Physics Letters B 663 (2008) 242–246

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Sweet spot supersymmetry and composite messengers

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ARTICLE INFO

Article history:
Received 28 February 2008
Received in revised form 9 April 2008
Accepted 10 April 2008
Available online 16 April 2008
Editor: G.F. Giudice

ABSTRACT

Sweet spot supersymmetry is a phenomenological effective Lagrangian of weak scale supersymmetry with a certain set of natural assumptions. This framework is designed to avoid problems in low-energy phenomenology and cosmology of supersymmetric models. We discuss a class of dynamical models of supersymmetry breaking and its mediation, whose low-energy effective description falls into this framework. Hadron fields in the dynamical models play a role of the messengers of the supersymmetry breaking. As is always true in the models of the sweet spot supersymmetry, the messenger scale is predicted to be 10^5 GeV $\lesssim M_{\rm mess} \lesssim 10^{10}$ GeV. Various values of the effective number of messenger fields $N_{\rm mess}$ are possible depending on the choice of the gauge group.

1. Introduction

If $\mathcal{N}=1$ supersymmetry is hidden in nature, it helps us to understand the hierarchy between the strength of gravity and weak interactions and also the variety of matter fields and gauge forces in the Standard Model. It is, however, not straightforward to correctly hide supersymmetry at low energy. Especially, there has been a trouble in making the Higgs sector suitable for electroweak symmetry breaking, i.e., the μ-problem. Also, the smallness of flavor mixing and CP violation has been considered as an unnatural aspect of the hypothesis.

Recently, the present authors carefully considered those problems, including cosmological one, and found a simple and realistic framework of supersymmetry breaking and mediation [1]. A small explicit breaking of the Peccei–Quinn (PQ) symmetry triggers supersymmetry to break down, and it induces μ-term through (a generalized version of) the Giudice–Masiero mechanism [2]. The explicit breaking term also makes messenger fields massive via classical supergravity effects [3]. Correct sizes of gaugino and sfermion masses are obtained through their loop diagrams (gauge mediation [4–7]). The dangerous proton-decay operators of the mass-dimensions four and five are forbidden by the PQ-symmetry. A mechanism of producing dark matter of the universe is built-in; non-thermally produced gravitinos through the decay of the Polonyi field naturally explains the correct abundance [8]. This framework solves many known problems in supersymmetric model building, and serves as a good phenomenological Lagrangian to calculate low-energy observables.

The framework, *the sweet spot supersymmetry*, is written in the language of the low-energy effective field theory. It is possible to construct various explicit models within this framework as ultraviolet (UV) completions, and each of those falls into a parameter point (or region) in the sweet spot supersymmetry. The parametrization can be done by four quantities: the number of messenger $N_{\rm mess}$, the μ-parameter, a gaugino mass, and the messenger scale $M_{\rm mess}$. Once we specify those parameters, we can calculate the spectrum of superparticles by a simple program described in Ref. [1]. Conversely, by measuring those quantities at low energy experiments, we can obtain information on UV models.

In a recent paper [9], an economical UV model in this framework was proposed, that addresses the origin of the small explicit breaking term of the PQ-symmetry. (It is called $U(1)_H$ symmetry in Ref. [9].) It is found that the term can be non-perturbatively generated in a QCD-like theory ($SU(N_c)$ gauge theory with N_c flavors), and the quark fields in that supersymmetric QCD play a role of the messenger fields. A parameter region of the sweet spot supersymmetry is identified for this UV completion: $N_{\rm mess}=5$ and 10^{11} GeV $\lesssim M_{\rm mess} \lesssim 10^{13}$ GeV. The lower bound on the messenger scale is obtained from a consistency of the analysis.

In this Letter, we argue that this class of models predicts $M_{\rm mess} \lesssim 10^{10}$ GeV, rather than $M_{\rm mess} \gtrsim 10^{11}$ GeV where the analysis in Ref. [9] is meaningful. Nevertheless, we find that there is a consistent effective description in terms of hadron fields in that case. We can find a supersymmetry breaking vacuum where gaugino/sfermion masses are generated by loop diagrams of hadronic messenger fields instead of elementary quarks. Gen-

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0370-2693
doi:10.1016/j.physletb.2008.04.023

eralizations with $\mathrm{Sp}(N_c)$ and $\mathrm{SO}(N_c)$ gauge theories are also discussed.

2. Model

We briefly review the framework of the sweet spot supersymmetry, and present a dynamical model of supersymmetry breaking which falls into this framework.

