Negative-parity baryon masses using an $\mathcal{O}(a)$-improved fermion action

M. Göckeler [a], R. Horsley [b,c], D. Pleiter [b], P.E.L. Rakow [a], G. Schierholz [b,d,1], C.M. Maynard [e,2], D.G. Richards [f,g,3]

[a] *Institut für Theoretische Physik, Universität Regensburg, D-93040 Regensburg, Germany*
[b] *Deutsches Elektronen-Synchrotron DESY, John von Neumann-Institut für Computing NIC, D-15735 Zeuthen, Germany*
[c] *Institut für Physik, Humboldt-Universität zu Berlin, D-10115 Berlin, Germany*
[d] *Deutsches Elektronen-Synchrotron DESY, D-22603 Hamburg, Germany*
[e] *Department of Physics & Astronomy, University of Edinburgh, Edinburgh EH9 3JZ, Scotland, UK*
[f] *Jefferson Laboratory, MS 12H2, 12000 Jefferson Avenue, Newport News, VA 23606, USA*
[g] *Department of Physics, Old Dominion University, Norfolk, VA 23529, USA*

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Abstract

We present a calculation of the mass of the lowest-lying negative-parity $J = 1/2^-$ state in quenched QCD. Results are obtained using a non-perturbatively $\mathcal{O}(a)$-improved clover fermion action, and a splitting is found between the mass of the nucleon, and its parity partner. The calculation is performed on two lattice volumes and at three lattice spacings, enabling a study of both finite-volume and finite lattice-spacing uncertainties. A comparison is made with results obtained using the unimproved Wilson fermion action.

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1. Introduction

The study of the excited nucleon spectrum can provide important clues to the dynamics of QCD and the nature of the interactions between its fundamental partons. The observed N^* spectrum raises many important questions, such as the nature of the Roper resonance, and whether the $\Lambda(1405)$ is a true three-quark state or a molecular state. For

these reasons, the study of the spectrum is an important element of the Jefferson Laboratory experimental programme. The phenomenological interest in the excited nucleon spectrum has been complemented by a flurry of activity in the lattice community. In particular, two calculations of the mass of the parity partner of the nucleon have appeared; the first employed the highly-improved D_{234} fermion action [1,2], whilst the second employed domain-wall fermions [3,4]. Both calculations exhibited a clear splitting between the masses of the $N^{1/2+}$ and $N^{1/2-}$ states, and the importance of chiral symmetry breaking in obtaining a non-zero mass splitting has been stressed [3].

E-mail address: dgr@jlab.org (D.G. Richards).

[1] QCDSF Collaboration.
[2] UKQCD Collaboration.
[3] LHPC Collaboration.

In this Letter, we present a calculation of the lowest lying negative parity nucleon using an $\mathcal{O}(a)$-improved Sheikholeslami–Wohlert (SW), or clover, fermion action; preliminary results were presented in Ref. [5]. By choosing the coefficient of the improvement term appropriately, all $\mathcal{O}(a)$ discretisation uncertainties are removed, ensuring that the continuum limit is approached with a rate proportional to a^2.

The calculation of the excited nucleon spectrum places particularly heavy demands on lattice spectroscopy. The excited nucleon states are expected to be large; the size of a state is expected to double with each increase in orbital angular momentum. Thus a lattice study of the excited nucleon spectrum requires large lattice volumes, with correspondingly large computational requirements. Furthermore, the states are relatively massive, requiring a fine lattice spacing, at least in the temporal direction. These requirements could be satisfied with much greater economy using the clover fermion action than using the domain-wall or overlap formulation. Thus it is important to establish that the negative parity states are indeed accessible to calculations using the clover action. Finally, by comparing the masses obtained using the clover action with a calculation, at a single quark mass, using the Wilson fermion action, we also gain insight into the nature of the interaction responsible for the splitting in the parity doublet.

