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The hexagon Wilson loop and the BDS ansatz for the six-gluon amplitude

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Abstract

As a test of the gluon scattering amplitude/Wilson loop duality, we evaluate the hexagonal light-like Wilson loop at two loops in $\mathcal{N}=4$ super-Yang–Mills theory. We compare its finite part to the Bern–Dixon–Smirnov (BDS) conjecture for the finite part of the six-gluon amplitude. We find that the two expressions have the same behavior in the collinear limit, but they differ by a non-trivial function of the three (dual) conformally invariant variables. This implies that either the BDS conjecture or the gluon amplitude/Wilson loop duality fails for the six-gluon amplitude, starting from two loops. Our results are in qualitative agreement with the analysis of Alday and Maldacena of scattering amplitudes with infinitely many external gluons.

1. Planar gluon amplitude/Wilson loop duality

With recent advances of the AdS/CFT correspondence, it became possible to study gluon scattering amplitudes in maximally supersymmetric Yang–Mills theory (SYM) both at weak and strong coupling.

At weak coupling, the conjecture was put forward by Bern et al. [1] that the maximally helicity-violating (MHV) amplitudes in $\mathcal{N}=4$ SYM have a remarkable all-loop iterative structure. The color-ordered planar n-gluon amplitude, divided by the tree amplitude, takes the following form,

$$\ln \mathcal{M}_n = [\text{IR divergences}] + F_n^{(\text{MHV})}(p_1,\dots,p_n) + O(\epsilon). \quad (1)$$

Here the first term on the right-hand side describes infrared divergences in the dimensional regularization scheme with $D = 4-2\epsilon$, while the second term is the finite contribution (dependent on the gluon momenta and the 't Hooft coupling $a = g^2N/(8\pi^2)$). The BDS conjecture provides an explicit expression for the finite part, $F_n^{(\text{MHV})} = F_n^{(\text{BDS})}$, for an arbitrary number n of external gluons, to all orders in the 't Hooft coupling. At present, the BDS conjecture has been confirmed up to three loops for F_4 [1] and up to two loops for F_5 [2]. An explicit verification of the conjecture for $n=6$ at two loops has not been performed up to now. However, we can at least say that were the BDS conjecture (1) to fail, it would have to be corrected by terms that satisfy an important additional consistency requirement. It originates from the known two-loop asymptotic behavior of the scattering amplitude in the collinear limit when the momenta of two neighboring on-shell gluons become collinear. In this limit, $\mathcal{M}_n$ factorizes into the product of the $(n-1)$-gluon amplitude and the universal splitting amplitude [3,4]. Since the BDS conjecture does have this property [1], any potential correction to $F_n^{(\text{BDS})}$ must vanish in this limit.

Recently, Alday and Maldacena proposed the strong coupling description of n-gluon scattering amplitudes [5] using the AdS/CFT correspondence. According to their proposal, $\ln \mathcal{M}_n$ is given by the minimal surface in AdS_5 attached to a contour C_n, made of n light-like segments $[x_i, x_{i+1}]$, with the coordinates x_i related to the on-shell gluon momenta, $x_i^\mu - x_{i+1}^\mu = p_i^\mu$,

$$\ln \mathcal{M}_n = -\frac{\sqrt{a}}{2\pi} A_{\min}(C_n). \quad (2)$$

Remarkably, for $n=4$ the corresponding minimal surface can be found explicitly and, after regularization, it leads to the same

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expression for $\ln\mathcal{M}_4$ as Eq. (1) with the finite part F_4 in agreement with the BDS ansatz. For $n\geqslant 5$ the practical evaluation of the solution of the classical string equations turns out to be difficult, but it simplifies significantly for n very large [6]. In the limit $n\to\infty$ the strong coupling prediction for $\ln\mathcal{M}_n$ disagrees with the BDS conjecture [6].

