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# A Branch-and-Price Algorithm to Solve a Quay Crane Scheduling Problem

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**Abstract**

As the maritime industry grows rapidly in size, more attention is being paid to to a wide range of aspects of problems faced at ports with respect to the efficient allocation of resources. A very important seaside planning problem that has received large attention in literature lately is the quay crane scheduling problem (QCSP). The problem involves the creation of a work schedule for the available quay cranes at the port to empty the containers from a vessel or given set of vessels. These optimization problems can be very complex and since they involve a large number of variables and constraints, the use of a commercial solver is impractical. In this paper, we reformulate a problem currently available in the literature to a Dantzig-Wolfe formulation that can be solved by column generation. We then develop a branch-and-price algorithm, which is an exact method, to effectively solve mixed integer programs with very large instances. The algorithm is first tested on a formulation currently available in literature with a small instance and will then be tested on large instances.


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*Keywords:* martime logistics, quay crane scheduling problem, dantzig-wolfe, column generation, branch-and-price

## 1. Introduction

According to the International Chamber of Shipping[1] more than 90% of the world's trade relies on maritime transport and logistics, which means that port operations have a direct impact on the global economy. These numbers are expected to increase annually with the continuous dependence on maritime transport as the main medium for international trade. This is why considerable attention is being given to these port operations in the literature.

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**Nomenclature**

| | |
|---|---|
| FA | set of all feasible crane to bay assignments, indexed by *fa* |
| J | set of all bays on a given vessel, indexed by *j* |
| N | number of cranes in the problem |
| $\alpha_j$ | dual variable associated with constraint (bay) *j* of the restricted master problem $\forall j \in J$ |
| $\theta_j$ | number of containers on bay *j* of the vessel $\forall j \in J$ |
| $\upsilon$ | safety distance that must be kept between two consecutive cranes |

Port operations are mainly governed by three activities; berth allocation of arriving vessels, crane assignment to already berthed vessels and crane operation on the assigned vessels. Berth allocation involves assigning appropriate berths to the currently waiting or arriving vessels and is often referred to as the berth allocation problem (BAP). Crane assignment involves assigning the available cranes to the already berthed vessels to load and unload the containers and is often referred to as the quay crane assignment problem (QCAP). The operations of the cranes on the assigned vessels involves the schedule that the cranes should follow when loading and unloading containers and is often referred to as the quay crane scheduling problem (QCSP).

The cost of constructing new berths or purchasing more cranes to accomplish the assigned jobs on time is very high and hence the majority of the literature focuses on optimizing these seaside operations in a sense that minimizes cost or operation time. In their paper, Al-Dhaheri and Diabat[2] develop a formulation for the QSCP that minimizes the overall time to accomplish all jobs, taking into consideration a safety margin between the cranes at all times.

Since the QCSP is a famous problem known to be NP-Complete (high level proof shown in section 3) and there is no algorithm so far that can solve it in polynomial time, the main contribution of this paper is to apply an exact method algorithm, called branch-and price, to the work of Al-Dhaheri and Diabat[2] that can solve the problem in a reasonable amount of time as opposed to commercial solvers that have failed to solve the problem for big instances of input data. Branch-and-price is a branch-and-bound method where the nodes are solved using column generation. The original problem in Al-Dhaheri and Diabat[2] is reformulated into a Dantzig-Wolfe formulation. The problem would then consist of a restricted master problem (RMP) and a sub-problem. The sub- problem would generate a new column to be entered into the RMP recursively until the sub- problem stops pricing out negatively. We then branch over the variable that is closest to 0.5 in its decimal and repeat the process until we reach an integer solution.

The remainder of the paper is organized as follows. Section 2 is the literature review. Section 3 is the model, in which the formulation and algorithm are explained. Section 4 is the numerical example and results, where two different instances are used in the model and their results are shown and discussed. Section 5 is the conclusion.

## 2. Literature Review

The optimization of seaside processes has attracted considerable attention in the literature especially around the three main problems described in section 1, namely BAP, QCAP and QCSP. Imai *et al.*[3] developed a multiobjective formulation for the static BAP, where the operations on the vessels only start after all vessels have arrived and are waiting at the bay. Their formulation assumed the assignment of vessels to bays was discrete, meaning only one vessel can be assigned to one bay at a time. Their goal was to minimize the total waiting and handling time of all the vessels. Imai *et al.*[4] developed a formulation similar to that of Imai *et al.*[3] except that it is a dynamic BAP where vessels may be arriving while other vessels are under operation. Simrin and Diabat[6] develop a formulation for the disceret dynamic BAP and then go on to solve it using a genetic algorithm to solve the instances they use. Simrin *et al.*[6] developed a Lagrangian-relaxation based heuristic using the cutting planes method to solve the discrete static BAP proposed by Imai *et al.*[3].

