Analytical Solution for Predicting In-plane Elastic Shear Properties of 2D Orthogonal PWF Composites

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Abstract

This paper proposes a new analytical solution to predict the shear modulus of a two-dimensional (2D) plain weave fabric (PWF) composite accounting for the interaction of orthogonal interlacing strands with coupled shear deformation modes including not only relative bending but also torsion, etc. The two orthogonal yarns in a micromechanical unit cell are idealized as curved beams with a path depicted by using sinusoidal shape functions. The internal forces and macroscopic deformations carried by the yarn families, together with macroscopic shear modulus of PWFs are derived by means of a strain energy approach founded on micromechanics. Three sets of experimental data pertinent to three kinds of 2D orthogonal PWF composites have been implemented to validate the new model. The calculations from the new model are also compared with those by using two models in the earlier literature. It is shown that the experimental results correlate well with predictions from the new model.

Keywords: textile composites; mechanical properties; shear modulus properties; plain weave fabric

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>a, a₁</td>
<td>Width of strand cross-section</td>
</tr>
<tr>
<td>b, b₁</td>
<td>Height of strand cross-section</td>
</tr>
<tr>
<td>A</td>
<td>Cross-sectional area of strand</td>
</tr>
<tr>
<td>A₁, A₂</td>
<td>Cross-sectional area of warp and weft strand</td>
</tr>
<tr>
<td>d</td>
<td>Ply thickness of textile composites</td>
</tr>
<tr>
<td>E₁</td>
<td>Elastic modulus of fibre strand in longitudinal direction</td>
</tr>
<tr>
<td>E₂</td>
<td>Elastic modulus of fibre strand in transverse direction</td>
</tr>
<tr>
<td>E_f</td>
<td>Elastic modulus of fibre</td>
</tr>
<tr>
<td>E_m</td>
<td>Elastic modulus of resin</td>
</tr>
<tr>
<td>g₁</td>
<td>Warp interstrand gap</td>
</tr>
<tr>
<td>g₂</td>
<td>Weft interstrand gap</td>
</tr>
<tr>
<td>G₁₁</td>
<td>In-plane shear modulus of fibre strand</td>
</tr>
<tr>
<td>G_c</td>
<td>In-plane shear modulus of PWF composites</td>
</tr>
<tr>
<td>G₁₂</td>
<td>In-plane shear modulus of fibre</td>
</tr>
<tr>
<td>G_m</td>
<td>In-plane shear modulus of resin</td>
</tr>
<tr>
<td>G_t</td>
<td>In-plane shear modulus of PWF</td>
</tr>
<tr>
<td>h₁</td>
<td>Undulated height of the warp strand</td>
</tr>
<tr>
<td>h₂</td>
<td>Undulated height of the weft strand</td>
</tr>
<tr>
<td>I</td>
<td>Inertia moment of fibre strand</td>
</tr>
<tr>
<td>I₁</td>
<td>Inertia moment of warp strand</td>
</tr>
<tr>
<td>I₂</td>
<td>Inertia moment of weft strand</td>
</tr>
<tr>
<td>l_p</td>
<td>Polar moment of inertia of strand</td>
</tr>
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1. Introduction