Alday and Maldacena pointed out [5] that their prescription (2) is mathematically equivalent to the calculation of a Wilson loop at strong coupling [7,8]. Inspired by this, in [9] three of us conjectured that a duality relation between planar gluon amplitudes and light-like Wilson loops also exists at weak coupling. We illustrated this relation by an explicit one-loop calculation for $n=4$. This was later extended to the case of arbitrary n at one loop in [10]. The duality relation reads

$$\ln\mathcal{M}_n=\ln W(C_n)+O(\epsilon),\tag{3}$$

with C_n the same contour as before. We have recently verified this duality at two loops for $n=4$ and $n=5$ and derived a conformal Ward identity for the light-like Wilson loop $W(C_n)$, valid to all orders in the coupling [11,12]. This Ward identity fixes the form of the finite part of the Wilson loop for $n=4$ and $n=5$, up to an additive constant, to agree with the conjectured BDS form for the corresponding gluon amplitudes. However, for $n\geqslant 6$ it allows for an arbitrary function of the conformal invariants in addition to the BDS form (for $n=6$ there are three such invariants). It is the purpose of the present Letter to determine this function for $n=6$ at two loops.

The basic object we consider is the Wilson loop in $\mathcal{N}=4$ SYM,

$$W(C_n)=\frac{1}{N}\langle 0|\operatorname{Tr}P\exp\left(i\oint_{C_n}dx^\mu\,A_\mu(x)\right)|0\rangle,\tag{4}$$

where $A_\mu(x)=A_\mu^a(x)t^a$ is a gauge field and t^a are the generators of the gauge group $SU(N)$ in the fundamental representation. We use the conventions of [11,12] and refer the interested reader to these papers for details. Even though $\mathcal{N}=4$ SYM is a finite gauge theory, the Wilson loop (4) has specific ultraviolet divergences due to the presence of cusps on the integration contour C_n [13–15]. To regularize these singularities we use dimensional regularization with $D=4-2\epsilon$. Like the scattering amplitude, the Wilson loop can be factorized into divergent and finite parts,

$$\ln W(C_n)=Z_n+F_n^{(\mathrm{WL})}.\tag{5}$$

Due to exponentiation of the cusp singularities to all loops, the divergent part Z_n has the special form [16]

$$Z_n=-\frac{1}{4}\sum_{l\geqslant 1}a^l\sum_{i=1}^n\left(-x_{i-1,i+1}^2\mu^2\right)^{l\epsilon}\left[\frac{\Gamma_{\mathrm{cusp}}^{(l)}}{(l\epsilon)^2}+\frac{\Gamma^{(l)}}{l\epsilon}\right],\tag{6}$$

where $\Gamma_{\mathrm{cusp}}^{(l)}$ and $\Gamma^{(l)}$ are the expansion coefficients of the cusp anomalous dimension and the so-called collinear anomalous dimension, respectively, defined in the adjoint representation of $SU(N)$:

$$\Gamma_{\mathrm{cusp}}(a)=\sum_{l\geqslant 1}a^l\Gamma_{\mathrm{cusp}}^{(l)}=2a-\frac{\pi^2}{3}a^2+O\big(a^3\big),$$

$$\Gamma(a)=\sum_{l\geqslant 1}a^l\Gamma^{(l)}=-7\zeta_3a^2+O\big(a^3\big).\tag{7}$$

In [11,12] we confirmed these relations by an explicit two-loop calculation of the divergent part of W_4 and W_5.

The duality relation (3) implies that upon a specific identification of the regularization parameters, the infrared divergences of the scattering amplitude $\mathcal{M}_n$ match the ultraviolet divergences of the light-like Wilson loop $W(C_n)$ and, most importantly, the finite parts of the two objects also coincide up to an inessential additive constant,

$$F_n^{(\mathrm{MHV})}=F_n^{(\mathrm{WL})}+\mathrm{const}.\tag{8}$$

While the former property immediately follows from the known structure of divergences of scattering amplitudes/Wilson loops in a generic gauge theory [15], the latter property (8) is extremely non-trivial.

In this Letter we report on the two-loop calculation of $F_6^{(\mathrm{WL})}$. We find that $F_6^{(\mathrm{WL})}\neq F_6^{(\mathrm{BDS})}$, with their difference being a nontrivial conformally invariant function of the gluon momenta. At the same time, $F_6^{(\mathrm{WL})}$ has the same collinear limit behavior as the six-gluon amplitude $F_6^{(\mathrm{MHV})}$.

