Rainbow Thermometry Based on EMD De-Noising and Signal Analysis

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Abstract

A novel Rainbow thermometry method is presented which is based on Empirical Mode Decomposition (EMD) de-noising and signal analysis, and permits one to get simultaneously the temperature and size of droplet under low Signal-to-Noise Ratio (SNR) condition. The noise is removed based on EMD and wavelet threshold method, and the inverse parameters are calculated according to the signal decomposition analysis based on EMD. A experiment is taken, and the measurement result shows the validity of the presented method.

Keywords: Empirical Mode Decomposition; Wavelet threshold; Signal decomposition; Rainbow thermometry; Temperature

1. Main text

Rainbow thermometry, which measures the droplet refractive index (hence temperature) and size by detecting the scattering light pattern at the rainbow region, has been of interest since early 1990s[1]. Compare with the forward scattering light, the rainbow light intensity is more weaker, which is easy to be effected by noise. To solve such problem, a rainbow signal de-noising and analysis method is presented in this paper. The noise is removed by EMD and wavelet threshold method, and the inversed parameters are calculated according to the decomposed structure. A cross section radius and temperature measurement of a water column is done to validate the accuracy of the presented method.
2. Rainbow signal de-noising and analysis

As a complex light scattering phenomena, rainbow signal contents different kinds of light interferences and diffraction. Both the frequencies of the Airy structure and Ripple structure of the rainbow are mainly influenced by the particle size, and the interference pattern positions are almost determined by the particle refractive index. So the two parameters can be inverted by rainbow pattern detection. As the measured rainbow signal is always corrupted by noise, it can be written as follows:

\[ I(t) = I_s(t) + N(t) \]  (1)

Where \( I(t) \) is the measured signal, and \( I_s(t) \) is real rainbow signal with \( N(t) \) representing the noise.

2.1. De-noising based on EMD

The Empirical Mode Decomposition (EMD) is a novel signal decomposition method which is presented in 1998[2]. Using this method, the signals are decomposed into a set of intrinsic components named intrinsic mode function (IMF) and a residual. In normal condition the noise will be firstly decomposed from the measured signal due to its high frequency, and the de-noising process is simply subtracting these noise IMFs from the signal. However, in some low SNR situation, when the measured droplet size is large which makes the frequencies of the interference patterns approach the noise frequency, some decomposed IMFs contain both the rainbow information and the noise, which can be expressed as follows:

\[ I(t) = \sum C_i(t) + \sum C_j(t) + \sum C_k(t) + r_L(t) \quad i=1,...M_1; \quad j=M_1+1,...M_2; \quad k=M_2+1,...L \]  (2)
Where $\sum C_i(t)$ are the IMFs contain noise; $\sum C_j(t)$ are the IMFs contain partial rainbow information and partial noise; $\sum C_k(t)$ is the IMF contain information only, and $r_L(t)$ is the residual. To distinguish such different kinds of IMFs, an autocorrelation analysis is taken, and in order to remove the noise contains in $\sum C_j(t)$, a wavelet threshold method is used.

- Firstly, a reference noise is calculated according to the IMFs which contain noise only. The variance of the reference noise is calculated by Donoho method[3], and then the noise variance of the IMFs which contain partial noise can be estimated by the following formula:

$$\sigma_j = \left( \frac{\rho}{\beta} \right)^{0.5} \sigma_0$$

where $\sigma_j$ is the noise variance of the IMFs which contain partial noise; $\sigma_0$ is the variance of the reference noise. According to Flandrin’s study[4], $\rho$ equals 2.01 and $\beta$ equals 0.719.

- Secondly, with the noise variance calculated in formula(3), the global threshold can also be estimated by Donoho method, and threshold function can be set according to the SCAD method which always used in wavelet threshold setting[5,6]:

$$C_j'(s_j^n) = \begin{cases} 
C_j(s_j^n) \max(0, |C_j(r_j^n)|) - T_j / |C_j(r_j^n)| & |C_j(r_j^n)| < 2 T_j \\
C_j(s_j^n)(\alpha - 1)|C_j(r_j^n)| / [(\alpha - 2)|C_j(r_j^n)|] & 2T_j < |C_j(r_j^n)| \leq \alpha T_j \\
C_j(s_j^n) & |C_j(r_j^n)| > \alpha T_j 
\end{cases}$$  \hspace{1cm} (4)

where $r_j^n$ is the extreme point of the nth small region bordered by two adjacent zero value in the jth IMF, with $s_j^n$ is the any sample points in the nth small region.

- Lastly, the signal can be rebuilt which is shown as follows:

$$I(t) = \sum C_j'(t) + \sum C_k(t) + r_L(t) \quad j = M_1 + 1, \ldots, M_2 \quad k = M_2 + 1, \ldots, L$$  \hspace{1cm} (5)

Fig. 1 shows the de-noising effect of the rainbow signal using the aforementioned method. The real rainbow signal is shown in Fig.1(a) which is simulated by Mie theory, and the rainbow signal plus random noise is shown in figure 1(b). The SNR of the random noise is 4, and it is shown the signal is almost ruined by the strong noise. The de-noising result is shown in Fig.1(c). In the figure, both the Airy structure and the Ripple structure is clearly visible. Although small loss and deformations of the high frequency structure appear in some place, the interference peak position changes little, and the low frequency part changes even more smaller. Fig. 2 is a comparison between the de-noising result shown in
Fig. 1 and the result which uses Symmlets wavelet de-noising. It is shown that the wavelet de-noising filters out most of the noise, but in some position, noise components still remains, and large deformations also occur in some place. The problem is mainly due to the unsuitable chosen wavelet. However, as a self-adaptive and fully signal driven data process, the EMD based method completely avoided such problems.