A precondition for application of textile composites in engineering structures (especially in primary aerospace structures) is the need to determine mechanical properties through either large number of experimental databases of engineering properties or suitable predictive models. A large body of research exists to characterize tensile [1,2], compression [3,4] and interlaminar shear [5,6] behaviour of plain weave fabric (PWF) composites. Recently, with potential for the wider applications of 2D PWF composite structures prepared by resin transfer moulding (RTM), there is growing interest in assessing the integrated and comprehensive mechanical properties and failure mechanisms for textile composites tested in uniaxial or biaxial tension, compression, three-point flexure, short-beam shear and in-plane shear [7-11]. Various analytical techniques have been developed to predict the elastic properties of representative unit cells (RUCs) of textile composites including 2D and 3D weave composites. Comprehensive reviews of the subject have been conducted by Bogdanovich [12], Whitcomb [13] and Potluri [14], et al. Some attempts to model the PWF are to homogenize the behavior of the underlying mesostructure and to approximate the fabric as an anisotropic planar continuum with two preferred material directions [15-19]. Though continuum models allow greater computational efficiency, the identification of appropriate homogenized material parameters can be a formidable challenge and rely both on empirical testing and on detailed finite element modeling without accounting for the effect of interactions between the yarn families. A large number of mesostructurally curved beam-based analytical models [5,6,20-23] have been developed to predict the mechanical response of the fabric and its component yarns in specific modes of deformation based on elastic beam theory with coupled yarn extension, bending and torsion effects. Mesostructural models can be used to quantify homogenized material properties for use in continuum models. Presently, a significant amount of research has been conducted by using the variational principles of strain energy [13,24-28] to analyze elastic properties of woven composites and detailed stress field throughout the RUC from the averaged mechanical properties of the constituent materials. In order to produce finite element (FE) models, straight inclined segments or continuous mathematical functions have been used to depict in more detail the idealized PWF geometry and to represent the yarn path and cross-sectional shape. One drawback of FE models though is their intensity and complexity due to the complexity of PWFs geometry. From the previous reviews, it is obvious that the analytical models and techniques to predict mechanical properties of textile composites have received much interest over the last several decades. However, it is also clear that there is a need for a more practical and expedient model for structural applications, particularly in the aerospace field.

In this paper an attempt is made to develop a novel model to predict the elastic shear property of PWF composites by using the minimum complementary energy principle based on apt geometrical approximation and assumptions for PWF composites through accounting for the interaction of orthogonal interlacing strands with coupled shear deformation modes including not only relative bending but also torsion. A new analytical solution for shear modulus calculation is established. The results from the model are compared with experimental observations and with formulations using the existing theories/models.
2. Modified Curved Beam Model Considering Interaction of Orthogonal Interlacing Strands

A biaxial PWF composite consists of stacked, pre-impregnated layers of woven fabric, which are cured and consolidated by a process similar to tape laminates. The fabric is composed of two sets of interlacing, mutually orthogonal (warp and weft) yarns in a regular sequence of one under and one over. Each yarn is a bundle of filaments (or fibers) and the yarn size is measured by the number of filaments in the yarn. The fabric is woven on a loom and its architecture is characterized by the interlacing pattern of the warp and weft yarns. The periodicity of the repeating pattern in a woven (or braided) fabric can be used to isolate a small RUC which is sufficient to describe the fabric architecture. The RUC in yarn interlacing pattern for a PWF is indicated in Fig. 1 by dark borders. As proposed by Naik, et al. [21], the fabric geometry should be so chosen that it should give the best possible properties for the application under consideration. The RUC for 2D PWF is shown in Fig. 2, which shows the sectional views of undulating warp and weft yarns as a yarn crosses over and under the other one. The undulating yarns are idealized as curved beams with a path depicted by using sinusoidal shape functions. As shown in Fig. 2, the suffix ‘1’ denotes the warp yarn or warp direction and the suffix ‘2’ for the weft yarn or weft direction.

![Fig. 1](image1.jpg) RUC in yarn interlacing pattern for a PWF [21].

![Fig. 2](image2.jpg) An idealized 2D orthogonal PWF lamina geometrical RUC [21].

The interlacement of two orthogonal yarns leads to a crimp or waviness in woven fabrics in the two orthogonal directions, causing a significant influence on the moduli and strength of PWF composites. The strand crimp and undulated length as well as the gap between adjacent strands can be determined from the strand cross-sectional geometry and fabric count (or the number of strands per unit length along the warp or weft direction) either by photomicrographs or by mathematical shape functions. The overall fibre volume fraction (alternatively, the ratio of fibre volume within a unit cell to the volume of the unit cell) can then be obtained. In addition to the above influencing factors, the stacking orientation angle of individual layers and relative shifting of strands within the adjacent layers in warp and/or weft directions also affect the mechanical properties of PWF composites. Consequently, the focus of this work is placed on an idealized 2D orthogonal PWF composites formed by stacking PWF layers one over the other without relative orientation angle and shifting. The individual (impregnated) yarns of a PWF composite can be treated as a transversely isotropic material, resulting in the specification of only four elastic constants, i.e. $E_{11}$, $E_{22}$, $\mu_{12}$, and $G_{12}$ with reference to an orthogonal material coordinate system. The elastic modulus of a 2D orthogonal PWF can be derived from that of the fibre by means of a strain energy approach. Then elastic modulus of PWF composites can be predicted from those of PWF and pure resin based on the rule of mixture formulations.