2. Finite part of the hexagon Wilson loop

The finite part of the hexagon Wilson loop, $F_6^{(\mathrm{WL})}$, does not depend on the renormalization scale and it is a dimensionless function of the distances x_{ij}^2. Since the edges of C_6 are light-like, $x_{i,i+1}^2=0$, the only nonzero distances are $x_{i,i+2}^2$ and $x_{i,i+3}^2$ (with $i=1,\dots,6$ and the periodicity condition $x_{i+6}=x_i$). We argued in [11,12] that the conformal symmetry of the Wilson loop in $\mathcal{N}=4$ SYM imposes severe constraints on $F_n^{(\mathrm{WL})}$. It has to satisfy the following Ward identity,

$$\sum_{i=1}^n\big(2x_i^\nu x_i\cdot\partial_i-x_i^2\partial_i^\nu\big)F_n=\frac{1}{2}\Gamma_{\mathrm{cusp}}(a)\sum_{i=1}^n\ln\frac{x_{i,i+2}^2}{x_{i-1,i+1}^2}x_{i,i+1}^\nu.\tag{9}$$

Specified to $n=6$, its general solution is given by [11]

$$F_6^{(\mathrm{WL})}=F_6^{(\mathrm{BDS})}+f(u_1,u_2,u_3).\tag{10}$$

Here, upon the identification $p_i=x_i-x_{i+1}$,

$$F_6^{(\mathrm{BDS})}=\frac{1}{4}\Gamma_{\mathrm{cusp}}(a)\sum_{i=1}^6\left[-\ln\left(\frac{x_{i,i+2}^2}{x_{i,i+3}^2}\right)\ln\left(\frac{x_{i+1,i+3}^2}{x_{i,i+3}^2}\right)+\frac{1}{4}\ln^2\left(\frac{x_{i,i+3}^2}{x_{i+1,i+4}^2}\right)-\frac{1}{2}\mathrm{Li}_2\left(1-\frac{x_{i,i+2}^2x_{i+3,i+5}^2}{x_{i,i+3}^2x_{i+2,i+5}^2}\right)\right],\tag{11}$$

while $f(u_1,u_2,u_3)$ is an arbitrary function of the three cross-ratios[3]

$$u_1=\frac{x_{13}^2x_{46}^2}{x_{14}^2x_{36}^2},\qquad u_2=\frac{x_{24}^2x_{15}^2}{x_{25}^2x_{14}^2},\qquad u_3=\frac{x_{35}^2x_{26}^2}{x_{36}^2x_{25}^2}.\tag{12}$$

[3] The last term in (11) is a function of cross-ratios only, but we keep it in $F_6^{(\mathrm{BDS})}$, because it is part of the BDS conjecture.