The rest of the Letter is laid out as follows. In the next section, we introduce the hadronic operators and correlators measured in the calculation, describe the fermion action, and provide the simulation parameters. Section 3 contains our results for the masses of the lowest lying positive- and negative-parity nucleon states using the SW fermion action. Detailed discussion, including a comparison with the Wilson fermion case, and our conclusions, are presented in Section 4.

2. Calculational details

2.1. Baryon operators

For a particle at rest, there are three local interpolating operators for the positive-parity $I = 1/2$ nucleon,

$$N_1^{1/2+} = \epsilon_{ijk}\left(u_i^T C\gamma_5 d_j\right)u_k, \tag{1}$$

$$N_2^{1/2+} = \epsilon_{ijk}\left(u_i^T C d_j\right)\gamma_5 u_k, \tag{2}$$

$$N_3^{1/2+} = \epsilon_{ijk}\left(u_i^T C\gamma_4\gamma_5 d_j\right)u_k. \tag{3}$$

The "diquark" part of both N_1 and N_3 couples upper spinor components, while that in N_2 involves the lower components and thus vanishes in the non-relativistic limit [1]. For some of the lattices in our calculation, the positive-parity nucleon mass is obtained using the non-relativistic quark operators [6], defined by

$$\psi \to \psi^{\mathrm{NR}} = \frac{1}{2}(1+\gamma_4)\psi, \quad \bar{\psi}^{\mathrm{NR}} = \bar{\psi}\,\frac{1}{2}(1+\gamma_4). \tag{4}$$

In practice, lattice calculations confirm the naive expectation that the operators N_1 and N_3 have a much greater overlap with the nucleon ground state than N_2, and therefore we do not use this operator in the fits.

The operators appropriate to negative-parity states are obtained from those of Eqs. (1)–(3) by simply multiplying by γ_5. However, correlators constructed from these operators receive contributions from both parities. The best delineation that can be achieved is that of forward-propagating positive-parity states and backward-propagating negative-parity states, or the converse, through the use of the parity projection operator $(1 \pm \gamma_4)$. On a lattice periodic or anti-periodic in time, the resulting correlators may be written:

$$C_{N_i^{+/-}}(t)$$
$$= \sum_{\vec{x}}\Big((1 \pm \gamma_4)_{\alpha\beta}\langle N_{i,\alpha}(\vec{x},t)\overline{N}_{i,\beta}(0)\rangle$$
$$\qquad + (1 \mp \gamma_4)_{\alpha\beta}\langle N_{i,\alpha}(\vec{x},N_t-t)\overline{N}_{i,\beta}(0)\rangle\Big), \tag{5}$$

where N_t is the temporal extent of the lattice. At large distances, when $t \gg 1$ and $N_t - t \gg 1$, the correlators behave as

$$C_{N_i^+}(t) \to A_i^+ e^{-m_i^+ t} + A_i^- e^{-m_i^-(N_t-t)}, \tag{6}$$

$$C_{N_i^-}(t) \to A_i^- e^{-m_i^- t} + A_i^+ e^{-m_i^+(N_t-t)}, \tag{7}$$

where m_i^+ and m_i^- are the lightest positive- and negative-parity masses, respectively, in channel i.

2.2. Fermion action

To leading order in a the Symanzik improvement programme amounts to adding the well-known Sheikholeslami–Wohlert term to the fermionic Wilson

action [7]

$$\delta S = -c_{\mathrm{SW}} \frac{i\kappa}{2} \sum_{x,\mu,\nu} \bar{\psi}(x)\sigma_{\mu\nu}F_{\mu\nu}(x)\psi(x). \tag{8}$$

Provided that c_{SW} is chosen appropriately, spectral quantities such as hadron masses approach the continuum limit with a rate proportional to a^2. Non-perturbative determinations of c_{SW} have been made in the quenched approximation to QCD in Refs. [8,9].

The Sheikholeslami–Wohlert term, Eq. (8), is of magnetic moment form, and it is well known that the use of the SW action results in hyperfine splittings that are closer to their experimental values than those obtained using the standard Wilson fermion action, see, for example, Ref. [10]. The SW term also removes the leading chiral-symmetry-breaking effects at finite a. In view of these considerations, we compare the splitting between the positive- and negative-parity states obtained using the Wilson fermion action at a single light-quark mass with the results obtained using the SW fermion action.