From the idealized curved beam unit cell (see Fig. 2) with an idealized cross-section (see Fig. 3) of a typical warp or weft yarn, it is possible to have five variables in the warp and weft yarns as the curved beam width ($a_1$ and $a_2$), the curved beam cross-section height ($b_1$ and $b_2$), the curved beam length ($2a_1 + 2g_2$ and $2a_1 + 2g_1$), the interstrand gap ($g_1$ and $g_2$) and sinusoidal crimp amplitude ($h_1/2$ and $h_2/2$). All geometric parameters e.g., $a$ and $b$, were determined by using optical microscope. For PWF, $h_1 = b_2, h_2 = b_1$. In general, the PWFs are tightly stacked in thickness direction, thus all layers can be considered to have the same shear strain, and the parallel model, rather than the conventional series model, can be used in rule of mixtures to calculate $G_{12}$. It is worth noting that in the above calculations, all material properties can be obtained either from material manuals, or by calculating using rule of mixture, and the volume fractions of fibres may be determined from manufacturing process.

From the idealized curved beam unit cell (see Fig. 2) with an idealized cross-section (see Fig. 3) of a typical warp or weft yarn, it is possible to have five variables in the warp and weft yarns as the curved beam width ($a_1$ and $a_2$), the curved beam cross-section height ($b_1$ and $b_2$), the curved beam length ($2a_1 + 2g_2$ and $2a_1 + 2g_1$), the interstrand gap ($g_1$ and $g_2$) and sinusoidal crimp amplitude ($h_1/2$ and $h_2/2$). All geometric parameters e.g., $a$ and $b$, were determined by using optical microscope. For PWF, $h_1 = b_2, h_2 = b_1$. In general, the PWFs are tightly stacked in thickness direction. From the engineering viewpoint, the gap between $b_1$ and $b_2$ is much small and neglectable, therefore the ply thickness $H = h_1 + b_1 = b_1 + b_2$. Letting $L_1 = 2a_2 + 2g_2, L_2 = 2a_1 + 2g_1$, the sinusoidal crimp paths for warp and weft yarns can be then expressed respectively as

$$z_1(x) = \frac{h_1}{2} \sin \frac{2\pi x}{L_1} \quad (1)$$
$$z_2(y) = \frac{h_2}{2} \sin \frac{2\pi y}{L_2} \quad (2)$$
Because the fabric of textile composites is fully filled with resin, the idealized curved beam unit cell with an idealized cross-section of a typical warp and weft yarns filled with resin can have a further approximation (see Fig. 3). In order to easily calculate the area and axial inertial moment of idealized cross-section of a typical warp and weft yarns shown in Fig. 3, a simplified cross-section (see Fig. 4) for warp yarn is implemented to determine the area and inertia moment as

\[ A = \frac{1}{4} \pi b^2 + b(a-b) \]  
(3)

\[ I_x = \frac{1}{64} \pi b^4 + \frac{1}{12} b(a-b)^2 \]  
(4)

\[ I_y = \frac{1}{64} \pi b^4 + \frac{1}{12} b(a-b)^2 + \frac{1}{16} \pi b^2(a-b)^2 \]  
(5)

Fig. 3 An idealized cross-section of a typical warp and weft yarns for 2D orthogonal PWF.

Fig. 4 An approximate cross-section of a typical warp and weft yarns filled with resin for matt weave fabric.

In the case of a shear loading (see Fig. 5), the relative bending and torsion occur between two orthogonal yarns in the longitudinal and transverse directions. In order to simplify the model, the interaction between two orthogonal interlacing strands within 2D orthogonal PWF composites is considered as a simple contact problem and the interfacial effects are ignored. Assuming external plain shear loading \( N_s \), with interaction internal force and constraint moments of warp and weft yarns being \( N_s, N_t, N_s, N_t, M_1, M_2, T_1 \) and \( T_2 \) respectively (see Fig. 6), it is possible to show that the resultant forces and moments at any cross-section for warp and weft yarns are respectively