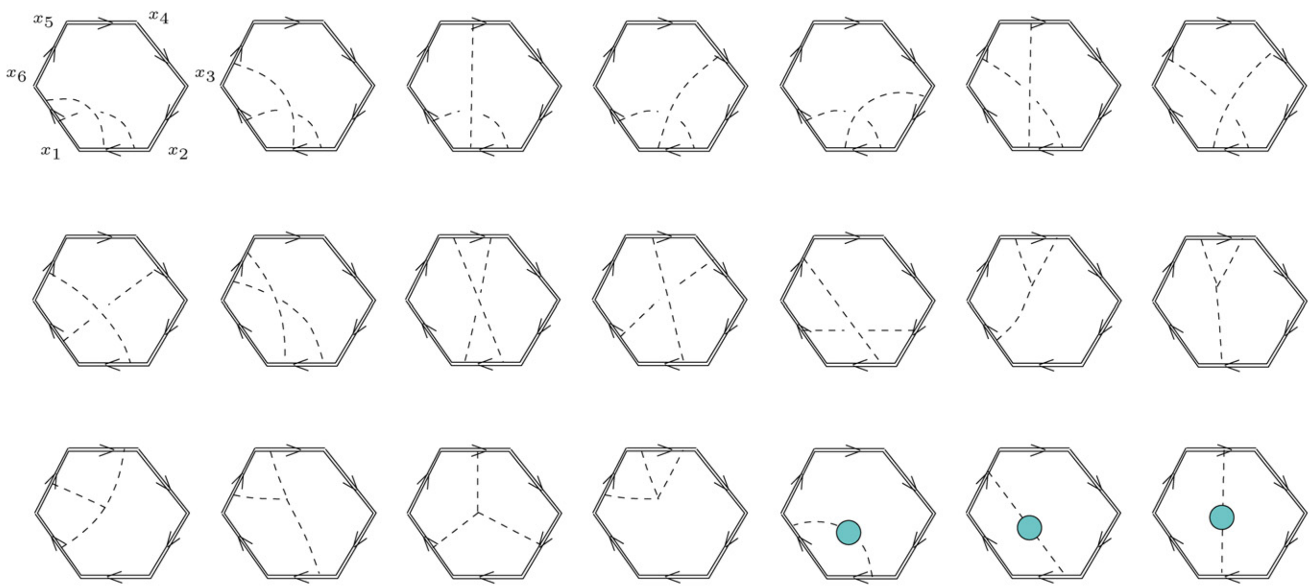

Fig. 1. The maximally non-Abelian Feynman diagrams of different topology contributing to $F_6^{(\mathrm{WL})}$. The double lines depict the integration contour C_6, the dashed lines—the gluon propagator and the blob—the one-loop polarization operator.

These variables are invariant under conformal transformations of the coordinates x_i^μ and, therefore, they are annihilated by the conformal boost operator entering the left-hand side of (9). In addition, the Wilson loop $W(C_6)$ is invariant under cyclic ($x_i \to x_{i+1}$) and mirror ($x_i \to x_{6-i}$) permutations of the cusp points [12]. This implies that $f(u_1, u_2, u_3)$ is a totally symmetric function of three variables.

Combining together (8) and (10), we conclude that were the BDS conjecture *and* the duality relation (8) correct for $n = 6$, we would expect that $f(u_1, u_2, u_3) = \text{const}$. The explicit two-loop calculation we report on here shows that this is not true.

For the sake of simplicity we performed the calculation of $W(C_6)$ in the Feynman gauge, in the DRED scheme. In addition, we made use of the non-Abelian exponentiation property of Wilson loops [17] to reduce the number of relevant Feynman diagrams. In application to $f(u_1, u_2, u_3)$ this property can be formulated as follows (the same property also holds for $F_6^{(\mathrm{WL})}$)

$$f = \frac{g^2}{4\pi^2} C_F f^{(1)} + \left(\frac{g^2}{4\pi^2}\right)^2 C_F N f^{(2)} + O\big(g^6\big), \tag{13}$$

where $C_F = t^a t^a = (N^2 - 1)/(2N)$ is the Casimir in the fundamental representation of the $SU(N)$. The functions $f^{(1)}$ and $f^{(2)}$ do not involve the color factors and only depend on the distances between the cusp points on C_6. At one loop, $f^{(1)}(u_1, u_2, u_3)$ is in fact a constant [10].

As explained in [11], the relation (13) implies that in order to determine the function $F_6^{(\mathrm{WL})}$ at two loops (and hence $f^{(2)}$) it is sufficient to calculate the contribution to $W(C_6)$ from two-loop diagrams containing the 'maximally non-Abelian' color factor $C_F N$ only. All relevant two-loop graphs are shown in Fig. 1. We derived parameter integral representations for all the Feynman graphs. The integrals are difficult to evaluate analytically and so we calculated them numerically for many different sets of values of x_{ij}^2.[4] We found that, firstly, for values of x_{ij}^2 related by conformal boosts (hence leaving u_1, u_2, u_3 invariant), the difference $F_6^{(\mathrm{WL})} - F_6^{(\mathrm{BDS})}$ remains constant. Thus, it only depends on the cross-ratios (12), in agreement with (10). Secondly, varying the values of the cross-ratios we found that $f^{(2)}(u_1, u_2, u_3) \neq \text{const}$ (see Figs. 2 and 3), i.e., it is a non-trivial function of u_1, u_2, u_3.

