Procedia Computer Science
Volume 66, 2015, Pages 166–173
YSC 2015. 4th International Young Scientists Conference on Computational Science

Different criteria of dynamic routing

Kurochkin[1*], Grinberg[1†]
[1] *Kharkevich Institute for Information Transmission Problems of Russian Academy of Sciences*
kurochkin@iitp.ru, greenjak@iitp.ru

Abstract
The mathematical model of functioning of a telecommunication network is developed. The analysis technique of the telecommunication networks, loading of networks revealing direct dependence on routing strategy is implemented. The primary goals which can be solved: check of efficiency of strategy of routing; determination of vulnerabilities in a telecommunication network; execution of estimation and the comparative analysis of various strategies of routing. Results of experiments on comparison of efficiency of heuristic algorithms of the routing directed on the maximum increase in the general traffic on networks are presented. Comparing of results of mathematical simulation by several criteria was allowed to range heuristic algorithms on a level of fitness to dynamic filling of telecommunication networks.

Keywords: Dynamic routing, telecommunication network, rational use of network resources, multicommodity problem, minimal cost path algorithm

1 Introduction

In the paper we present our results of studies of the problem which is very important in telecommunication industry. We consider telecommunication network consisting of commutation nodes and communication links of limited capacities. Then, suppose that a number of network subscribers' pairs generate a sequence of messages (requests) which have to be sent from one subscriber to another. How should these messages be routed to maximize the number of fulfilled requests?

Prior to mathematical formalization we will present brief survey of methods of similar problems' solving.

We think that well-known in the network flow theory so called multicommodity problem (Fulkerson & Ford, 1962), (Hu, 1970). It is stated as the admissibility problem for commodity specified intensity flows organization in a network with specified capacitated links. It is true that in such traditional statement the requirements for intensity of commodity flows among subscribers are

[*] Masterminded EasyChair and created the first stable version of this document
[†] Created the first draft of this document

Selection and peer-review under responsibility of the Scientific Programme Committee of YSC 2015

doi:10.1016/j.procs.2015.11.020

specified at once, immediately. Our problem will be reduced to a traditional statement, if all requests among subscribers are summarized at a certain moment and the flow distribution problem is solved in line with the admissibility problem. It should be noted that there are no special effective methods of multicommodity problem solving, in respect of computation it is a linear programming problem.

The queuing network method is the main method of telecommunication network analysis. Many works are devoted to this method (Ivnitsky, 2004) (Martynov, Vatutin, & Titiov, 2014). Nevertheless we don't know works within the framework of this method devoted to search of routing algorithms maximizing or at least increasing network total capacity as compared to any known algorithm. The prevailing routing algorithm in Internet is the minimum path-length algorithm (used for example in the network layer RIP-protocol) or the minimum cost path algorithm where costs are assigned to communication lines in accordance with various criteria – delay factor, transmission bandwidth, reliability etc. The numerical values of criteria in the OSPF network layer protocol are called "metrics".

Qualitative method presented in this work includes the following. At each certain moment a telecommunication network is characterized by links "residual" capacities. Next information channel should be developed so that as much as possible of "resource" could be left in the network (Grinberg, Kurochkin, & Korkh, TPSA, 2012). By "resource" we shall meat two criteria – link residual capacity and minimal-cut minimal capacity of all subscriber pairs. Accordingly two groups of algorithms will be offered.

2 Problem statement and brief description

As a mathematical model of telecommunication network we take a connected un-oriented graph, where vertices represents network commutation nodes and edges (Grinberg & Kurochkin, The study of the results of mathematical modeling of networks with sequential filling of the cluster topology, 2009) (Grinberg & Kurochkin, Mathematical modeling of dynamic networks of sequential filling of streams due, 2009). Initially, i.e. the empty network is characterized by edge capacities. Network subscribers are represented by a subset of vertices (further we'll call them poles). Those network nodes that are not included in the poles set are commutation ones. On the subscribers' request a path (communication channel) is developed between a pole pair passing along the graph edges and a commodity unit-intensity flow is allowed along this path. Hereupon capacities of the edges, along which the path passes, are reduced by a unit. Then another pair of poles is selected and the procedure is repeated. The problem is: how to select paths between pole pairs to fulfill maximum number of requests. In other words which should be the path-tracing algorithm - routing algorithm – to fulfill maximum possible number of requests for all types of networks and all variants of requests among poles.