\[ N_s = N_s - 2N_s \sin(\theta/2) \]  
(6)

\[ N_t = 2N_s \sin(\theta/2) \]  
(7)

\[ M_1 = N_s a_1 \cos(\theta/2) \]  
(8)

\[ M_2 = N_s a_2 \cos(\theta/2) \]  
(9)

\[ T_1 = N_s a_1 \]  
(10)

\[ T_2 = N_s a_2 \]  
(11)

where \( \theta \) is the included angle between the warp and weft yarns. In the case of orthogonal yarns in the longitudinal and transverse directions, \( \theta \) is equivalent to 90°, and the resultant bending and torsion moments for warp yarn then respectively become

\[ X_{s1} = \frac{h_i}{4} \left( \cos \theta \frac{a_1 + g_2}{L_1} - \cos \frac{2\pi x}{L_1} \right) \]  
(12)

\[ X_{s2} = \frac{a_1}{2} \left( \frac{\pi h_i^2}{2L_i^2} \sin \frac{2\pi x}{L_1} \right) \]  
(13)

\[ X_{s3} = \frac{a_1}{2} \left( \frac{\pi h_i^2}{2L_i^2} \sin \frac{2\pi x}{L_1} \right) \]  
(14)

\[ X_{s1} = \frac{h_i}{4} \left( \cos \frac{\pi a_1 + g_2}{L_1} - \cos \frac{2\pi x}{L_1} \right) \]  
(15)

\[ X_{s2} = \frac{a_1}{2} \left( \frac{\pi h_i^2}{2L_i^2} \sin \frac{2\pi x}{L_1} \right) \]  
(16)

\[ X_{s3} = \frac{a_1}{2} \left( \frac{\pi h_i^2}{2L_i^2} \sin \frac{2\pi x}{L_1} \right) \]  
(17)

where

\[ X_{s1} = \frac{h_i}{4} \left( \cos \frac{\pi a_1 + g_2}{L_1} - \cos \frac{2\pi x}{L_1} \right) \]  
(18)

\[ X_{s2} = \frac{a_1}{2} \left( \frac{\pi h_i^2}{2L_i^2} \sin \frac{2\pi x}{L_1} \right) \]  
(19)
\[ X_{11} = \frac{h}{4} \left[ \cos \left( \frac{\alpha_1 + g_2}{L_1} \right) - \cos \left( \frac{\alpha_1 + 3g_2}{L_1} \right) \right] \left( 1 - \frac{\pi^2 k_2^2}{2L_1^2} \sin^2 \left( \frac{2\pi x}{L_1} \right) \right) \]

\[ X_{41} = \frac{h}{4} \left[ \cos \left( \frac{\alpha_1 + g_2}{L_1} \right) - \cos \left( \frac{\alpha_1 + 3g_2}{L_1} \right) \right] \left( 1 - \frac{\pi^2 k_2^2}{2L_1^2} \sin^2 \left( \frac{2\pi x}{L_1} \right) \right) \]

\[ \frac{\pi h}{L_1} \left( \frac{2\pi x}{L_1} \right) \sin^2 \left( \frac{2\pi x}{L_1} \right) \left( 1 - \frac{\pi^2 k_2^2}{2L_1^2} \sin^2 \left( \frac{2\pi x}{L_1} \right) \right) \]

Fig. 5 External shearing force and deformation for PWF.

Using the analogy of Eqs. (12)-(13), it is possible to have the resultant bending and torsion moments for weft yarn as

\[ M_{x2} (x) = \begin{cases} 
- (N_1 - N_2) X_{22} & 0 \leq x < (a_1 + g_1)/2 \\
- (N_1 - N_2) X_{22} + 2N_s X_{22} - 2N_s X_{s2} & (a_1 + g_1)/2 \leq x < (a_1 + 3g_1)/2 \\
- (N_1 - N_2) X_{22} + 4N_s X_{22} - 2N_s X_{s2} & (a_1 + 3g_1)/2 \leq x < L_2/2 
\end{cases} \]

\[ T_{x2} (x) = \begin{cases} 
(N_1 - N_3) X_{x12} & 0 \leq x < (a_1 + g_1)/2 \\
(N_1 - N_3) X_{x12} - 2N_s X_{x12} + 2N_s X_{x2} & (a_1 + g_1)/2 \leq x < (a_1 + 3g_1)/2 \\
(N_1 - N_3) X_{x12} - 4N_s X_{x12} + 2N_s X_{x2} & (a_1 + 3g_1)/2 \leq x < L_2/2 
\end{cases} \]

where the transformation variables \( X_{12} \) to \( X_{82} \) can be determined by analogy aid of Eqs. (14)-(21).