2.1. Sweet spot supersymmetry

The Lagrangian of the sweet spot supersymmetry is written in terms of the fields in the minimal supersymmetric Standard Model (MSSM) (chiral superfields $\Phi_{\rm MSSM}$ and gauge fields W_α), the Goldstino field S, and the messenger fields f and $\bar f$ that have quantum numbers of the Standard Model gauge group.[1] It is defined by the supergravity Lagrangian with a Kähler- and a superpotential, K and W:

$$\begin{aligned} K = {} & \Phi^\dagger_{\rm MSSM}\Phi_{\rm MSSM} + S^\dagger S + f^\dagger f + \bar f^\dagger \bar f \\ & - \frac{(S^\dagger S)^2}{\Lambda^2} + \left(\frac{c_\mu S^\dagger H_u H_d}{\Lambda} + \text{h.c.}\right) \\ & - \frac{c_H S^\dagger S (H_u^\dagger H_u + H_d^\dagger H_d)}{\Lambda^2}, \end{aligned} \tag{1}$$

$$W = W_{\rm Yukawa}(\Phi_{\rm MSSM}) + m^2 S + kS f\bar f + w_0. \tag{2}$$

In the Kähler potential, there are direct interaction terms between the Goldstino fields S and the Higgs fields H_u and H_d suppressed by a 'cut-off' scale Λ. (c_μ and c_H are O(1) coefficients.) These interactions are responsible for generating the μ-term and the soft mass terms for the Higgs fields.

One may think that it is unnatural to assume the absence of the terms $S^\dagger S Q^\dagger Q/\Lambda^2$ where Q is the matter chiral superfields. This assumption is indeed essential to solve the flavor problem. To achieve this, the matter fields need to have a higher 'cut-off' scale than S, H_u and H_d. As is discussed in Refs. [1,10] this situation is reasonable rather than unnatural. As we see later, we need to set the scale Λ to be the grand unification scale. The presence of Λ-suppressed operators indicates that S and the Higgs fields get strongly coupled and/or new states which couple to S and the Higgs fields appear above the GUT scale. We claim that it is completely reasonable since the Higgs fields must couple to the GUT breaking sector in order to achieve the doublet–triplet splitting. On the other hand, we have no strong reason to assume that the GUT breaking sector couples to the matter fields directly as they form complete multiplets of a GUT gauge group. In a concrete model in Ref. [10], S and the Higgs fields are composite fields of a strong dynamics which breaks GUT gauge group dynamically whereas the matter fields are elementary thereby explains the difference of the 'cut-off' scale. One can also imagine a situation that S and the Higgs fields live in the bulk of a warped extra-dimension but the matter fields are confined on an ultraviolet brane. Again, this structure is a somewhat natural set-up for realistic GUT models. Independent of GUT models, what we have assumed is that the Higgs fields are somewhat special, which most of the particle theorists would agree in many respects. We further proposed a unified picture that the Higgs field to break supersymmetry (S) shares the same speciality. The two different 'cut-off' scales are analogous to the relation between $\Lambda_{\rm QCD}$ and M_W in the Standard Model. Below the scale $\Lambda_{\rm QCD}$, the effective Lagrangian for hadron fields contains many kinds of interaction terms suppressed by $\Lambda_{\rm QCD}\sim 1$ GeV. Meanwhile, the lepton-to-hadron couplings are suppressed by the W boson mass $M_W \sim 80$ GeV. For detailed discussion taking into account Yukawa interactions, see Ref. [1].

In the superpotential, a linear term of S represents the source term of the F-component of S. The interaction term between S and messenger fields f and $\bar f$ is responsible for gauge mediation. The constant term w_0 is needed to cancel the cosmological constant such that $w_0 = m_{3/2}M_{\rm Pl}^2 = m^2 M_{\rm Pl}/\sqrt{3}$, where $m_{3/2}$ is the gravitino mass. This is the most general Lagrangian with the PQ-symmetry, $PQ(S)=2$, $PQ(H_u)=PQ(H_d)=1$, and $PQ(m^2)=-2$, where m^2 represents the small explicit breaking parameter. Smallness of the supersymmetry breaking scale and also of the μ-parameter are controlled by this parameter. The assumption made here is that the whatever dynamics at the scale Λ should possess the (approximate) PQ-symmetry with the above charge assignment.