Let us call algorithms, realizing path selection in sequence as per each request, sequential algorithms. It is supposed that it is impossible to change the selected path. Let us call multicommodity problem solving algorithms, when edge flows and paths identification is made in accordance with pole interchange intensity specified summarized requirements, synchronous algorithms. One of sequential algorithms is the minimum path-length algorithm (as per edge number). Let us call this algorithm a simple algorithm. One of synchronous algorithms is the linear programming algorithm or LP-algorithm. The LP- algorithm is absolute one in the sense that in the network load of requirements can be complied with when and only when admissibility problem is solved. Let us call the sequential algorithm optimal one, if impossibility to fill the next request per this algorithm means admissibility problem insolubility for the entire load of requirements.

We think that it is hardly possible to develop an optimal sequential algorithm due to the fact that at each certain moment there is uncertainty of requirement final distribution among pole pairs. The objective of the present work is to offer sequential algorithms increasing network summarized traffic as compared to a simple algorithm.

Two groups of algorithms will be offered. One of these is based on the concept that the next path to be developed along the edges with this moment maximum "residual" capacities. Another group is based on computation of capacities of minimal cuts among all pole pairs on the basis of data about network edge "residual" capacities as well as on acceptance of probability law for requirement distribution among pole pairs.

3 Sequential filling algorithms

Let G be a network consisting of N vertices $A_1, A_2,\ldots,A_N$, where A_i, $i=1,2,\ldots N_1$, $N_1 \le N$ are poles, and K edges $B_1,\ldots,B_K$, with b_k, $k=1,2,\ldots,K$, capacities. Let us arrange somehow all pole pairs. The total number of various pole pairs in the G network is equal to $M = N_1(N_1-1)/2$.

Let us assume that commodity flow among pole pairs organization requirements follow the probability law, i.e. there are M nonegative numbers p_m, $m = 1,2,\ldots,M$, such that $\sum_{m=1}^{M} p_m = 1$. These numbers are probabilities of request for commodity flow between the m-th pole pair in common requests flow.

We suppose also that request completion means organization of unit intensity flow.

3.1 Fundamental minimal-cut algorithm

Let us introduce the following designations.

R_m – capacity of the minimal cut between the m-th pole pair equal, in accordance with the maximal flow and minimal cut, to the maximal flow between this pole pair;

$\overline{R} = \frac{1}{M}\sum_{m=1}^{M} R_m$ - minimal cut capacity average among all pole pairs.

Let us generate the following value, which will be later called "χ criterion" or "irregularity measure":

$$\chi = \sum_{m=1}^{M} \left(\frac{1}{M}\frac{R_m}{\overline{R}} - p_m\right)^2, \tag{1}$$

This value is the "mismatch measure" between minimal cuts (normalized in a certain manner) and probabilities of this pole pair requests. The more mismatch is the bigger value is. Meanwhile it is clear that one should expect that the "better matched" minimal cuts and probabilities are, the more requests can be filled. Thus the next path is to be selected so that this value should be minimal in the network with edge capacity residual values. This is equivalent to this value increment upon the next path selection should be minimal (in the algebraic sense). In the special case of request equiprobable distribution as per pole pairs, i.e. with $pm = 1/M$, (1) it is simplified and takes on the following form:

$$\chi = \sum_{m=1}^{M} \left(\frac{R_m}{\overline{R}} - 1\right)^2. \tag{2}$$

(The $1/M^2$ constant factor is not essential and is not taken into consideration.)

The next request fill is identification of the path between the pole pair and "allowing" the unit intensity flow along it. Herewith the G network is changed as b_k. arc capacities are changed. This change is to result in the χ value minimal increment. Let us express the χ value and its increment in terms of b_k. value. For this purpose we will need to know all minimal cuts among all pole pairs in the G network.