Due to the symmetry of the warp yarn, with the curved beam subject to the symmetric external loading and internal forces, from Eqs. (12)-(21), one has the complementary strain energy of the warp yarn within a RUC as

\[ U_1^* = \frac{1}{G_{12} I_{pl}} \int_0^{l_{1/2}} T_{x2}^2 (x) dx + \frac{1}{E_{11} I_{sl}} \int_0^{l_{1/2}} M_{x1}^2 (x) dx \]

Fig. 6 Forces of warp and weft yarns for weave fabric.

Using the analogy of Eqs. (12)-(13), it is possible to have the resultant bending and torsion moments for weft yarn as

\[ M_{x2} (x) = \begin{cases} 
- (N_1 - N_2) X_{22} & 0 \leq x < (a_1 + g_1)/2 \\
- (N_1 - N_2) X_{22} + 2N_s X_{22} - 2N_s X_{s2} & (a_1 + g_1)/2 \leq x < (a_1 + 3g_1)/2 \\
- (N_1 - N_2) X_{22} + 4N_s X_{22} - 2N_s X_{s2} & (a_1 + 3g_1)/2 \leq x < L_2/2 \end{cases} \]

\[ T_{x2} (x) = \begin{cases} 
(N_1 - N_3) X_{x12} & 0 \leq x < (a_1 + g_1)/2 \\
(N_1 - N_3) X_{x12} - 2N_s X_{x12} + 2N_s X_{x2} & (a_1 + g_1)/2 \leq x < (a_1 + 3g_1)/2 \\
(N_1 - N_3) X_{x12} - 4N_s X_{x12} + 2N_s X_{x2} & (a_1 + 3g_1)/2 \leq x < L_2/2 \end{cases} \]

Where
Using the well-known rule of mixtures, $E_1$ and $G_{12}$ are determined as

$$E_1 = E_f V_{f1} + E_m (1 - V_{f1})$$

(27)

$$G_{12} = G_f V_{f1} + G_m (1 - V_{f1})$$

(28)

Letting $X_{u1} = \sqrt{1 + \frac{h_1^2 \pi^2}{E_1} \sin^2 \left( \frac{2\pi x}{L_1} \right)}$, then Eq. (24) becomes

$$U_1' = \left( N_1 - N_1 \right)^2 J_{11} + 4 N_2^2 J_{21} + 4 N_1 N_3 J_{31} + 4 (N_1 - N_3) N_3 J_{41} + 4 (N_1 - N_3) N_3 J_{51} + 8 N_1 N_2 J_{61}$$

(29)

where

$$J_{11} = \frac{1}{G_{12} I_{pl}} \int_0^{L_1} X_{11} X_{u1} \, dx + \frac{1}{E_1 I_{pl}} \int_0^{L_1} X_{21}^2 X_{u1} \, dx$$

(30)

$$J_{21} = \frac{1}{G_{12} I_{pl}} \left( \int_{(a_1 + \beta_1)/2}^{(a_1 + \beta_1)/2} X_{21} X_{u1} \, dx \right)$$

(31)

$$J_{31} = \frac{1}{G_{12} I_{pl}} \left( \int_{(a_1 + \beta_1)/2}^{(a_1 + \beta_1)/2} X_{31}^2 X_{u1} \, dx \right)$$

(32)

$$J_{41} = \frac{1}{G_{12} I_{pl}} \left( \int_{(a_1 + \beta_1)/2}^{(a_1 + \beta_1)/2} X_{41}^2 X_{u1} \, dx \right)$$

$$J_{51} = \frac{1}{G_{12} I_{pl}} \left( \int_{(a_1 + \beta_1)/2}^{(a_1 + \beta_1)/2} X_{51}^2 X_{u1} \, dx \right)$$

$$J_{61} = \frac{1}{G_{12} I_{pl}} \left( \int_{(a_1 + \beta_1)/2}^{(a_1 + \beta_1)/2} X_{61}^2 X_{u1} \, dx \right)$$