Obviously, there is a supersymmetric vacuum in this model where

$$\langle S\rangle = 0, \qquad \langle f\bar f\rangle = -m^2/k. \tag{3}$$

However, we can find a local minimum with broken supersymmetry if the value of k is small enough [3]. From the above K and W, we obtain a scalar potential for the S field:

$$\begin{aligned} V(S) = {} & m^4\left(\frac{4}{\Lambda^2}|S|^2 + \frac{k^2 N}{(4\pi)^2}\log\left(\frac{k^2|S|^2}{\Lambda^2}\right)\right) \\ & - \left(2m_{3/2}m^2 S + \text{h.c.}\right). \end{aligned} \tag{4}$$

The logarithmic term is a loop correction from the interaction term, $kSf\bar f$, and N is a number of fields running in the loop. For example, $N=5$ if the messenger fields f and $\bar f$ transform as 5 and $\bar 5$ representations under SU(5) symmetry ($(3,1)_{-1/3}\oplus(1,2)_{1/2}$ and $(\bar 3,1)_{1/3}\oplus(1,2)_{-1/2}$ under the Standard Model gauge group). The linear term, $2m_{3/2}m^2 S$, is a supergravity effect; this is a soft supersymmetry breaking term associated with the linear term in the superpotential in Eq. (2). Once we ignore the logarithmic term, the minimum is at

$$\langle S\rangle = \frac{\sqrt{3}\Lambda^2}{6M_{\rm Pl}}. \tag{5}$$

This makes the messenger fields massive, and thus stabilizes the $f\bar f$ direction. Supersymmetry is broken by $F_S \simeq m^2 + k\langle f\bar f\rangle = m^2$. For a large value of k, however, this local minimum disappears because the quantum correction becomes stronger than the supergravity effects. As we will see in Section 3, the condition that there is a meta-stable supersymmetry breaking vacuum provides an upper bound on the messenger scale, $M_{\rm mess} = k\langle S\rangle$.

By integrating out those massive messenger fields, we obtain terms responsible for the gaugino and sfermion masses (gauge mediation) [11]:

$$f_{\rm kin} \ni -\frac{N_{\rm mess}}{(4\pi)^2}\log S W^\alpha W_\alpha, \tag{6}$$

for the gauge kinetic function, and

$$K \ni -\frac{4g^4 N_{\rm mess}}{(4\pi)^4} C_2(R)\left(\log|S|^2\right)\Phi^\dagger_{\rm MSSM}\Phi_{\rm MSSM}, \tag{7}$$

with $N_{\rm mess}$ the number of the messenger fields ($N_{\rm mess}=1$ for a pair of 5 and $\bar 5$ representations of SU(5)). With the non-vanishing value of F_S and $\langle S\rangle$, we obtain gaugino/sfermion masses through the above interaction terms.

There are two dimensionful parameters in this model: Λ and $m_{3/2}(= m^2/(\sqrt{3}M_{\rm Pl}) = w_0/M_{\rm Pl}^2)$. The interesting discovery in

[1] The Lagrangian of the sweet spot supersymmetry is presented in Ref. [1] as the one after integrating out the messenger fields f and $\bar f$. The original form contains terms in Eqs. (6) and (7) instead of those involving f and $\bar f$. They are, of course, equivalent.

Ref. [1] is that there is a sweet spot in the two-dimensional parameter space $(m_{3/2}, \Lambda)$ where everything works out fine. The choice is $(m_{3/2}, \Lambda) \sim (1\ \text{GeV}, 10^{16}\ \text{GeV})$ with which we obtain correct sizes of the μ-term, gaugino and sfermion masses, and the abundance of gravitino dark matter. The fact that Λ is at the grand unification scale is also an interesting coincidence.

2.2. A model of dynamical supersymmetry breaking

A part of the above Lagrangian,

$$K \ni S^\dagger S - \frac{(S^\dagger S)^2}{\Lambda^2},$$
$$W \ni m^2 S, \tag{8}$$

provides an effective description of a quite general class of supersymmetry breaking models. As long as the Goldstino superfield (a field or a combination of fields which gets F-component VEV) is weakly coupled in the actual supersymmetry breaking model, the above Lagrangian is obtained by integrating out other massive fields in the model.[2] The Λ parameter represents the strength of the self interactions of the Goldstino and m^2 is the size of the supersymmetry breaking. We consider a simple example of such models where the superpotential terms in Eq. (2), $m^2 S + kSf\bar{f}$, are replaced with a single term,

$$W \ni kS(F\bar{F}), \tag{9}$$

where F and $\bar{F}$ transform as 5 and $\bar{5}$ representations of an $SU(5)_F$ group (which contains the Standard Model gauge group as a subgroup), respectively. They also have quantum numbers (5 and $\bar{5}$) of another gauge group $SU(5)_H$ which becomes strongly coupled at an energy scale Λ_{dyn}. In the above term, the indices of $SU(5)_H$ and $SU(5)_F$ are contracted in $(F\bar{F})$.

