delta_{BL} = 15 \theta_{BL}, \quad \theta_{BL} = \frac{Re_{ci} \mu}{U}$$

where $U$ is the magnitude of velocity.

The transition Reynolds number in the source term is computed from freestream turbulence intensity and local pressure gradient parameter:

$$Re_{ci} = \begin{cases} \left( \frac{1173.51 - 589.428 T_{\infty}}{0.2196 T_{\infty}^{1/2}} \right) F(\lambda), & T_{\infty} \leq 1.3 \\ 331.5(T_{\infty} - 0.5658)^{-0.671} F(\lambda), & T_{\infty} > 1.3 \end{cases}$$

(8)

$$F(\lambda) = \begin{cases} 1 - (-12.986 \lambda - 123.66 \lambda^2 - 405.689 \lambda^3) \exp\left( -\left( \frac{T_{\infty}^{1/3}}{15} \right)^4 \right), & \lambda < 0 \\ 1 + 0.275(1 - e^{-10\lambda}) \exp(-2T_{\infty}), & \lambda > 0 \end{cases}$$

(9)

where $T_{\infty}$ is the freestream turbulence intensity and $\lambda$ is the pressure gradient parameter, they are calculated by

$$T_{\infty} = \frac{100 \sqrt{2k_{\infty}/3}}{U_{\infty}}, \quad \lambda = \frac{\rho \phi^2}{\mu} \frac{dU}{ds}$$

Correlations for critical Reynolds numbers and transition length function in the original $\gamma - Re_{ci}$ transition model are

$$Re_{ci} = \left\{ \begin{array}{cl} 3.96 - 1.21 \times 10^{-2} Re_{ci} + 8.68 \times 10^{-4} Re_{ci}^2 & Re_{ci} < 400 \\ 263.4 - 1.24 Re_{ci} + 1.95 \times 10^{-1} Re_{ci}^2 - 1.02 \times 10^{-3} Re_{ci}^3 & 400 \leq Re_{ci} < 956 \\ 0.5 - (Re_{ci} - 956) \times 3.0 \times 10^{-4} & Re_{ci} \geq 956 \end{array} \right.$$  

(10)

$$F_{\text{length}} = \begin{cases} 39.8 - 1.19 \times 10^{-2} Re_{ci} - 1.33 \times 10^{-4} Re_{ci}^2, & Re_{ci} < 400 \\ 263.4 - 1.24 Re_{ci} + 1.95 \times 10^{-1} Re_{ci}^2 - 1.02 \times 10^{-3} Re_{ci}^3, & 400 \leq Re_{ci} < 956 \\ 0.5 - (Re_{ci} - 956) \times 3.0 \times 10^{-4} & Re_{ci} \geq 956 \end{cases}$$

(11)

where $Re_{ci}$ is the critical Reynolds number and $F_{\text{length}}$ is the length function.

The combination of SST turbulence model and $\gamma - Re_{ci}$ transition model is achieved by multiplying the effective intermittency factor to the production and destruction terms of the turbulent kinetic energy equation, shown as follows:

$$\frac{\partial (\rho \phi)}{\partial t} + \frac{\partial}{\partial x_j} \left[ \mu \frac{\partial \phi}{\partial x_j} \right] = \tilde{P}_k + \tilde{D}_k$$

(12)

$$\tilde{P}_k = \gamma_{eff} P_k, \quad \tilde{D}_k = \min\left( \max\left( \gamma_{eff}, 0.1 \right), 1.0 \right) D_k$$

where $P_k$ and $D_k$ are the production and destruction terms of the original SST turbulence model. Specially, a correction is proposed for separation-induced transition:

$$\gamma_{sep} = \min\left( 2 \max\left( \frac{Re_{ci}}{3.235 Re_{ci}} - 1.0, 0 \right) F_{\text{reattach}, 2} \right)$$

(13)
the effective intermittency factor is defined as $\gamma_{\text{eff}} = \max (\gamma, \gamma_{\text{app}})$. Model coefficients are $c_{a1} = 2.0$, $c_{a2} = 0.06$, $c_{a3} = 1.0$, $c_{b} = 50.0$, $c_{w} = 0.03$, $\alpha = 1.0$ and $\alpha_2 = 2.0$.