(33)

Again, using Eq. (29) as an analogy, it is possible to have the complementary strain energy $U_2'$ of the weft yarn within a RUC from Eqs. (22)-(23). And the total complementary energy of the RUC can thus be obtained:

$$\Pi^* = U_1'^* + U_2'^* = \left( N_1 - N_3 \right)^2 J_1 + 4 N_2^2 J_2 + 4 N_1 J_3 + 4 (N_1 - N_3) N_3 J_4 + 4 (N_1 - N_3) N_3 J_5 + 8 N_1 N_2 J_6$$

(34)

where

$$J_k = J_{k1} + J_{k2} \quad (k = 1, 2, \ldots, 6)$$

(35)

By minimizing the total complementary energy function of the unit cell, one has

$$\frac{\partial \Pi^*}{\partial N_2} = 8 N_2 J_2 + 4 (N_1 - N_3) J_4 + 8 N_1 J_6 = 0$$

(36)

$$\frac{\partial \Pi^*}{\partial N_4} = -2 (N_1 - N_3) J_1 + 8 N_2 J_3 - 4 N_1 J_4 + 4 N_1 J_5 - 8 N_2 J_6 + 8 N_1 J_6 = 0$$

(37)

Solving Eqs. (38)-(39) gives the solutions of $N_2$ and $N_3$:

$$N_2 = \frac{-J_4 - \left( 2J_4 - J_4 \right)}{2J_2}$$

(38)

$$N_3 = \frac{-2J_4 + J_4 + J_4 (2J_4 - J_4) / J_2}{J_4 + 4 J_5 - 4 J_4 - \left( 2J_6 - J_6 \right) J_4 / J_2}$$

(39)
Based on the potential energy principle, the relative shifts $A_1$ and $A_2$ at both ends of orthogonal yarns under the external shear force $N_1$, internal forces $N_2$ and $N_3$ can be deduced.

$$\Delta_1 = \frac{\partial U^*}{\partial (N_1 - N_3)} = \frac{1}{G_1 J_{11}} \int_0^L T(x) \frac{\partial T(x)}{\partial (N_1 - N_3)} \, dx + \frac{1}{E J_{12}} \int_0^L M_2(x) \frac{\partial M_2(x)}{\partial (N_1 - N_3)} \, dx = \frac{(2J_{11} + 4D_2 J_{41} + 4D_4 J_{51}) (N_1 - N_3)}{L_2}$$

$$\Delta_2 = \frac{\partial U^*}{\partial (N_1 - N_3)} = \frac{1}{G_1 J_{12}} \int_0^L T(x) \frac{\partial T(x)}{\partial (N_1 - N_3)} \, dx + \frac{1}{E J_{12}} \int_0^L M_2(x) \frac{\partial M_2(x)}{\partial (N_1 - N_3)} \, dx = \frac{(2J_{12} + 4D_2 J_{42} + 4D_4 J_{52}) (N_1 - N_3)}{L_2}$$

where intermediate variables $D_1$ and $D_3$ are

$$D_1 = \frac{-2J_1 + J_2 (2J_6 - J_4)}{J_1 + 4J_1 - 4J_3 - 2J_6 - J_4}$$

$$D_3 = \frac{-J_4 - (2J_6 - J_4) D_1}{(2J_4)}$$

The torsion angle between orthogonal yarns under the external loading $N_1$ can thus be obtained:

$$\gamma = \frac{A_1}{L_1} + \frac{A_2}{L_2} = \frac{2J_{11} + 4D_2 J_{41} + 4D_4 J_{51}}{L_2} + \frac{2J_{12} + 4D_2 J_{42} + 4D_4 J_{52}}{L_2} (N_1 - N_3)$$

Therefore, the shearing modulus of 2D orthogonal PWF can be deduced as

$$G_s = \frac{(N_1 - N_3) / (A \gamma)}{L_1 L_2}$$

$$\{A(2J_{11} + 4D_2 J_{41} + 4D_4 J_{51}) L_2 + (2J_{12} + 4D_2 J_{42} + 4D_4 J_{52}) L_1 \}$$

Again, using the well-known rule of mixtures, the shearing modulus $G_s$ of PWF composites can then be obtained from the shear moduli $G_i$ and $G_m$ of PWF and resin.