2.3. Simulation details

The calculation is performed in the quenched approximation to QCD using lattices generated by the UKQCD and QCDSF collaborations; calculations of the light hadron spectrum using these lattices have appeared in Ref. [11] and [12,13], respectively. Propagators on the UKQCD lattices were computed from both local and fuzzed sources to both local and fuzzed sinks; the fuzzing procedure is described in Ref. [14]. The parameters used in the calculation are listed in Table 1. Propagators on the QCDSF lattices were computed using Jacobi smearing at both source and sink, described in Ref. [15].

In the case of the UKQCD data, the same numbers of configurations are used for each of the quark masses for a given $\beta \equiv 6/g^2$ and lattice volume; here the errors on the fitted masses are computed using a bootstrap procedure. In the case of the QCDSF configurations, different numbers of configurations are used at different quark masses even at the same β and volume. There the errors on the masses are also obtained using a bootstrap procedure. For the chiral extrapolations, a simple uncorrelated χ^2 fit is performed, with the uncertainties computed from the variation in the χ^2.

3. Results

The masses of the lowest-lying $N^{1/2+}$ and $N^{1/2-}$ states are obtained from a simultaneous, four-parameter fit to the positive- and negative-parity correlators of Eq. (5), constructed using fuzzed sources and local sinks (UKQCD) or using smeared sources and smeared sinks (QCDSF), using the fit functions of Eqs. (6) and (7). We see a clear signal for the mass of the negative-parity states, and the quality of a fit is illustrated in Fig. 1; the contamination of the negative-parity correlator from the lighter, backward-propagating, positive-parity state is clear both in the fits and in the data. The masses of the lightest particle of positive and negative parity as a function of m_π^2 on each ensemble are shown in Figs. 2–4.

For the chiral extrapolations of the hadron masses, we adopt the ansatz

$$(am_X)^2 = (aM_X)^2 + b_2(am_\pi)^2, \tag{9}$$

where we use upper-case letters to denote masses obtained in the chiral limit, and X is either $N^{1/2+}$ or

Table 1

The parameters of the lattices used in the calculation. The labels J and F refer to use of Jacobi and "fuzzed" quark sources, respectively. Lattice sizes in physical units are quoted using r_0 to set the scale [18]

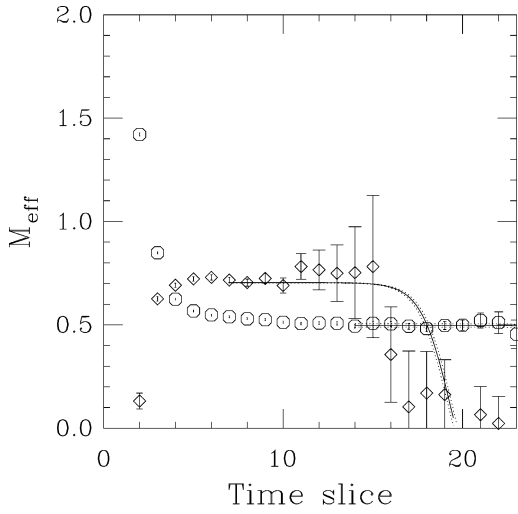| β	c_{SW}	$L^3 \times T$	L (fm)	κ	Smearing
6.4	1.57	$32^3 \times 48$	1.6	0.1313, 0.1323, 0, 1330, 0.1338, 0.1346, 0.1350	J
6.2	1.61	$24^3 \times 48$	1.6	0.1346, 0.1351, 0.1353	F
		$24^3 \times 48$	1.6	0.1333, 0.1339, 0.1344, 0.1349, 0.1352	J
		$32^3 \times 64$	2.1	0.1352, 0.1353, 0.13555	J
6.0	1.76	$16^3 \times 48$	1.5	0.13344, 0.13417, 0.13455	F
		$16^3 \times 32$	1.5	0.1324, 0.1333, 0.1338, 0.1342	J
		$24^3 \times 32$	2.2	0.1342, 0.1346, 0.1348	J