The transport equation of transition momentum thickness Reynolds number is used to calculate the critical Reynolds number $Re_{\text{th}}$ and transition length function $F_{\text{length}}$, as shown in Eqs. (10) and (11). If proper correlations for $Re_{\text{th}}$ and $F_{\text{length}}$ can be supplied, the transport equation can also be removed. According to Ge, they proposed a correlation for the critical Reynolds number using the parameter $T_1$ and freestream turbulence intensity:

$$Re_{\text{th}} = 900 - 810 \min [T_1(0.285T_{18}^{0.526} + 0.35)/4.0, 1.0]$$

where

$$T_1 = R_{\text{f}} \frac{\Omega}{\omega}$$

The $T_1$ parameter behaves, to some extent, similarly to the $H_t$ parameter, due to the existence of the magnitude of vorticity. The critical Reynolds number decreases with the increase of $T_1$ and has a value between 90 and 900. The transition length function controls the length of transition region and has some effect on the transition onset location. Previous research indicated that it is appropriate to formulate the function only by the freestream turbulence intensity and a value between 0.1 and 100 is suitable for the function. This leads to the new transition length function as

$$F_{\text{length}} = \max (0.1, 30.0 \ln (T_{18}^{0.526} + 0.8997))$$

All of the constants in the correlations in Eqs. (14) and (16) are empirically determined by numerical calculations based on the transitional flat plate. It is noteworthy that the abandonment of $Re_{\text{th}}$ may result in the inability of prediction for separation-induced transition, as shown in Eq. (13). Although cases with separation are conducted in this paper, no special treatment at present has been made for separation correction, which will be studied in the succeeding work. In addition, the transport equation for intermittency factor and the connection with SST turbulence model are identical to the original $\gamma - Re_{\text{th}}$ model.

For clear and complete acquaintance of the model, the governing equations and empirical correlations for the simplified transition model are summarized as

$$\begin{align*}
\frac{\partial \rho u}{\partial x} + \frac{\partial \rho u}{\partial y} &= \rho u \frac{\partial u}{\partial x} + \left( \mu + \sigma_i \mu_i \frac{\partial u}{\partial x} \right) = P_k - \beta \rho \rho \frac{\partial \rho \rho}{\partial x} \\
\frac{\partial \rho \rho u}{\partial y} &= \rho \frac{\partial u}{\partial y} + \frac{\partial \rho \rho \frac{\partial u}{\partial x}}{\partial x} = \frac{\partial \rho \rho \frac{\partial u}{\partial x}}{\partial x}
\end{align*}$$

with

$$Re_{\text{th}} = 900.0 - 810.0 \min [T_1(0.285T_{18}^{0.526} + 0.35)/4.0, 1.0]$$

$F_{\text{length}} = \max (0.1, 30.0 \ln (T_{18}^{0.526} + 0.8997))$

3. Computational method

The new transition model in this work is implemented into an in-house CFD solver, which is based on finite-volume method and uses multi-block structured grid for simulations. The solver, which is parallelized by message passing interface (MPI), provides Euler, Navier–Stokes, RANS and even Burnett equations for both perfect gas and chemical reacting gas. In this paper, only the RANS equations and perfect gas assumption are used for calculations. Concretely, the advection upstream splitting method by pressure-based weight functions (AUSMPW+25) is used to calculate the inviscid fluxes and the viscous fluxes are centrally discretized. The monotone upstream centered scheme for conservation laws (MUSCL), along with the van Almba limiter function is adopted for achievement of second-order accuracy. Notably, the turbulence model and transition model equations are also solved by the second-order scheme for the consideration of high-resolution. Steady-state solutions are finally obtained by a fully coupled implicit lower–upper symmetric Gauss–Seidel (LU-SGS)26 time-marching method, which solves all governing equations simultaneously at each iteration step. Additionally, the source terms of turbulence model and transition model are implicitly treated for the diminution of stiffness problem. Precisely, the negative part of the source terms can be written as:

$$S_{+1} = S + \int \left( \frac{\partial S}{\partial Q} \right) dQ$$

where $S$ is the source term and $Q$ the conservative variables. The Jacobian matrix of source term is defined as