Let us assume that $b_s^{lm}, s = 1,2,...,S_{lm}$, the s-th edge of the l-the minimal cut between the m-th pole pair, S_{lm} – this cut edge number. Then

$$R_m = \frac{1}{L_m}\sum_{l=1}^{L_m}\sum_{s=1}^{S_{lm}} b_s^{lm}, \tag{3}$$

where L_m – total number of minimal cuts between the m-th pole pairs. Let us convert this formula by defining $u_{mk}, m = \overline{1,M}, k = \overline{1,K}$ numbers as such, which designate the number of various minimal cuts between the m-th pole pair, which include the B_k. arc. Then it is evident that

$$R_m = \frac{1}{L_m}\sum_{k=1}^{K} u_{mk} b_k. \tag{4}$$

With account of (4) the network irregularity (1), (2) appears to be expressed in terms of the b_k. values. Upon the next elementary requirement is filled the G network is changed as related to the b_k, value change, therefore the χ value will also be changed. In the case if the b_k value relative change is minor, the network irregularity change is determined by the next formula:

$$\Delta\chi = \sum_{k=1}^{K}\frac{\partial\chi}{\partial b_k}\Delta b_k = \sum_{k=1}^{K}\left(\frac{\partial\chi}{\partial\overline{\overline{R}}}\frac{\partial\overline{R}}{\partial b_k} + \sum_{m=1}^{M}\frac{\partial\chi}{\partial R_m}\frac{\partial R_m}{\partial b_k}\right)\Delta b_k, \tag{5}$$

Later we have:

$$\frac{\partial\chi}{\partial\overline{\overline{R}}} = -\frac{2}{\overline{\overline{R}}}\left[\frac{\overline{R^2}}{\overline{\overline{R}}^2} - 1\right], \tag{6}$$

$$\frac{\partial\overline{R}}{\partial b_k} = r_k = \frac{1}{M}\sum_{m=1}^{M}\frac{u_{mk}}{L_m}, \tag{7}$$

$$\frac{\partial\chi}{\partial R_m} = \frac{2}{M\overline{\overline{R}}}\left(\frac{1}{M}\frac{R_m}{\overline{\overline{R}}} - p_m\right), \tag{8}$$

$$\Delta\chi = \frac{2}{M\overline{\overline{R}}}\sum_{k=1}^{K}\Delta b_k\left[\sum_{m=1}^{M}\left(\frac{R_m}{\overline{\overline{R}}}r_k - r_{mk}\right)p_m - \frac{\overline{R^2}}{\overline{\overline{R}}^2}r_k + \frac{1}{M\overline{\overline{R}}}\sum_{m=1}^{M}R_m r_{mk}\right], \tag{9}$$

where $r_{mk} = \frac{u_{mk}}{L_m}$, $\sum_{m=1}^{M} r_{mk} = M r_k$.

Equation (9) implies that network irregularity measure increment is given as the linear form as per arc capacity increments. The problem is to make this increment minimal (in the algebraic sense, i.e. increment negative values are possible) while filling the next elementary requirement. Thus we come to the problem of minimal cost path search.

If the elementary requirement flow is equiprobable for all pole pairs, then (9) is simplified and becomes:

$$\Delta\chi = \frac{2}{M\overline{R}}\sum_{k=1}^{K}\Delta b_k\left[-\frac{\overline{R^2}}{\overline{R}^2}r_k + \frac{1}{M\overline{R}}\sum_{m=1}^{M}R_m r_{mk}\right]. \quad (10)$$

Let us write out explicitly the formula with account that for any path all Δb_k increments are negative (or equal to zero):

$$W_k = \sum_{m=1}^{M}\left(-\frac{R_m}{\overline{R}}r_k + r_{mk}\right)p_m + \frac{\overline{R^2}}{\overline{R}^2}r_k - \frac{1}{M\overline{R}}\sum_{m=1}^{M}R_m r_{mk}, \quad (11)$$

in the general case and

$$W_k = \frac{\overline{R^2}}{\overline{R}^2}r_k - \frac{1}{M\overline{R}}\sum_{m=1}^{M}R_m r_{mk}. \quad (12)$$

for requests equiprobable distribution.

The formulas (11), (12) include values, fully determined by network edge capacities, which are changed upon each next elementary requirement fill. Thus in general terms edge cost total re-calculation is needed at each step for determination of the optimal path according to this criterion. [4][5]

3.2 Fundamental edge algorithm

We want to recall that this algorithm idea is that each next path should be allowed along maximum capacity network edges. The problem of this section is to formalize this idea so that this path search appeared to be minimal cost path search.