Fig. 1. The effective masses for the $N^{1/2+}$ channel (circles) and the $N^{1/2-}$ channel (diamonds) at $\beta = 6.2$ with $\kappa = 0.1351$. The lines are from a simultaneous fit to both parities.
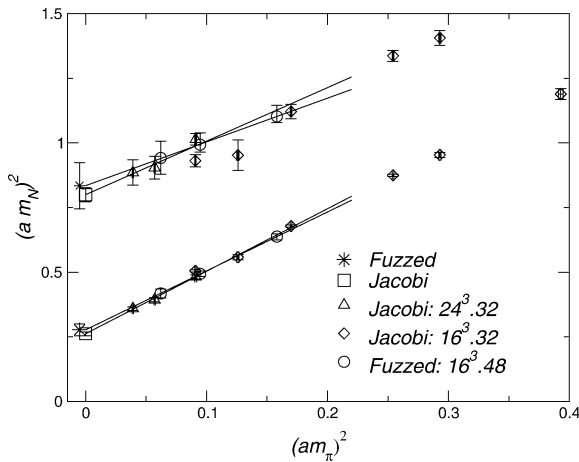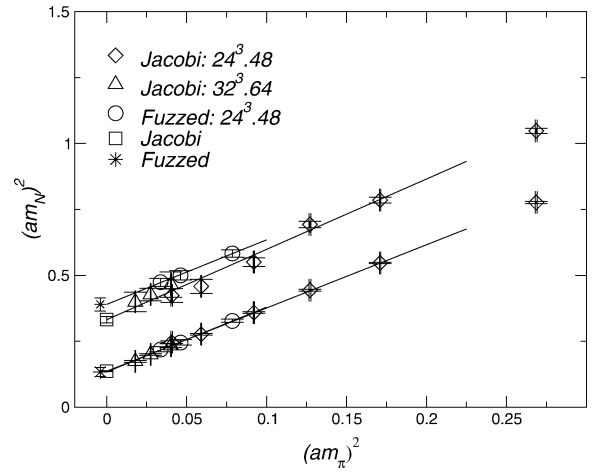

Fig. 3. The masses in lattice units of the lowest-lying positive- and negative-parity nucleons at $\beta = 6.2$. The curves are from independent fits to the Jacobi-smeared and fuzzed data for m_X^2 using Eq. (9).

Fig. 2. The masses in lattice units of the lowest-lying positive- and negative-parity nucleons on the lattices at $\beta = 6.0$. The curves are from independent fits to the Jacobi-smeared and fuzzed data for m_X^2 using Eq. (9), as described in the text.
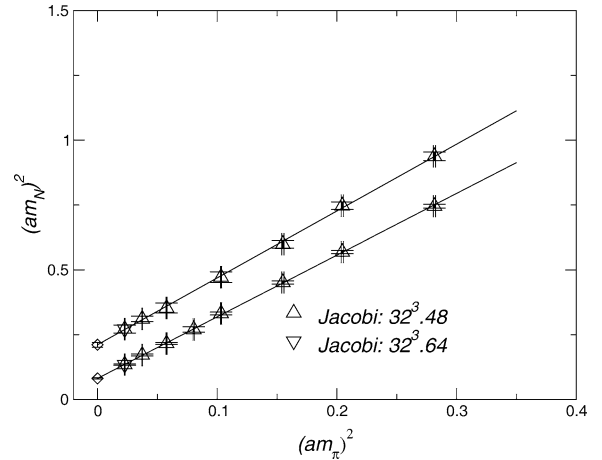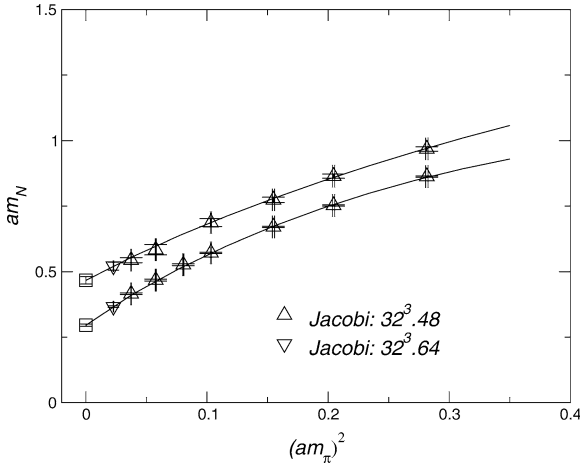

