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Preface

Special issue on mathematical imaging, Part II

The journal *Applied and Computational Harmonic Analysis* (**ACHA**) is committed to chronicling important advances in interdisciplinary research in various areas related to applied and computational harmonic analysis as they occur, and particularly to highlighting research showing the way to cross disciplinary development in both theory and applications. To do so, the journal occasionally publishes special issues, such as this second issue of the "Special Issues on Mathematical Imaging" (SIMI), to compile some central papers on new developments in certain exciting research areas. All papers published in the special issues are rigorously refereed under the same strict guidelines and policy that apply to regular issues of **ACHA**.

Mathematical imaging is currently a very active and exciting research area that has captured the attention and interests of many researchers in several related disciplines. The development has led to many sophisticated mathematical models and theories. The aim of SIMI is to chronicle the ongoing explosion of research work and disseminate the important findings in an organized way to the wide spectrum of **ACHA** readership. Papers in SIMI are in-depth research papers from different areas of mathematical imaging, including computational harmonic analysis, partial differential equations, and numerical linear algebra.

The guest editors of the two issues of SIMI are R. Chan, T. Chan, and Z. Shen. The first issue of SIMI was published in July 2007, in Volume 23/1, pages 1–152. The present publication is the second issue of SIMI. This issue consists of six regular papers, with the first three papers focusing on the application of frames to the mathematics of imaging. While the theory of both wavelet and Gabor frames has been an active research area during last two decades, its application to image processing is relatively recent.

The first paper, by J. Cai, R. Chan, and Z. Shen, is concerned with the fundamental problem of digital image inpainting in image processing. The main objective is to give a complete analysis, including the study of convergence and optimal properties, of the recently introduced iterative algorithm based on tight frames. In particular, it is proved that the limit function of the algorithm minimizes a certain cost functional that balances the approximation to the given data, sparsity, and smoothness, under the framework of convex analysis and optimization theory. To enhance sparsity, the ℓ_1-norm on the tight frame coefficients is minimized, while continuity is promoted by penalizing the distance between the tight frame coefficients and the corresponding canonical frame coefficients. This paper also discusses the relationship of this method with other wavelet-based approaches and illustrates the performance of the proposed algorithm with numerical experiments.

The second paper, contributed by F.J. Herrmann, P. Moghaddam, and C. Stolk, is devoted to the study of seismic image recovery by using curvelet frames. A nonlinear singularity-preserving solution to seismic image recovery with sparseness and continuity constraints is proposed. The method explicitly explores the curvelet transform as a directional frame expansion that allows for a stable recovery of the migration amplitudes from noisy data. The solution is formulated as a nonlinear optimization problem, where sparsity in the curvelet domain and continuity along the imaged reflectors are jointly promoted. Here again, the ℓ_1-norm on the curvelet coefficients is minimized to enhance sparsity, while continuity is promoted by minimizing an anisotropic diffusion norm on the image. The performance of the recovery scheme is evaluated with a time-reversed "wave-equation" migration code on synthetic datasets.

In the third paper, C. Sagiv, N.A. Sochen, and Y.Y. Zeevi introduce and study a certain system that combines wavelet and Gabor transforms for image analysis and synthesis. In particular, Gabor-type wavelets are generated by applying the action of the affine group that allows translations, rotations, and dilations, on top of the translation

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operation in the frequency domain, as well as independent dilations in x and y directions. This approach adopts certain optimality criteria on minimal space feature uncertainty and tightness of the frame tessellating in the combined space. Various properties of the system, including frame bound estimates, are derived, and numerical illustrations are given to support the theory.

In the fourth paper, M. Welk, G. Steidl, and J. Weickert extend their earlier one-dimensional results, on the equivalence between wavelet soft shrinkage and some numerical scheme for solving the TV diffusion equation, to higher dimensions. In particular, they develop certain novel finite difference schemes for solving isotropic and anisotropic diffusion equations in higher dimensions. These schemes are explicit, unconditionally stable, shift invariant, "approximately rotational invariant," and less dissipative than a "standard explicit scheme," but conditionally consistent. They also establish the equivalence of their finite difference schemes to certain single-scale Haar wavelet shrinkage rules. Generalization to multiscale shrinkage rules is also considered in this paper.

The fifth paper is by F. Arandiga, A. Cohen, R. Donat, N. Dyn, and B. Matei. It is the latest in a series of papers by these authors on nonlinear, edge-adapted image representation by using the ENO framework introduced by Harten. The core in Harten's approach is the prediction operator P, which lifts coarse-scale data to its refined version. The operator P equals AR, where R is an interpolant, which reconstructs a continuous function from discrete data; and A is an averaging operator, which projects a continuous function to its fine-scale averages. The paper details how to properly construct an edge-adaptive R for 2-D images. The authors also show that it is possible to obtain optimal approximation for some special classes of images, namely the piecewise C^2 functions with C^2 edges for which curvelets have optimal approximation capabilities. This justifies much of the complexity and nonlinearity that are involved in the ENO scheme.

The final paper, authored by R. Molina, M. Vega, J. Mateos, and A.K. Katsaggelos, proposes a super-resolution approach to processing multispectral low-resolution images. The main application of multispectral images is to allow for better data signature recognition and classification. But since the image resolution is usually low, information on the shape and texture of the objects may be lost. This paper develops a super-resolution image reconstruction algorithm for multispectral images based on Bayesian estimation and posterior approximation. The Bayesian framework incorporates prior knowledge of the expected characteristics of the multispectral images, including information on the unknown parameters in the model in the form of hyperprior distributions. The required probability distribution is estimated via the Kullback–Leibler divergence learning methodology. The algorithm is tested on the set of Landsat ETM+ images.

We express our sincere appreciation to the authors for submitting their very interesting work for publication in this special issue and to the referees for their conscientious reviews that are essential to maintaining the high quality of **ACHA**. We are truly grateful to these referees for their thoughtful and well-prepared reviews, knowing that reading and composing helpful critiques for interdisciplinary research papers require tremendous time and efforts. Special thanks are also due to Charles Chui for his suggestion and encouragement for publishing this series of special issues on mathematical imaging in **ACHA.**

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Guest Editors