Let us introduce the following definitions. Let us distribute all network edges by classes, in each of which let us place the same capacity edges, and let us arrange all classes in the order of included edge capacity increase. Let us assume that $B_1, \ldots, B_q, \ldots, B_Q$ – are designations of introduced classes (the fact the edge class designation coincides with the G network edges themselves designation as a matter of fact there's no risk of ambiguity), $K_1, \ldots, K_Q$ – are edge number in these classes, $b_1, \ldots, b_Q$ – are the capacities value of the edge included in these classes.

Now let C be the path from one pole to another and this path includes c_1 edges of the B_1, … class, c_Q edges of the B_Q class. By definition all c_q, $c_q \geq 0$, are integers, moreover, $\sum_{q=1}^{Q} c_q \leq N-1$, as any path cannot include more than c edges. Let us treat each path as a Q-bit number, i.e. $c_1, c_2, \ldots, c_Q$, where each "digit" is an integer less or equal to N-1. It enables us to arrange elements of path set in numerical order. Each path corresponds to unique number, but the same number can represent different paths.

Now it is possible to state path optimality criterion with the next request fill the minimal path is the optimal path, i.e. the path determining the minimal number

The problem is that it is necessary to assign such costs to the edges that the number-path having a unit in a bit and zero in previous ones, should be more than any other path-number having zero in all these bits. One of the possible methods of such costs determination is the following one:

$$W_q = (N-1)^{Q-q}, \qquad q = 1,\ldots Q \quad (13)$$

The path cost in this case is just a number, which it determines, understood to be a N-1-based positional number.

3.3 Other minimal-cut algorithms

Preliminary calculations in accordance with the (12) formulas showed that edge costs can be both positive and negative, herewith negative cycles are not rare ones. In these conditions minimal cost path selection is the NP-full problem which is inacceptable for numerical experiments. Therefore we introduce fundamental minimal-cut algorithm modification, which is as follows. Among all edges having in accordance with (12) formulas negative values we will select the maximum per modulus value and will designate it with W_0 . Let us introduce new costs in accordance with the formula

$$W_k' = W_k + W_0 + \varepsilon, \qquad \varepsilon > 0 \tag{14}$$

Such transformation makes all costs positive and deletes the problems of minimal cost paths search, but it is of course arbitrary. Such transformation effectiveness can be justified or invalidated only by numerical experiments. Let us call the algorithm based on such transformation the *additive mincut algorithm*.

Another algorithm is based solely on the simple concept that if a G network edge enters the minimal cuts, then its cost should be as higher as less is the minimal capacity of all minimal cuts, where this edge enters. Some preliminary researches resulted in the following method of edge cost determination:

$$W_m = N - 1 + (M - h_m + 1)^4 , \tag{15}$$

where h_m – is the serial number of the R_m, minimal cut value, which are ordered in accordance with capacity increase and M – is the total number of various per capacity minimal cuts. Let us call this algorithm *empirical mincut algorithm*.

3.4 Other edge algorithm

The fundamental edge algorithm is the "contrasting" algorithm, in which the next path is allowed along less capacity edges only in the case, if it is impossible to allow it along more capacity edges. This algorithm can be "softened" by introducing, like in empirical minimal-cut algorithm, continuous function of weight assigning. I.e. let us assume that edge costs are determined by the formulas

$$W_m = N - 1 + (T - a_m + 1)^4 , \tag{16}$$

where a_m – is the serial number of the class of the edges, which are arranged in accordance with capacity increase and T – is the number of edge classes.

This function was selected in accordance with the principle: the less is the arc capacity, more "expensive" it is. This algorithm was called the empirical edge algorithm.

4 Results and discussion

We developed a computer model of all main stages of telecommunication network functioning – generation of nonoriented graph with specified number of vertices, poles, edges and capacities thereof, generation of request flow, computation of the network irregularity criteria, edge cost calculation – for each algorithm there is its own calculation method – determination of minimal cost path, unit intensity flow passing and corresponding change of the network edge capacities.