This means that either the BDS conjecture, or the gluon amplitude/Wilson loop duality (or both) is not correct for $n = 6$, starting from two loops. At this stage, we cannot discriminate between the different scenarios. Nevertheless, we can show that $F_6^{(\mathrm{WL})}$ has the same collinear limit behavior as $F_6^{(\mathrm{BDS})}$ at two loops, i.e., $f^{(2)}(u_1, u_2, u_3)$ tends to a constant in the collinear limit.

We recall that for the six-gluon amplitude $\mathcal{M}_6$ depending on light-like momenta, $\sum_{i=1}^6 p_i^\mu = 0$ and $p_i^2 = 0$, the collinear limit amounts to letting, e.g., p_5^μ and p_6^μ be nearly collinear (see, e.g., [3] for more details), so that $(p_5 + p_6)^2 \to 0$ and

$$p_5^\mu \to z P^\mu, \qquad p_6^\mu \to (1 - z) P^\mu, \tag{14}$$

with $P^2 = 0$ and $0 < z < 1$ being the momentum fraction. Using the identification $p_i^\mu = x_i^\mu - x_{i+1}^\mu$, we translate these relations into properties of the corresponding Wilson loop $W(C_6)$. We find that the cusp at point 6 is 'flattened' in the collinear limit and the contour C_6 reduces to one with five cusps. In terms of the distances x_{ij}^2, the collinear limit

[4] One should bear in mind that the allowed values of x_{ij}^2 have to obey kinematical constraints. They originate from the six gluon momenta p_i^μ satisfying $p_i^2 = 0$ and $\sum_{i=1}^6 p_i^\mu = 0$. Solving these constraints is not a trivial task. We are grateful to Fernando Alday for sharing with us his numerical solutions for the kinematical configurations.

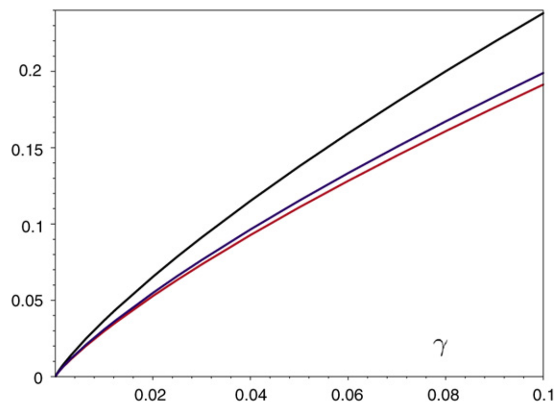

Fig. 2. The γ-dependence of the function $\hat{f}^{(2)}(\gamma, u, 1-u)$, Eq. (18), for different values of the parameter $u = 0.5$ (lower curve), $u = 0.3$ (middle curve) and $u = 0.1$ (upper curve).

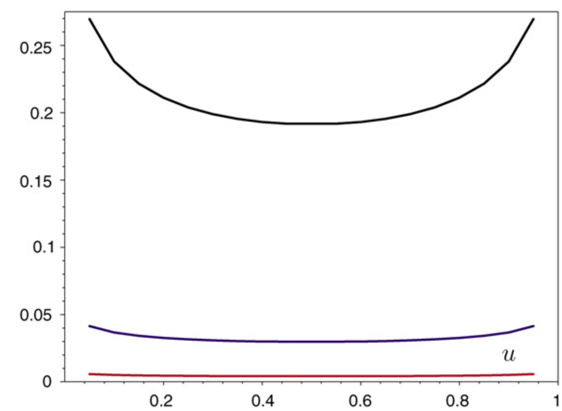

Fig. 3. The u-dependence of the function $\hat{f}^{(2)}(\gamma, u, 1-u)$, Eq. (18), for different values of the parameter $\gamma = 0.001$ (lower curve), $\gamma = 0.01$ (middle curve) and $\gamma = 0.1$ (upper curve).