2.2. Signal analysis and parameter inversion

According to Debye theory, the rainbow pattern is a superposition result of different types of light interference and diffraction. The main interference which always used for particle size and refractive index inversion is Airy structure. In previous studies, Airy structure is always got by FFT filtering with a empirical cut-off frequency and inverse FFT rebuilding. As the frequency of the Airy structure changes with the particle size, empirical cut-off frequency is not always correct, and the signal rebuilding process also makes some inevitable shifts which also increase the error. In this paper, EMD is used to decompose the rainbow signal, and a decomposition result is shown in Fig. 3. The decomposed signal is simulated by Mie theory plus some random noises. It is shown that after the noise component IMF1 decomposed, the next appearing two signal IMF2 and IMF3 indicate the Ripple structure, with the IMF4 representing the Airy structure, and the residual shows the light diffraction effect on the particle. The decomposed result agrees with the physical mechanism of rainbow formation which can explained by Debye theory and geometric optics. So using EMD method, the Airy structure can be separated.

When the Airy structure is determined, the maximum peak position and the secondary peak position can be extracted from the signal. According to Airy theory and geometric optics, two formulas can be derived, which are shown as follows:

\[
a = \frac{\lambda}{8\left\{(4m^2)/(m^2-1)\right\}^{0.5} \left\{(m^2-1)^3/(4m^2)\right\}^{0.5} \left[2.37959/(\theta_2-\theta_1)*180^\circ/\pi\right]^{3/2}}
\]

\[
\theta_1 = 4\cos^{-1}\left\{((4m^2)/3m^2)^{0.5} - 2\sin^{-1}\left\{((m^2-1)/3)^{0.5}\right\} + 1.0845/(4(4m^2)^{0.5})(3\lambda^2(4m^2)^{0.5}/a^2)^{1/3}\right\}^{*180^\circ/\pi}
\]

where \(a\) is the particle radius; \(m\) is the refractive index of the particle; \(\lambda\) is the wavelength of the incident light; \(\theta_2\) is the extracted secondary peak position; and \(\theta_1\) is the maximum peak position. To solve the two formula, a assumptive refractive index is taken into formula(6) to calculate the particle radius, then the refractive index is estimated according to formula(7). The above process is repeated until the best value of refractive index and radius are obtained. As the airy theory is a approximate theory compared with exact
light scattering theory, the difference between the two theory increases with the particle size decreasing. So when the particle size is small, a inversion algorithm based on Debye theory is used[7].

3. Experiment and results discussion

To verify the presented method, a water column size and temperature experiment is taken. In order to avoid the measurement error brought by droplet non-sphericity, the liquid column is measured instead of the droplet. As according to Debye theory, the scattering pattern of the infinite column is similar to the spherical particle scattering. The experiment setup is depicted in Fig. 4. The water is pumped to the reservoir at the top of the setup. When the water level and the flow rate are controlled, the pressure can force water to form a stable column after flowing through the capillary. The column is illuminated by a continuum laser with the wavelength of 532nm and power of 14mW. The rainbow light is collected by a lens with large diameter and received by a linear CCD placed in the focus of the lens. An adjustable mirror is used to change the angle of the laser beam, with the polarization direction and the intensity of the incident light regulated by polarizer1 and polarizer 2 respectively.

The water column refractive index and size at different temperature are measured. The temperature is controlled in the reservoir, and a reference temperature is measured by a thermocouple placed under the measuring region which is taken as exact temperature value. With the measured refractive index, the temperature is got by using empirical formula which mentioned in reference[8].

The measured temperature results are depicted in Fig. 5. It is shown when the temperature is less than 60°C, the measured temperature has the same trend with the exact temperature, and the deviation between two temperature is very small which is only about 2°C. This deviation is thought to be brought by the system error during the scattering angle calibration process. However, when the temperature is higher than 60°C, although the measured temperature increases with the exact temperature growing, the upward trend gradually decrease, and the temperature curve gradually deviated from the exact value. Such error is thought to be caused by heat loss of evaporations at high temperature. Moreover, the exact temperature which measured by thermocouple are not completely synchronized with rainbow thermometry, and the aerosols formed by the water vapor bring a lot of miscellaneous light. All this above factors make the measurement errors in at high temperature.

The cross section radius of the water column is also measured. As the cross section of the water column is nearly round and the radius is close to the inner radius of the capillary tube which is 1100μm. We measure the water column radius with the measuring area 2.5cm from the orifice of the capillary tube, and the measured radius is 1069μm, which is within the error range of the parameter inversion.

Fig. 4. A sketch of experiment setup                                       Fig. 5 Measured temperature result
4. Conclusion

A rainbow signal de-noising and analysis method is presented in this paper. The noise is removed by EMD and wavelet threshold method, and the Airy structure used for parameter inversion is rebuilt from the signal decomposition using EMD method. A water column cross section radius and temperature measurement is done to validate the accuracy of the presented method, and the result shows good accuracy compared with reference value, except some error occurs at high temperature condition, and the factors which brought error are analyzed.

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References