$$G_s = G_m (1 - V_m) + G_m (1 - V_w)$$

As mentioned above, it is clear that the modified model presented in this paper considers the interaction forces between the warp and weft yarns. Fewer geometry characteristic parameters of warp and weft yarns and the elastic moduli of fibre and resin are required to predict shearing modulus of PWF composites than with other models given in literature (Raun and Chou, 1995 [17] Nguyen, et al., 1999 [5]. Page and Wang, 2000 [28], Xue, et al, 2003 [18], King, et al., 2005 [19], Sun and Pan, 2005 [20], and Zhu, et al., 2007 [11]).

3. Experimental Validation and Comparison Between Predictions and Experiments

There are three sets of experiments to verify the validation of modified model, with two sets of data from Ref. [5] and one set from new experimental data generated in this work. In Ref. [5], two sets of experimental moduli of PW(1K) and PW(3K) are 2.5 GPa and 1.4 GPa respectively. In this work, 2D orthogonal EW220/5284 PWF composites are employed to obtain the new experimental results to validate the modified model. The fabric specifications and mechanical properties of the above three kinds of textile composites are listed in Table 1. One type of composites panel specimens were fabricated for use in the shear property tests of 2D orthogonal EW220/5284 PWF composites. Overall, 5 specimens were tested (see Table 2). The geometry and dimensions of the specimen are shown in Fig. 7. All specimens were preformed using 2D orthogonal EW220 glass fibre PWFs, infused with 5284 epoxy resin, and then consolidated by RTM in a closed steel mould. During the consolidating process, the composite performs were first heated up to 160 °C, at a rate of 5 °C/min and at which temperature the specimen was held for 1 h; subsequently, it was heated up, again at the same rate, to the desired processing temperature of 180 °C with the specimen being held at this temperature for 2 h. Next, the mould was cooled to room temperature and the consolidation was carried out.

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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_p$ ($10^{-4}$ m$^4$)</td>
<td>224</td>
<td>224</td>
<td>414</td>
</tr>
<tr>
<td>$I_L$ ($10^{-5}$ m$^4$)</td>
<td>112</td>
<td>112</td>
<td>207</td>
</tr>
<tr>
<td>$a$ (mm)</td>
<td>1.20</td>
<td>1.16</td>
<td>1.51</td>
</tr>
<tr>
<td>$h$ (mm)</td>
<td>0.08</td>
<td>0.08</td>
<td>0.11</td>
</tr>
<tr>
<td>$g$ (mm)</td>
<td>0.08</td>
<td>0.08</td>
<td>0.11</td>
</tr>
<tr>
<td>$L$ (mm)</td>
<td>3.15</td>
<td>3.04</td>
<td>4.26</td>
</tr>
<tr>
<td>$d$ (mm)</td>
<td>0.16</td>
<td>0.22</td>
<td>0.17</td>
</tr>
<tr>
<td>$A/mm^2$</td>
<td>0.118</td>
<td>0.16</td>
<td>0.087</td>
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</tbody>
</table>

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>$E_I$ (GPa)</td>
<td>5.5</td>
<td>5.5</td>
<td>8.4</td>
</tr>
<tr>
<td>$E_Y$ (GPa)</td>
<td>22.9</td>
<td>22.9</td>
<td>22.9</td>
</tr>
<tr>
<td>$G_{12}$ (GPa)</td>
<td>0.34</td>
<td>0.34</td>
<td>0.24</td>
</tr>
<tr>
<td>$E_m$ (GPa)</td>
<td>3.2</td>
<td>3.2</td>
<td>3.2</td>
</tr>
<tr>
<td>$G_m$ (GPa)</td>
<td>1.1</td>
<td>1.1</td>
<td>1.1</td>
</tr>
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</table>