By taking into account non-perturbative effects of the $SU(5)_H$ gauge interaction, there appears a supersymmetry breaking vacuum which is stabilized by interaction terms in the supergravity action as we will see below. The supersymmetry breaking model above has first been discussed in Ref. [12], although there is a little difference that it is a model of gravity mediation and the $SU(5)_F$ group is not gauged. In Ref. [12], it has been assumed that the leading contribution to the Kähler potential term, $(S^\dagger S)^2$, comes from the $SU(5)_H$ dynamics itself, rather than the 'cut-off' suppressed operator in Eq. (1). In this case, unfortunately, there is no gravitationally stabilized local minimum corresponding to Eq. (5) once we take into account the logarithmic term in Eq. (4) generated by loop diagrams of light particles. We can find an extremum at $\langle S\rangle \neq 0$ when the coupling constant k is small enough, but that is in fact a local maximum in the $\text{Im}\, S$ direction [3]. In a recent paper [9], a model with a (partly) gauged $SU(5)_F$ symmetry has been considered. Once the S field gets the supersymmetry breaking VEV, the quark fields F and $\bar{F}$ mediate it to the gauge/matter fields via gauge mediation. They also introduced the Kähler terms in Eq. (1) so that it fits to the program of the sweet spot supersymmetry.

We assume here that the gauge interaction of $SU(5)_H$ becomes strong in the regime where the masses of messenger particles, $M_{\text{mess}} (\equiv k\langle S\rangle)$, are not important, i.e., $M_{\text{mess}} < \Lambda_{\text{dyn}}$.[3] In fact, in Ref. [9], it has been claimed that this regime is incompatible with the mechanism of supersymmetry breaking and mediation since the meta-stable vacuum in Eq. (5) disappears. We show, however, that we still have the gravitationally stabilized vacuum where supersymmetry is spontaneously broken. Furthermore, we will see in the next section that M_{mess} is almost always lower than the dynamical scale Λ_{dyn} for the vacuum to be meta-stable.

Below the scale Λ_{dyn}, there is an effective description of the theory in terms of meson $M_{ij} \sim F_i \bar{F}_j$ and baryon fields $B \sim F^5$ and $\bar{B} \sim \bar{F}^5$. The indices $i, j (= 1-5)$ are those of $SU(5)_F$. The effective superpotential is given by

$$W = kS \cdot \text{Tr}\, M + X\big(\det M - B\bar{B} - (\Lambda_{\text{dyn}}^2/5)^5\big), \tag{10}$$

where a Lagrange multiplier X is introduced in order to ensure the quantum modified constraint to be satisfied [13]. We can find a meta-stable vacuum in the meson branch, $\det M = (\Lambda_{\text{dyn}}^2/5)^5$. By solving the constraint around the point $M_{ij} = \Lambda_{\text{dyn}}^2 \delta_{ij}/5$, we obtain

$$\text{Tr}\, M = \Lambda_{\text{dyn}}^2 + \frac{1}{2}\frac{\text{Tr}\,\delta M^2}{\Lambda_{\text{dyn}}^2/5} + \frac{B\bar{B}}{(\Lambda_{\text{dyn}}^2/5)^4} + \cdots, \tag{11}$$

where δM is the traceless part of the matrix M. We have neglected higher order terms in the field expansion. The effective superpotential below the scale Λ_{dyn} is then given by

$$W_{\text{eff}} = k\Lambda_{\text{dyn}}^2 S + S\left(\frac{\hat{k}_M}{2}\text{Tr}\,\delta\hat{M}^2 + \hat{k}_B \hat{B}\hat{\bar{B}}\right). \tag{12}$$

The fields $\delta\hat{M}$, $\hat{B}$, and $\hat{\bar{B}}$ are canonically normalized fields. Through this normalization procedure, O(1) uncertainties arise in the coupling constants $\hat{k}_M \sim \hat{k}_B \sim k$.