amounts to

$$x_{15}^2 \to 0, \qquad x_{36}^2 \to z x_{13}^2 + (1-z) x_{35}^2,$$
$$x_{46}^2 \to x_{14}^2, \qquad x_{26}^2 \to (1-z) x_{25}^2, \tag{15}$$

while the other distances $x_{13}^2, x_{24}^2, x_{25}^2, x_{35}^2$ remain unchanged. For the conformal cross-ratios the relation (15) implies

$$u_1 \to u, \qquad u_2 \to 0, \qquad u_3 \to 1-u, \tag{16}$$

with $u = z x_{13}^2/(z x_{13}^2 + (1-z) x_{35}^2)$ fixed. As was already mentioned, the relation (10) is consistent with the collinear limit of the six-gluon amplitude provided that, in the limit (16), the function $f(u_1, u_2, u_3)$ approaches a finite value independent of the kinematical invariants. The same property can be expressed as follows (we recall that the function $f(u_1, u_2, u_3)$ is totally symmetric)

$$f(0, u, 1-u) = c, \tag{17}$$

with c being a constant. Using our two-loop results for the finite part F_6, we performed thorough numerical tests of the relation (17) for different kinematical configurations of the contour C_6.

We found that, in agreement with (17), the limiting value of the function $f^{(2)}(\gamma, u, 1-u)$ as $\gamma \to 0$ does not depend on u. Since the duality relation (8) is not sensitive to the value of this constant, it is convenient to subtract it from $f^{(2)}(\gamma, u, 1-u)$ and introduce the function

$$\hat{f}^{(2)}(\gamma, u, 1-u) = c - f^{(2)}(\gamma, u, 1-u), \tag{18}$$

which satisfies $\hat{f}^{(2)}(0, u, 1-u) = 0$. To summarize our findings, in Fig. 2 we plot the function $\hat{f}^{(2)}(\gamma, u, 1-u)$ against γ for different choices of the parameter $0 < u < 1$ and in Fig. 3 the same function against u for different choices of the parameter γ. The important region for the collinear limit is where γ is close to zero. We also give numerical values for a range of values of γ such that one can see how the function $f^{(2)}(u_1, u_2, u_3)$ varies in the particular parametrization $u_1 = \gamma$, $u_2 = u$, $u_3 = 1-u$.

3. Conclusions

Given the results we have presented in this Letter, it is urgent to know the six-gluon amplitude at two loops. Depending on the outcome of this calculation, we can envisage the following three scenarios:

- If the duality between amplitudes and Wilson loops persists for the six-gluon amplitude at two loops, then the BDS conjecture fails and the difference will be given by the function $f^{(2)}(u_1, u_2, u_3)$ that we have found.
- If the BDS conjecture holds, then the duality between amplitudes and Wilson loops breaks down for $n = 6$ at two loops.
- If the gluon amplitude disagrees with both the BDS ansatz and the Wilson loop, then it would be very interesting to verify whether it still respects dual conformal symmetry [11,12,18] (i.e., the difference is a function of the conformal cross-ratios).

The finite part of the one-loop MHV amplitude involves functions of the kinematical invariants of transcendentality 2 (double logs and dilogs). We expect that this is a general feature, i.e., the finite part of $\ln \mathcal{M}_n$ should have maximal transcendentality 2ℓ at ℓ loops. This is indeed true for the BDS ansatz (11), where the non-trivial functions are of transcendentality 2 and the factor $\Gamma_{\rm cusp}(a)$ is supposed to supply the remaining transcendentality $2(\ell-1)$. There are a priori no reasons why functions of higher transcendentality should not appear at higher loops, provided that they have the general analyticity properties of gluon amplitudes, including the correct collinear limit behavior. An example for this is the constant term in the finite part of $\ln W(C_n)$, as we have demonstrated by explicit two loop calculations for $n = 4$ and $n = 5$. We conjecture that the same property holds for arbitrary n to all orders. In particular, we expect that our two-loop function $F_6^{(\rm WL)} - F_6^{(\rm BDS)} = f(u_1, u_2, u_3)$ has transcendentality 4. Needless to say, it would be very interesting to identify its analytical form.

Independently of the outcome of the two-loop calculation of the six-gluon amplitude, we have presented an example of a non-trivial function which is not captured by the BDS ansatz and which has the right collinear limit properties to appear in the final two-loop expression for the six-gluon amplitude.

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