Fig. 4. The masses in lattice units of the lowest-lying positive- and negative-parity nucleons at $\beta = 6.4$. The curves are from fits to m_X^2 using Eq. (9).

$N^{1/2-}$. In order to avoid large discretisation errors, we include in the extrapolation only those masses which are less than of order unity, in lattice units. We include data at different volumes, but at the same β, in the chiral extrapolations, but treat the fuzzed and Jacobi-smeared data independently. The parameters of the fit for the positive- and negative-parity states are given in

Table 2, and the chiral extrapolation, together with the extrapolated masses, shown in Figs. 2–4.

The fit to m_X^2, rather than m_X, gives a sensible behaviour in the heavy-quark limit, whilst being formally the same at light pseudoscalar masses. The data for m_X at $\beta = 6.4$, shown in Fig. 5, exhibit clear curvature compared with those in Fig. 4, and a satisfactory fit to the data requires the addition of a non-analytic term

Table 2
The parameters of the fit of the lightest positive- and negative-parity states to Eq. (9)

β	6.4	6.2		6.0	
	Jacobi	Jacobi	Fuzzed	Jacobi	Fuzzed
$aM_{N^{1/2+}}$	0.285(4)	0.369(3)	0.37(2)	0.512(6)	0.53(2)
b_2	2.38(2)	2.40(2)	2.5(3)	2.42(3)	2.3(2)
$aM_{N^{1/2-}}$	0.46(1)	0.58(1)	0.62(2)	0.89(2)	0.91(5)
b_2	2.58(6)	2.67(7)	2.4(4)	2.1(2)	1.7(7)

Fig. 5. The masses in lattice units of the lowest-lying positive- and negative-parity nucleons at $\beta = 6.4$. The curves are fits to m_X using Eq. (10), as described in the text.

Fig. 6. The masses of the lowest-lying positive- and negative-parity baryons in units of r_0^{-1} [17,18] against a^2 in units of r_0^2. The lines are linear fits in a^2/r_0^2 to the positive- and negative-parity baryon masses. Also shown are the physical values.

Table 3
A comparison of the masses of the lowest-lying negative-parity states on the small-volume and large-volume lattices at $\beta = 6.0$ and $\beta = 6.2$, at a pseudoscalar mass $am_\pi = 4.8/(L/a)$

β	am_π	$am_{N^{1/2-}}$ (small)	$am_{N^{1/2-}}$ (large)
6.0	0.3	0.97(1)	1.01(1)
6.2	0.2	0.664(9)	0.69(4)

$$am_X = aM_X + b_2(am_\pi)^2 + b_3(am_\pi)^3. \qquad (10)$$

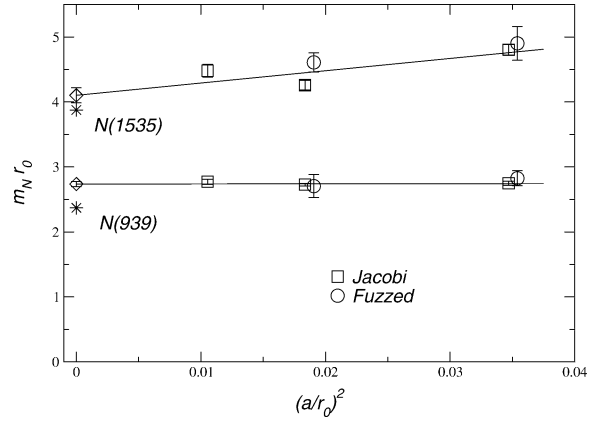The fit to Eq. (10) at $\beta = 6.4$ yields