The effective superpotential above is exactly the one in Eq. (2) by the identifications of $m^2 \sim k\Lambda_{\text{dyn}}^2$, $k \sim \hat{k}_M$, $f \sim \delta\hat{M}$ and $\bar{f} \sim \delta\hat{M}$. The baryon fields do not contribute to the gaugino/sfermion masses since they are singlet under the Standard Model gauge group. The field $\delta\hat{M}$, on the other hand, transforms as the adjoint representation under the $SU(5)_F$ flavor group. (The quantum numbers under the Standard Model gauge group are $(8,1)_0 \oplus (1,3)_0 \oplus (3,2)_{-5/6} \oplus (\bar{3},2)_{5/6} \oplus (1,1)_0$.) By integrating out those meson fields, the terms in Eqs. (6) and (7) are obtained with $N_{\text{mess}} = 5$. The fact that N_{mess} did not change from the elementary picture can be understood by looking at the $U(1)_R$–$SU(5)_F$–$SU(5)_F$ anomaly, where $R(S) = 2$ and $R(F) = R(\bar{F}) = 0$. When we integrate out the F and $\bar{F}$ field in the elementary picture, we obtain terms in Eq. (6) where the factor N_{mess} reflects the anomaly, i.e., a phase rotation of S shifts the θ-term. Since the $SU(5)_H$ gauge interaction does not violate $U(1)_R$ symmetry even at the non-perturbative level, this structure remains unchanged after taking into account the strong dynamics.

The linear term $k\Lambda_{\text{dyn}}^2 S$ violates the PQ-symmetry which we discussed before: $PQ(S) = 2$. This is due to the fact that the PQ-symmetry is anomalous to the $SU(5)_H$ gauge interaction. As we have seen already, the linear term triggers to break supersymmetry and induces the μ-term.

We need to make sure that the strong dynamics does not destabilize the vacuum in Eq. (5). Through the interaction term, $kSF\bar{F}$, it is expected to appear higher-dimensional operators in the Kähler potential such as

$$\delta K \sim \frac{N}{(4\pi)^2}\frac{|kS|^4}{\Lambda_{\text{dyn}}^2}, \tag{13}$$

[2] In general, if there is no (approximate) symmetry under which S is charged in a supersymmetry breaking model, there can be a cubic term in the Kähler potential, $K \ni S^\dagger S^2 + \text{h.c.}$ However, such terms can be shifted away by an appropriate field redefinition $S \to S + c$. In the case of the sweet spot supersymmetry, the presence of the approximate PQ-symmetry is assumed. That restricts the form of Kähler and superpotential to be the ones in Eqs. (1) and (2).

[3] Precisely speaking, the corresponding inequality should be $M_{\text{mess}} \lesssim 4\pi\Lambda_{\text{dyn}}/\sqrt{N_C}$ with $N_C = 5$ according to the naive dimensional analysis. The dynamical scale Λ_{dyn} will be defined more clearly (but still implicitly) in Eq. (10). In the following analysis, we will ignore a factor of $4\pi/\sqrt{N_C}$. Obviously, inclusion of those factors will just make the assumption milder.

where $N = 25$. However, the effect of this term is smaller than that of the term $-(S^\dagger S)^2/\Lambda^2$ in Eq. (8) if

$$k \lesssim 3 \times 10^{-3} \left(\frac{N}{25}\right)^{-1/5} \left(\frac{m_{3/2}}{1\ \text{GeV}}\right)^{1/5} \left(\frac{\Lambda}{1 \times 10^{16}\ \text{GeV}}\right)^{-2/5}. \tag{14}$$

Here we have used a relation, $m^2 = k\Lambda_{\text{dyn}}^2 = \sqrt{3} m_{3/2} M_{\text{Pl}}$. As we will see later, the above condition is always satisfied when $M_{\text{mess}} < \Lambda_{\text{dyn}}$ that we have already assumed.