$$\mathbf{T} = \frac{\partial S}{\partial \mathbf{Q}} = \begin{bmatrix} T_{11} & T_{12} & T_{13} \\ T_{21} & T_{22} & T_{23} \\ T_{31} & T_{32} & T_{33} \end{bmatrix}$$

with

$$T_{11} = -2\beta \rho \rho$$

$$T_{22} = -2\beta \rho \rho - 2F_1 \frac{\partial \rho \rho}{\partial x}$$

$$T_{33} = \left[ F_{\text{length}} c_{a1} S (\gamma_{\text{onset}})^{0.5} \right] (\gamma^{0.5} - 1.5) + F_{\text{turb}} (1.0 - 2.0c_{a2} \gamma) c_{a2} \Omega$$

$$f(x) = \begin{cases} 0 & x \geq 0 \\ 0 & x < 0 \end{cases}$$

The boundary conditions for turbulent kinetic energy and specific dissipation rate at inlet are $k_\infty = 1.5(T_{18}/100U_\infty)^2$ and $\omega_\infty = p_\infty k_\infty/\mu_\infty$, while their values on the wall are $k = 0$ and $\omega = 60R_{\text{th}}/\rho \beta \gamma^2$ respectively. The freestream values for $\gamma$ and $Re_{\text{th}}$ are given as 1 and 0.0001 respectively, and both of them have a zero flux ($\partial \gamma/\partial n = 0$) on the wall. Besides, all freestream conditions are set as the initial conditions for the whole computational domain.

4. Results and discussion

4.1. Zero pressure gradient flat plate

The commonly used Schubauer and Klebanoff case27 is a zero pressure gradient flat plate with a Mach number of 0.15. It is a
standard test case for the evaluation of performance of new transition models in natural transition. The freestream velocity is 50.1 m/s, freestream turbulence intensity is 0.18%, and unit Reynolds number is $3.4 \times 10^6$. Final results of skin friction coefficient along the flat plate from three sets of grid and experimental data are shown in Fig. 1. Dimensions for Grid 1 to Grid 3 are $154 \times \frac{50}{1}, 194 \times \frac{90}{3}$ and $234 \times \frac{130}{3}$ respectively. All grids are clustered near the solid wall, and the spacing of the first grid to the wall is $1 \times 10^{-6}$ m, which corresponds to a $y^+$-plus value of about 0.15. Similarly, the $y^+$-plus values for all the following cases are guaranteed to be less than one. We can see from Fig. 1 that grid convergence is obtained, since all the following cases are guaranteed to be less than one. We can see from Fig. 1 that grid convergence is obtained, since the transition location changes little with the refinement of grid. The present new transition model predicts the transition location, skin friction values in laminar part and in turbulent part accurately except the transition length. The deficiency of transition length may largely relate to the empirical correlation shown in Eq. (16). In general, the transition model performs much better than the fully laminar and fully turbulent results, especially in the latter part of the flat plate, where the turbulence model under-estimates the skin friction with a value of about 20%.

4.2. Aerospatial-A airfoil

The Aerospatiale-A airfoil, shown in Fig. 2, is another typical test case for transition models. The experimental results obtained from ONERA F1 wind tunnel at $13.1^\circ$ angle of attack are used in this work. Simulations are carried out at Mach number 0.15, Reynolds number $2.1 \times 10^6$ and freestream turbulence intensity 0.2%. Experiment turns out that a laminar separation bubble forms at 12% of the suction side of the chord, resulting in a turbulent boundary-layer downstream. Details of the flow field can be witnessed from the Mach number contour in Fig. 3(a), and the separation-induced transition can be clearly seen from Fig. 3(b), in which the intermittency factor $\gamma$ develops rapidly when turbulence occurs. Specifically, separation occurs at about 80% of the chord and intermittency factor increases significantly when transition occurs. The computed skin friction coefficient $C_f$ along the suction side of the airfoil is shown in Fig. 4(a). We can see that boundary layer transition is accurately predicted from the sharp increase of skin friction obtained by transition models, and good agreement with experimental data is obtained by both the original $\gamma - Re_h$ and new transition models. The fully turbulent calculation, however, fails to capture the transition onset feature. Fig. 4(b) shows the pressure coefficient $C_p$ around the airfoil. We can see that the transition models outperform the fully turbulence model significantly except in the trailing edge. The under-prediction of pressure near the trailing edge region can be attributed to the inability of the transition model for simulation of strong flow separation, which occurs at about 83% of the chord as observed in experiment. A closer look at the figure shows that the presented new transition model performs a bit better for the skin friction and the pressure coefficient in the trailing edge than the original model. Nevertheless, the difference between the two transition models is slight in this case.