$$aM_{N^{1/2+}} = 0.296(5), \qquad (11)$$

$$aM_{N^{1/2-}} = 0.47(1) \qquad (12)$$

to be compared to the fits of Eq. (9)

$$aM_{N^{1/2+}} = 0.285(4), \qquad aM_{N^{1/2-}} = 0.46(1).$$

The addition of such a non-analytic term makes only a small difference to the extrapolated masses in

the fits to m_X^2. There has been considerable study of the contribution of non-analytic terms arising from pion-induced baryon self energies [16]. We do not investigate the effect of these contributions in this Letter.

In order to isolate the magnitude of the possible finite-volume uncertainties in our results, we choose not to examine the masses in the chiral limit, but rather at a value of the pseudoscalar mass for which we have data on both smaller and larger volumes. Specifically, we compare the masses obtained from the Jacobi-smeared data at $\beta = 6.0$ and $\beta = 6.2$ with a pseudoscalar mass given by

$$am_\pi = 4.8/(L/a), \qquad (13)$$

where L is the spatial extent of the smaller of the lattices at each β. The results of this analysis are provided in Table 3; the mass of the negative-parity state on the larger lattices is higher than on the smaller

lattices at both lattice spacings, by an amount of order 5%.

In order to look at the discretisation uncertainties in our data, we show in Fig. 6 the masses in units of r_0 against the a^2/r_0^2, where $r_0 = 0.5$ fm is the hadronic scale [17,18]. For the lightest positive-parity state, we obtain entirely consistent results at the different lattice spacings. For the case of the negative-parity states, there is some trend towards decreasing mass at finer lattice spacings. In order to quantify the discretisation uncertainties, we perform uncorrelated χ^2 fits in a^2/r_0^2:

$$(M_X r_0)(a) = M_X r_0(a=0) + c\left(\frac{a}{r_0}\right)^2, \qquad (14)$$

where X is either $N^{1/2+}$ or $N^{1/2-}$, yielding

$$M_{N^{1/2+}} r_0 = 2.74(4), \qquad M_{N^{1/2-}} r_0 = 4.1(1). \qquad (15)$$

The fits and extrapolated masses are shown on the figure, together with the experimental values of the masses in units of r_0. We estimate the difference between the values obtained in the continuum limit and that obtained on our finest lattice, $\beta = 6.4$, as a measure of the discretisation uncertainties in our calculation; this is negligible for the positive-parity state, and 10% for the negative parity state.

Though there is a noticeable discrepancy between the lattice and physical values, a direct experimental measurement of r_0 is unavailable, and r_0 is a better scale for comparing data at different lattice spacings than for comparing data with experiment. Therefore, we choose as our final result the mass ratio of the negative- and positive-parity masses in the quenched approximation

$$M_{N^{1/2-}}/M_{N^{1/2+}} = 1.50(3), \qquad (16)$$

where the quoted error is purely statistical, and we estimate systematic uncertainties of 5% due to finite volume effects, and 10% due to discretisation uncertainties. This is to be compared with the physical ratio of 1.63.

4. Discussion and conclusions

The calculation exhibits a clear mass splitting between the positive- and negative-parity states, in agreement with calculations using the highly-improved and domain-wall fermion actions. In view of the dearth of studies of negative-parity baryon masses using the Wilson fermion action, it is instructive to compare the mass splitting obtained with the clover fermion action with that obtained from the standard Wilson fermion action. In order to study this, Wilson quark propagators were computed on the UKQCD $24^3 \times 48$ lattices at $\beta = 6.2$ at a quark mass corresponding to $m_\pi/m_\rho = 0.7$; the smearing and fitting procedures were the same as those employed in the SW calculation.