On top of the gauge mediation effects via loop diagrams of $\delta\hat{M}$, there can be uncalculable gauge mediation effects, i.e., generation of couplings between S and the Standard Model fields through the Standard Model gauge interaction mediated by heavy degrees of freedom (of the order of Λ_{dyn}). Those contributions are uncalculable, but suppressed by the scale Λ_{dyn}. Therefore, the effects are smaller than the calculable gauge mediation effects when $M_{\text{mess}} < \Lambda_{\text{dyn}}$ and thus negligible. For example, a term proportional to $(kS)^\dagger (kS)(H_u^\dagger H_u + H_d^\dagger H_d)/\Lambda_{\text{dyn}}^2$ may appear in the Kähler potential. It gives contribution to the soft terms of the Higgs fields in addition to the two comparable ones from $(S^\dagger S)(H_u^\dagger H_u + H_d^\dagger H_d)/\Lambda^2$ in Eq. (1) and from gauge mediation ($\delta\hat{M}$ loops). It is clear that the additional contribution is always smaller than the $\delta\hat{M}$-loop effects, $\propto \log|(kS)|^2 (H_u^\dagger H_u + H_d^\dagger H_d) \sim (kS)^\dagger (kS)(H_u^\dagger H_u + H_d^\dagger H_d)/|\langle kS\rangle|^2$, for $\langle kS\rangle \equiv M_{\text{mess}} < \Lambda_{\text{dyn}}$. Potentially dangerous terms such as $kS^\dagger H_u H_d/\Lambda_{\text{dyn}}$ and $(kS)^\dagger (kS) H_u H_d/\Lambda_{\text{dyn}}^2$ which induce the μ- and μB-terms do not appear as is usually the case in gauge mediation models. Those terms carry a charge, e.g., $Q(H_u) = Q(H_d) = 1$ and $Q(S) = Q(F) = Q(\bar{F}) = 0$, which is conserved as far as the $SU(5)_H$ and the Standard Model gauge interactions are concerned.

Uncalculable higher order Kähler terms involving hadron fields $\delta\hat{M}$, $\hat{B}$ and $\hat{\bar{B}}$ are not important in our discussion. Although the effects would become sizable when their classical values approach Λ_{dyn}, we are interested in the vacuum at $\delta\hat{M} = \hat{B} = \hat{\bar{B}} = 0$ and $S \neq 0$. This vacuum remains stable even in the presence of the higher order terms. First, a linear term of $\delta\hat{M}$, $\hat{B}$ or $\hat{\bar{B}}$ in the potential cannot appear because of the unbroken symmetries ($SU(5)_F$ and $U(1)_B$). With the VEV of S in Eq. (5), those fields obtain supersymmetric masses. Above two facts ensure the stability of the vacuum in the meson and baryon directions. Once we set $\delta\hat{M} = \hat{B} = \hat{\bar{B}} = 0$, the only term we should worry about is Eq. (13) that we have already discussed. The cubic terms $kS^\dagger (kS)^2/\Lambda_{\text{dyn}} + \text{h.c.}$ are not allowed by the non-anomalous R-symmetry we discussed.

There is a lower bound on the messenger scale by a condition that the messenger fields should not be tachyonic:

$$M_{\text{mess}}^2 = \hat{k}_M^2 \langle S\rangle^2 > \hat{k}_M F_S. \tag{15}$$

Thus, we obtain

$$M_{\text{mess}} = \hat{k}_M \langle S\rangle > \frac{F_S}{\langle S\rangle} = 3 \times 10^5\ \text{GeV} \left(\frac{m_{3/2}}{1\ \text{GeV}}\right) \left(\frac{\Lambda}{1 \times 10^{16}\ \text{GeV}}\right)^{-2}. \tag{16}$$

We will examine in the next section whether we have a consistent parameter region.

3. Upper bound on the messenger scale

We derive an upper bound on the messenger scale from the stability of the vacuum in Eq. (5). From the discussion, we will learn that the messenger scale is almost always lower than the dynamical scale Λ_{dyn}, consistent with the assumption made in the previous section. In order to derive an upper bound on the messenger scale we first consider a region with $M_{\text{mess}} > \Lambda_{\text{dyn}}$. In this case, the quark fields F and $\bar{F}$ can be integrated out without considering the non-perturbative effects. The phenomenon of supersymmetry breaking can be understood in a slightly different way in this regime. Below the scale M_{mess}, the theory matches to the pure supersymmetric $SU(5)_H$ gauge theory. Eventually at a scale Λ_{eff}, the superpotential acquires a contribution from the gaugino condensation, $W \ni \Lambda_{\text{eff}}^3$. Now, by a matching condition of the gauge coupling constant at the scale M_{mess}, we can see that this term has a dependence on the field value of S: $\Lambda_{\text{eff}}^3 = M_{\text{mess}} \Lambda_{\text{dyn}}^2 = kS\Lambda_{\text{dyn}}^2$. This is the linear term of S in Eq. (2) which causes supersymmetry breaking by $F_S = m^2 = k\Lambda_{\text{dyn}}^2$ [9].