The numerical experiment in general for one network is the numerical realization of this network filling process independently for each algorithm. The general factor for these experiments is, first, the initial ("empty") network and, second, the requests sequence. In all presented in this work experiments the request sequence was equiprobable for all pole pairs.

The experiment to be done in two modes – static and dynamic. In the static mode the path developed between the pole pair and the unit flow along it are kept until the experiment completion.

Thus such network resource like all its edges capacity monotonically decreases. In the dynamic mode the communication channel "lifetime" is finite and follows the probability law – we accepted the Gaussian offset law. In this mode the network filling process can include (or even completely consist of) stationary areas with failure various levels.

In the stationary mode filled requests number to first failure is registered as well as total number of filled requests and total number of failures, and the experiment is completed either when failure intensity is over a certain critical level or upon "time" expiry, i.e. of total number of the network requirements.

Networks with topology connected clusters are considered and stochastic topology of the Network consist of 30-45 nodes, have the initial capacity of arches from 30 to 500 and have 7-45 couples poles (couples a source drain). Filling is made by single flows. Mathematical modeling of process of filling of a network happens before the following criteria:

• criterion of the first refusal (carrying out the next demand is impossible, one of couples a source drain is incoherent; minimum from a set of the minimum cuts between couples of poles it is equal 0);

• criterion of full refusal (drains aren't achievable from sources, for all couples of poles; all minimum cuts between couples of poles are equal 0);

• an exit to the stationary mode when the criterion of full refusal isn't achievable, but rather large number of demands is already processed

Dynamic filling of a telecommunication network assumes that demands through certain time release network resources, that is have some time of life after which the capacities of arches belonging to the carried-out flow of the demand increase by the size of the carried-out stream.

Depending on distribution of time of life of demands filling of a network can proceed in several modes, however the stationary mode when simultaneous number of demands in a network big, sometimes there are refusals is most of all interesting, the size of the maximum stream is sometimes reached, but mass refusals aren't present; unevenness measure size depending on the carried-out demands – a constant.

By results of filling of networks with various algorithms of filling it is possible to draw a conclusion that use of algorithms of mincut and arc approach will be more effective for networks with a big initial capacity of arches (more than 150) and for a sufficient large number of couples of poles (more than 15). Duration of the stationary mode and time of effective network functioning is more when using mincut algorithms.

5 Conclusions

The methodology of the analysis of telecommunication networks (and not only, and in general, stream networks) allowing to investigate characteristics of a network in general, in particular, its total capacity depending on application of this or that policy of routing is created. The methodology is realized in the form of the mathematical model imitating process of consecutive filling of a telecommunication network with communication channels by means of various algorithms of routing. Algorithms of routing are original and in fact heuristic. All of them are directed on the maximum increase in a total general stream on a network and divided into two groups – "*minimal cut*" in which belonging of edges of a network to the minimum cuts between the corresponding couples of poles, and "*arc*" in which only the current capacity of edges of a network is considered is considered.

References

Fulkerson, D. R., & Ford, L. R. (1962). *Flows in networks*. New Jersey: Princeton University Pres.

Grinberg, Y. R., & Kurochkin, I. I. (2009). Mathematical modeling of dynamic networks of sequential filling of streams due. *Problems of computations in the distributed media: Proceedings of ISA RAS*(vol.46), p.233-258.

Grinberg, Y. R., & Kurochkin, I. I. (2009). The study of the results of mathematical modeling of networks with sequential filling of the cluster topology. *Problems of computations in the distributed media: Proceedings of ISA RAS*(vol.46), p. 198-232.

Grinberg, Y. R., Kurochkin, I. I., & Korkh, A. V. (2012). Theory and Practice of the System Analysis. *Selection of clusters in networks with dynamic flows filling*, *vol.1*, pp. 116-126. Rybinsk.

Hu, T. C. (1970). *Integer Programming and Network Flows*.

Ivnitsky, V. A. (2004). *Theory of queueing network*. Moscow: Physmatlit.

Martynov, I. A., Vatutin, E. I., & Titiov, V. S. (2014). Eighth World Conference on Intelligent Systems for Industrial Automation. *Hardware oriented classification of binary relations of graph-schemes of parallel algorithms*. Tashkent.