4.3. S809 airfoil

The S809 airfoil, designed by the National Renewable Energy Laboratory (NREL), is a primary airfoil used for wind turbine applications. Detailed experimental data, including lift $L$, drag $D$ and transition locations for different angles of attack $\alpha$ can be found from Somers. The computational grid is shown in Fig. 5, and simulations are performed at Mach number 0.1, Reynolds number $2.0 \times 10^6$ and freestream turbulence intensity 0.2%. Fig. 6 shows the transition locations $x/c$ on the pressure side and suction side of the airfoil. Specifically, the transition location on the pressure side moves downstream steadily from about 50% to 60% of the chord as the angle of attack increases, while the transition location on the suction side moves forward sharply at approximate 5° angle of attack due to the adverse pressure gradient. Computed lift coefficient $C_L$ and drag coefficient $C_D$ are depicted in Fig. 7, in which we can see that the transition models show apparent improvement for both of the coefficients, especially for the drag. As expected, transition models possess no distinct advantage over turbulence model at high angles of attack, due to the strong separation. On the whole, good agreement with experimental data is achieved by transition models, and the simplified model seems to perform better than the original model.
4.4. Hypersonic flat plate

The test case considered for validation of the new transition model in high speed flow is from Mee.\textsuperscript{30} Experiments were carried out in the T4 free-piston shock tunnel at University of Queensland for a 1.5 m long and 0.12 m wide flat plate. Frauholz et al.\textsuperscript{31} studied the case extensively based on the $\gamma - Re_h$ transition model with modified correlations, and satisfactory results were obtained. The freestream conditions for computation are listed in Table 1. The Stanton number is defined as

$$St = \frac{q_w}{\rho_{\infty} U_{\infty} (T_{0,\infty} - T_w)}$$

where $\rho_{\infty}$, $U_{\infty}$ and $T_{0,\infty}$ are the freestream density, velocity and total temperature respectively, $q_w$ and $T_w$ are the heat flux and temperature on the wall. It should be noted that the Stanton number measured in experiment has an uncertainty of $\pm 18\%$, as estimated by Mee.\textsuperscript{30} Since the real freestream turbulence intensity in the wind tunnel is unknown, preliminary simulations are performed to tune the transition location, as shown in Fig. 8, where $TI_{\infty}$ represents the freestream turbulence intensity, and the increase of $TI_{\infty}$ promotes the transition of boundary layer. We can also find that the transition location and transition length are strongly affected by the freestream turbulence intensity. The four computed conditions are summarized in Fig. 9, and significant improvement of prediction accuracy has been achieved by the transition models, together with the results obtained by Frauholz et al.,\textsuperscript{31} in which the authors predicted the transitions by using a modified

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Fig. 3 Contours of Mach number and intermittency factor for the Aerospatial-A airfoil.

Fig. 4 Skin friction and pressure coefficients distribution along the Aerospatial-A airfoil.

Fig. 5 Computational grid for S809 airfoil.
Reh model. Good agreement with experimental data can be witnessed clearly, and it is not an easy task to judge between the two transition models at present. A notable improvement should be mentioned that the new transition model not only predicts the transition onset accurately, but also gets closer to experimental data than the turbulence model on the fully turbulent part of the flat plate.

4.5. Hypersonic double wedge

A more complicated case for the transition model is the hypersonic double wedge, tested by Neuenhahn and Olivier in the TH2 shock tunnel. The shape considered at present has a slightly blunted leading edge of 0.5 mm. The first ramp is 178 mm long and has an angle of 9°, while the second ramp is 200 mm long and has an angle of 20.5°. Two-dimensional flow is guaranteed with a width of 270 mm for the geometry. The freestream flow conditions are listed in Table 2 and computational grid is shown in Fig. 10. Unlike the previous cases, the hypersonic double wedge encounters severe shock wave/boundary-layer interaction, which can be demonstrated by the Mach number contour obtained by the new transition model in Fig. 11. Details of the flow can be noted that a detached bow shock from the blunted leading edge interacts with the separation-induced shock in the corner, followed by a combination with the reattachment shock.