$\mu_s$ = 0.42
Table 2  Experiments for EW220/5284 PWF composites

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Shearing modulus/GPa</th>
<th>Shearing strength/MPa</th>
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<tbody>
<tr>
<td>1</td>
<td>6.46</td>
<td>110.91</td>
</tr>
<tr>
<td>2</td>
<td>6.43</td>
<td>110.53</td>
</tr>
<tr>
<td>3</td>
<td>6.19</td>
<td>110.97</td>
</tr>
<tr>
<td>4</td>
<td>6.58</td>
<td>111.42</td>
</tr>
<tr>
<td>5</td>
<td>6.58</td>
<td>111.63</td>
</tr>
<tr>
<td>Mean</td>
<td>6.45</td>
<td>111.09</td>
</tr>
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</table>

Table 3  Comparison between experiments and predictions for shearing moduli

<table>
<thead>
<tr>
<th>PWFs type</th>
<th>Experiment result/GPa</th>
<th>Modified model</th>
<th>Model in Ref [5]</th>
<th>Model in Ref [6]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Prediction result/GPa</td>
<td>Relative deviation/%</td>
<td>Prediction result/GPa</td>
<td>Relative deviation/%</td>
</tr>
<tr>
<td>EW220/5284</td>
<td>6.45</td>
<td>6.81</td>
<td>5.58</td>
<td>3.6</td>
</tr>
<tr>
<td>PW(1K)</td>
<td>2.5 [5]</td>
<td>2.04</td>
<td>22.5</td>
<td>1.9</td>
</tr>
<tr>
<td>PW(3K)</td>
<td>1.4 [5]</td>
<td>1.18</td>
<td>18.6</td>
<td></td>
</tr>
</tbody>
</table>

4. Conclusions

The focus of this paper has been to present a modified micromechanical curved beam model for elastic shear modulus prediction of 2D PWF composites. A new analytical solution for shear modulus calculation of this model is respectively applied to predicting the shear moduli of the above three kinds of textile composites. Predictions of the elastic shear moduli are listed in Table 3. From this it is clear that the calculations from the new model presented in this paper have reasonable agreement with the experiments, while the predicted values for EW220/5284 are much better than those for PW(1K) and PW(3K). A possible reason is that there is a lack of detailed information on some material properties and geometric parameters for PW(1K) and PW(3K) in Ref. [5] and Ref. [6], and one has to implement the approximations of the above material properties and geometric parameters for predicting the shear moduli of PW(1K) and PW(3K) by means of the modified curved beam model in this paper, which leads to greater prediction deviations for PW(1K) and PW(3K) than for EW220/5284.

Table 3 also demonstrates that the prediction from the modified curved beam model presented in this paper correlates better with three sets of experimental data than those using the models in Ref. [5] and Ref. [6]. Actually, the curved beams were employed to simulate fabric yarns in Nguyen and Sun’s models [5-6], however, the yarn torsion resulted from out of plane component of interaction force between warp and weft yarns was neglected and only the in-plane component of interaction force was taken into account in Nguyen’s model [5]; both in-plane and out of plane components of interaction force were simplified to concentrically act at the middle point of beam in Sun’s model [6]. Greater calculation inaccuracy of yarn deformation and shear modulus from Nguyen and Sun’s models [5-6] than from the new curved beam-based model presented in this paper resulted from the neglect and simplification of components of interaction force between warp and weft yarns in Nguyen and Sun’s models [5-6]. Thus it is argued that the new curved beam-based model presented in this paper is a valid and rational basis for shear modulus analysis of PWF composites. Using this model, the shear properties of woven fabrics could be predicted without any additional fabric level experimental investigation, i.e. only the input of basic woven fabric properties is needed to predict the material shear properties of the woven fabrics.
shear modulus analysis of PWF composites.

The modified curved beam model presented in this paper can be extended to predict the shear strength of textile composites. From Eqs. (40)-(41), it is possible to obtain the maximum stress of orthogonal interlacing strands by means of beam theory, and then the shear strength of PWF ply can be determined using the shear rule of fibre. After this, from classic laminate theory, the constitutive relationship of textile composites can be established by using the shear moduli of PWF and pure resin plies, and the internal forces of PWF and pure resin plies can be calculated. Finally, from the shear strengths of PWF and pure resin plies, the failure of textile composites can be evaluated and the shear strength can be assessed.

References


Biography:

Xiong Junjiang is a professor in Beihang University. His main research interest is fatigue and fracture reliability engineering.

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