The quantum corrections to the scalar potential of S can be calculated perturbatively in the picture where F and $\bar{F}$ are elementary fields. It is simply the logarithmic term in Eq. (4) with $N = 25$. A condition to have a local minimum in the scalar potential (4) is

$$\frac{1}{3M_{\text{Pl}}^2} - \frac{4}{\Lambda^2} \frac{k^2 N}{(4\pi)^2} > 0, \tag{17}$$

from which the bound on k is obtained to be

$$k < 3 \times 10^{-3} \left(\frac{N}{25}\right)^{-1/2} \left(\frac{\Lambda}{1 \times 10^{16}\ \text{GeV}}\right). \tag{18}$$

Therefore, with the VEV of S in Eq. (5), we obtain the upper bound on the messenger scale to be

$$M_{\text{mess}} < 4 \times 10^{10}\ \text{GeV} \left(\frac{N}{25}\right)^{-1/2} \left(\frac{\Lambda}{1 \times 10^{16}\ \text{GeV}}\right)^3. \tag{19}$$

On the other hand, the dynamical scale Λ_{dyn} has a relation to the m^2 parameter:

$$m^2 = k\Lambda_{\text{dyn}}^2 = \sqrt{3} m_{3/2} M_{\text{Pl}}. \tag{20}$$

From this, we obtain

$$\Lambda_{\text{dyn}} = 4 \times 10^{10}\ \text{GeV} \left(\frac{k}{3 \times 10^{-3}}\right)^{-1/2} \left(\frac{m_{3/2}}{1\ \text{GeV}}\right)^{1/2}. \tag{21}$$

From Eqs. (18), (19) and (21), we conclude that the messenger scale is lower than the dynamical scale unless the bound in Eq. (18) is saturated. Note that we cannot go far from the sweet spot values of $m_{3/2}$ and Λ, otherwise the natural solution to the μ-problem is spoiled.

This discussion justifies the assumption $M_{\text{mess}} \lesssim \Lambda_{\text{dyn}}$. Since we have a weakly coupled description of the theory even in the $M_{\text{mess}} \lesssim \Lambda_{\text{dyn}}$ regime in terms of mesons and baryons, we can reliably estimate the quantum correction to the potential [14]. That is simply the logarithmic term in Eq. (4) with k replaced by $\hat{k}_M$ and $\hat{k}_B$. Therefore, Eq. (18) should be understood as a condition for the coupling constants $\hat{k}_M$ and $\hat{k}_B$ rather than for the fundamental coupling constant k. Then, by a relation $\hat{k}_M \sim \hat{k}_B \sim k$, the inequality in Eq. (19) just results in a consistency condition: $M_{\text{mess}} \lesssim \Lambda_{\text{dyn}}$. The bound in Eq. (18) (barring O(1) ambiguities in the relation between k and $\hat{k}_M$) is identical to the previously obtained constraint in Eq. (14) which ensures the stability of the potential against uncalculable corrections from the strong dynamics.

In summary, we have obtained a consistent region

$$10^5\ \text{GeV} \lesssim M_{\text{mess}} \lesssim 10^{10}\ \text{GeV}, \tag{22}$$

for the messenger scale, where the hadron picture is appropriate for the analysis. Note, however, that this prediction is generally true in any models of the sweet spot supersymmetry. The only non-trivial prediction of this model is $N_{\text{mess}} = 5$. In the next section, we examine the same class of models with different strong gauge groups. We find those models predict different values of N_{mess}.

4. Sp(N_c) and SO(N_c) models

The mechanism of supersymmetry breaking and its mediation works also in Sp(N_c) and SO(N_c) gauge theories instead of SU(5)$_H$. In order for the matching condition, $\Lambda_{\rm eff}^3 = M_{\rm mess}\Lambda_{\rm dyn}^2$, to hold, gauge groups are determined to be Sp(4) or SO(12).

The discussion is almost the same for the Sp(4) case. We introduce F and $\bar{F}$, that are $(8,5)$ and $(8,\bar{5})$ under Sp(4)$_H \times$ SU(5)$_F$ group. Again, the Standard Model gauge group is a subgroup of the SU(5)$_F$ global symmetry. We assume an interaction term, $W \ni kS(F\bar{F})$, where both Sp(4)$_H$ and SU(5)$_F$ indices are appropriately contracted in $(F\bar{F})$. Below the dynamical scale of the Sp(4)$_H$ gauge theory, $\Lambda_{\rm dyn}$, the theory is described by meson fields, M. The superpotential is

$$W = kS\cdot {\rm Tr}\, M_{F\bar{F}} + X\big({\rm Pf}\, M - \big(\Lambda_{\rm dyn}^2/5\big)^5\big), \tag{23}$$

with X a Lagrange multiplier [15]. The meson field M is a 10×10 antisymmetric matrix:

$$M = \begin{pmatrix} M_{FF} & M_{F\bar{F}} \\ -M_{F\bar{F}}^T & M_{\bar{F}\bar{F}} \end{pmatrix}. \tag{24}$$