Results of pressure coefficient and Stanton number are shown in Fig. 12. We can see that generally good agreement, especially of the pressure coefficient, is obtained by the transition model. The fully laminar simulation under-estimates the Stanton number significantly, and has a slightly larger separation zone. On the contrary, the fully turbulent calculation over-estimates the Stanton number and has no separation at

![Fig. 6](image)

**Fig. 6** Transition location for S809 airfoil.

![Fig. 7](image)

**Fig. 7** Lift and drag coefficients at different angles of attack for S809 airfoil.

![Fig. 8](image)

**Fig. 8** Stanton number distribution of hypersonic flat plate under Condition 1.

![Table 1](image)

**Table 1** Freestream parameters under different conditions.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Ma_{\infty})</td>
<td>5.3</td>
</tr>
<tr>
<td>(T_{\infty}) (K)</td>
<td>570</td>
</tr>
<tr>
<td>(Re_{\infty})</td>
<td>1.7 (\times 10^6)</td>
</tr>
<tr>
<td>TL(\infty) (%)</td>
<td>2.6</td>
</tr>
</tbody>
</table>
all. For comparison, the Stanton number predicted by the original $\gamma = Re_{h}$ transition model is extracted from You\textsuperscript{14} and shown in Fig. 12(b). We can see that results of the new transition model outperform that of the fully laminar and turbulent assumptions. Specifically, the new simplified model has a relatively lower value of Stanton number and bigger separation zone as compared with the original model. However, discrepancies with experiment still exist for both of the transition models. The absence or inappropriate correction in the transition model for separation flow may be one possible explanation, since the research of You\textsuperscript{14} demonstrated that a more physical meaningful effective intermittency involved pressure gradient performed better. Moreover, some of our previous numerical attempts indicated that the transition length function and critical Reynolds number correlations have non-negligible effects on the flow features of transition. Anyway, these aspects are of great importance and will be the key points of the further work.

**Table 2** Freestream parameters for hypersonic double wedge.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Ma_{\infty}$</td>
<td>8.1</td>
</tr>
<tr>
<td>$Re_{\infty}$</td>
<td>$3.8 \times 10^6$</td>
</tr>
<tr>
<td>$u_{\infty}$ (m/s)</td>
<td>1635</td>
</tr>
<tr>
<td>$p_{\infty}$ (Pa)</td>
<td>520</td>
</tr>
<tr>
<td>$T_{\infty}$ (K)</td>
<td>106</td>
</tr>
<tr>
<td>$TL_{\infty}$ (%)</td>
<td>0.9</td>
</tr>
</tbody>
</table>

**Fig. 9** Stanton number distribution of hypersonic flat plate under different conditions.

**Fig. 10** Computational grid for double wedge.
5. Conclusions

(1) A simplified local-correlation-based transition model has been developed by the removal of the momentum thickness Reynolds number equation in the original $\gamma - Re_{th}$ transition model, along with the new transition length function and critical Reynolds number correlation.

(2) The new transition model is implemented into an in-house CFD solver and tested for some typical cases, ranging from low speed flows to hypersonic flows. Results from simulations in this work show that the accuracies of the flows are significantly improved by the proposed transition model, compared with the fully laminar and fully turbulent calculations.

(3) It is shown that the simplified transition model is comparable to the original $\gamma - Re_{th}$ model for the simulations presented in this work. The new model appears to be promising and deserves further research due to its satisfactory results and less computational cost than the original model.

(4) It is, however, necessary to validate the new model with more complex test cases for the verification of its practicability. Future attempts will be made to the separation-induced transition correction. Besides, the correlations in the model need to be calibrated in detail by wind tunnel data.

Acknowledgement

This study was supported by the State Key Development Program for Basic Research of China (No. 2014CB340201).

References


Xia Chenchao is a Ph.D. candidate at School of Aeronautics and Astronautics, Zhejiang University. His main research intersects are computational fluid dynamics and aerodynamic shape optimization.

Chen Weifang is a professor and Ph.D. supervisor at School of Aeronautics and Astronautics, Zhejiang University. His current research interests are gas dynamics and aircraft design.