The splitting between the masses of the positive- and negative-parity states obtained with the Wilson fermion action, together with that obtained with the SW fermion action is shown in Fig. 7. The splittings obtained with the two actions are entirely consistent, which we will now argue is reasonable. Under the $SU(6)$ spin-flavour symmetry, the low-lying negative-parity baryons, up to around 2 GeV, can be assigned to a $l = 1$ $\underline{70}$-plet. Similarly, the low-lying positive-parity baryons can be assigned to an $l = 0$ $\underline{56}$-plet. Thus the splitting in the parity doublets is analogous to the P–S splitting in the meson sector, which we know is relatively faithfully reproduced using the Wilson fermion action. Indeed an earlier lattice calculation demonstrating that the P-wave baryons are accessible to lattice calculation is contained in Ref. [19].
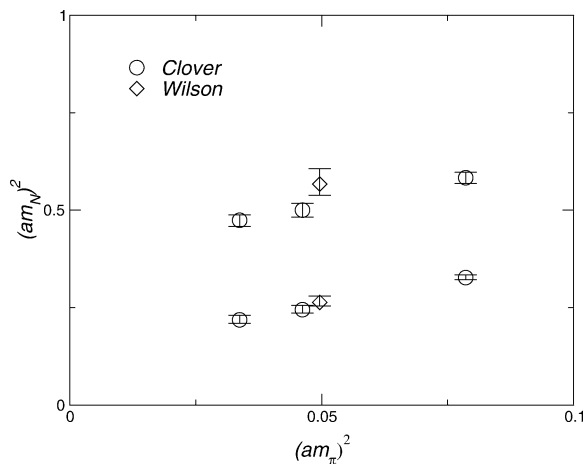

Fig. 7. The masses in lattice units of the lowest-lying positive- and negative-parity nucleons using the SW-clover action at $\beta = 6.2$ on the $24^3 \times 48$ lattices (circles). Also shown is the corresponding results obtained using the Wilson fermion action (diamonds) on the same ensemble of configurations.

Within the $l = 1$ $\underline{70}$-plet, calculations of the masses of these states both in the quark model [20], and in large N_C [21], suggest that the spin–orbit contribution is surprisingly small, whilst the hyperfine contribution is of normal size, and there is an effective interaction carrying the quantum numbers of pion exchange [22]. It is within a multiplet that we might expect the choice of action to play a rôle.

The continuation of the calculation to lighter quark masses will require careful consideration. A baryon can change parity through the emission of a π or η; the latter process is accessible even in the case of quenched QCD, where the π and η are degenerate in mass, as illustrated in Fig. 8.[4] The non-unitary behaviour associated with such processes has been observed in the scalar correlator [23–25]. Whilst no evidence for non-unitary behaviour is observed in this calculation, the lightest quark mass is indeed close to the $N^* \to N\pi$ threshold.

In this Letter it has been shown that both the SW-clover and Wilson fermion actions are capable of resolving the splitting between the positive- and negative-parity baryon masses in the quenched approximation to QCD, and therefore that these are accessible to relatively economical calculation. We obtain a ratio for the masses of the negative- and positive-parity states of 1.49(2), where the error is purely statistical, compared with the experimental value of 1.63. A more extensive study of the spectrum including the case of non-degenerate quark masses, and for the lowest-lying $I = \frac{3}{2}$ states will appear in a longer paper [26].
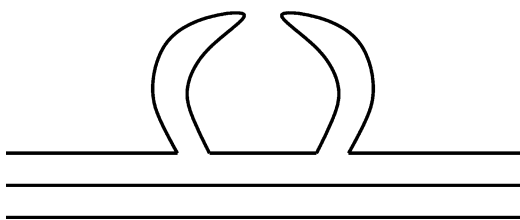

Fig. 8. Diagram contributing to the decay $N^{1/2-} \longrightarrow N^{1/2+}\eta$ in quenched QCD.

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[4] We are grateful to Robert Edwards and Chris Michael for this observation.

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