The submatrices M_{FF}, $M_{\bar{F}\bar{F}}$, and $M_{F\bar{F}}$ transform as 10, $\overline{10}$, and $1+24$ under the SU(5)$_F$ flavor group, respectively. By solving the constraint and canonically normalizing the fields, we obtain

$$W_{\rm eff} = k\Lambda_{\rm dyn}^2 S + S\left(\hat{k}_{FF}\,{\rm Tr}(\hat{M}_{FF}\hat{M}_{\bar{F}\bar{F}}) + \frac{\hat{k}_{F\bar{F}}}{2}\,{\rm Tr}\,\delta\hat{M}_{F\bar{F}}^2\right). \tag{25}$$

The effective number of messengers are $N_{\rm mess}=8$ in this case.

The case with an SO(12) gauge group is essentially the same, yet a little bit more complicated. The quarks F and $\bar{F}$ transform as $(12,5)$ and $(12,\bar{5})$ this time. Below the dynamical scale, the effective theory is a U(1) gauge theory with superpotential:

$$W = kS\cdot {\rm Tr}\, M_{F\bar{F}} + \big(\det M - \big(\Lambda_{\rm dyn}^2/5\big)^{10}\big)E^+E^-, \tag{26}$$

near a point $\det M = (\Lambda_{\rm dyn}^2/5)^{10}$. The fields $E^\pm$ are dyons [16]. The meson field M is a 10×10 symmetric matrix:

$$M = \begin{pmatrix} M_{FF} & M_{F\bar{F}} \\ M_{F\bar{F}}^T & M_{\bar{F}\bar{F}} \end{pmatrix}. \tag{27}$$

The submatrices M_{FF}, $M_{\bar{F}\bar{F}}$, and $M_{F\bar{F}}$ transform as 15, $\overline{15}$, and $1+24$ under the SU(5)$_F$ flavor group, respectively. By turning on the VEV of S, the minimum of the potential is at

$$\langle M_{F\bar{F}}\rangle = \big(\Lambda_{\rm dyn}^2/5\big)\delta_{ij}, \tag{28}$$

$$\langle E^+E^-\rangle = -\frac{kS}{2(\Lambda_{\rm dyn}^2/5)^9}. \tag{29}$$

In this vacuum, the U(1) symmetry is Higgsed at the scale $(k\langle S\rangle\Lambda_{\rm dyn})^{1/2}$ and the trace part of $M_{F\bar{F}}$ and $E^\pm$ obtain masses (or eaten by the U(1) gauge field) and decouple. The effects of those massive particles on the S potential are always smaller than the term $K \ni -(S^\dagger S)^2/\Lambda^2$ for $M_{\rm mess} < \Lambda_{\rm dyn}$. Below the scale $(M_{\rm mess}\Lambda_{\rm dyn})^{1/2}$, the effective superpotential is

$$W_{\rm eff} = k\Lambda_{\rm dyn}^2 S + S\left(\hat{k}_{FF}\,{\rm Tr}(\hat{M}_{FF}\hat{M}_{\bar{F}\bar{F}}) + \frac{\hat{k}_{F\bar{F}}}{2}\,{\rm Tr}\,\delta\hat{M}_{F\bar{F}}^2\right). \tag{30}$$

Here, we canonically normalized fields. Again, this is the superpotential of the sweet spot supersymmetry. As anticipated, the effective number of messengers is $N_{\rm mess}=12$ in this SO(12) model.

In fact, there is another branch in the SO(12) model where the superpotential is given by

$$W = kS\cdot {\rm Tr}\, M_{F\bar{F}} + M_{ij}q_i^+q_j^-, \tag{31}$$

where q's are monopoles. There is no supersymmetry breaking vacuum in this branch. This is consistent with the fact that gaugino condensation cancels and $W_{\rm eff}=0$ in this case.

Although these models are similar to the IYIT model of supersymmetry breaking [17], there are essential differences. In the IYIT model, we need to introduce gauge singlet fields for each flat direction in order to kill all the supersymmetric vacuum. In the model presented in this Letter, we introduced only one singlet field S. Therefore, there is a supersymmetric vacuum at $S=0$ since we do not fix all the flat directions. However, by the help of an external dynamics, i.e., supergravity interactions, S can be stabilized away from the supersymmetric vacuum.

Acknowledgements

The work of M.I. was supported by the US Department of Energy under contract number DE-AC02-76SF00515.

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