Fu and Diabat[7] develop a formulation for an integrated quay crane assignment and scheduling problem. They then go on to solve the model using a Lagrangian-relaxation based heuristic using the cutting planes method. Their results show that an integrated problem gives better results in terms of minimizing the makespan than solving the QCAP and QCSP each independently. Fu *et al.*[8] also develop a formulation for an integrated quay crane assignment and scheduling problem. They develop a genetic algorithm to solve the proposed model. Diabat and Theodorou[9] developed a formulation for an integrated quay crane assignment and scheduling problem and then go on to solve it using a genetic algorithm. Their results show that an improvement of approximately 31% is achieved when solving the integrated problem than when solving each problem independently. Theodorou and Diabat[10] develop an integrated quay crane assignment and scheduling problem formulation and then go on to provide a solution using a Lagrangian-relaxation based heuristic using the cutting planes method. Their solution methodology proves to be very efficient when solving small to medium sized instances and they mostly do not require a feasibility restoration process while solving large instances results in a big optimality gap. Zeng *et al.*[11] develop a mixed-integer formulation for a dual cycling operation in ports. This allows for the loading and unloading of containers from vessels simultaneously. They go on to solve it by developing a bi-level genetic algorithm and their results show that a dual cycling operations yields more efficient results than scheduling separate loading and unloading processes.

## 3. Model

Lee *et al.*[12] showed that the QCSP is NP-Complete. The proof is a little bit involved, so only a high level explanation is provided. To show that a problem is NP-Complete, one needs to show that it is both NP and NP-Hard. First, QCSP is NP; given a quay crane schedule, its feasibility can be checked in quadratic time i.e. polynomial time. Second, QCSP is NP-Hard; the PARTITION problem, a known NP-Complete problem (Garey and Johnsong[13]), can be reduced in polynomial time to the QCSP as shown by Lee *et al.*[12]. The PARTITION problem is about the possibility of partitioning a set S whose elements sum up to a number D into two disjoint subsets, in which the elements of each sum up to D/2.

### *3.1. Dantzig-Wolfe Reformulation for the restricted master problem*

Decision Variables

$z_{fa}$ The number of times a feasible solution is chosen for the schedule $fa \in FA$

Formulation

$$\min \sum_{fa \in FA} z_{fa} \tag{1}$$

$$\text{s.t.} \quad \sum_{fa \in FA} z_{fa} \geq \theta_j \quad \forall j \in J \tag{2}$$

$$z_{fa} \geq 0 \qquad \forall fa \in FA \tag{3}$$

The objective function (1) minimizes the total number of assignments used to unload the vessel completely. In a sense, the value of the objective function is the total number of time units needed to complete all jobs (empty all bays). Constraints (2) make sure that the cranes are assigned in such a way that all bays on the vessel are essentially emptied. Constraints (3) define the nature of our decision variables.

### *3.2. Sub-problem*

The sub-problem is used to generate feasible assignments that will be inserted as columns into the restricted master problem. The formulation of the sub-problem is the following

Decision Variables

$$x_j = \begin{cases} 1 & \text{if a crane is assigned to bay j} \quad \forall j \in J \\ 0 & \text{Otherwise} \end{cases}$$

Formulation

$$\min \quad 1-\sum_{j\in J}\alpha_j x_j \tag{4}$$

$$\text{s.t.} \quad \sum_{i=j}^{j+\upsilon} x_i \le 1 \qquad \forall j = 1,\ldots,|J|-(N-1) \tag{5}$$

$$\sum_{j\in J} x_j = N \tag{6}$$

$$x_j = \{0,1\} \qquad \forall j \in J \tag{7}$$

The objective function (4) of the sub-problem prioritizes which bays require cranes assigned to them depending on the value of the dual variables of the constraint in the restricted master problem. Constraints (5) take into consideration the safety margin between two cranes i.e. the number of bays between any two consecutive cranes should not be less than υ. Constraint (6) ensures that each crane should be assigned to a bay at every time interval. Constraints (7) define the decision variable to be binary, taking either a value of 0 or 1.

*3.3. The branch-and-price algorithm*

A clear description of the branch-and-price algorithm is provided in figure 1. The code was written on MATLAB R2014b.

```
Data: The nuber of cranes and bays are entered
Result: Each crane is assigned a schedule to empty the vessel in the shortest period of time possible
initialization;
while At least one node is not fathomed do
        Solve the RMP;
        Use the dual variables to solve the sub- problem;
        if Sub- problem prices out negatively then
                add the solution as a new colum to the RMP;
        else
                check solution to the RMP;
                if Solution to the RMP us non- integer and current
                  objective value is less than or equal to all stored valued then
                        branch on the variables with a solution
                        closest to the 0.5 decimal place;
                else
                        fathom the node and store objective value
                end
        end
end
```

Figure 1. Branch-and-price algorithm used to solve the Dantzig-Wolfe reformulation

## 4. Numerical example and results

All computations were done on an Intel Core i5 2.60 GHz processor with 8.00 GB RAM DELL computer running on Windows 7 Professional 64-bit. To test the strength of the branch-and-bound algorithm, one small instance and one large instance was chosen. Details of the small instance are provided in section 4.1 while details of the large instance are provided in section 4.2.

### *4.1. Small Instance*

The small instance consisted of a single vessel with six bays and two quay cranes working on it with a safety margin of one bay between the cranes. The load distribution is shown in figure 2. Commercial software such as CPLEX or GAMS were able to solve this instance in Al-Dhaheri's[2] original formulation to optimality. Thus, the branch-and-price algorithm was tested on the same instance and provided the same results which confirms that the code written was correct. The results are shown in figure 3.

### *4.2. Large Instance*

The large instance consisted of a single vessel with ten bays using three quay cranes and a safety margin of one bay between the cranes. The load distribution is shown in figure 2. Commercial software such as CPLEX or GAMs were not able to solve this instance in Al-Dhaheri's[2] original formulation to optimality while the branch-and-price algorithm developed provided a solution within a matter of seconds. The results are shown in figure 3.

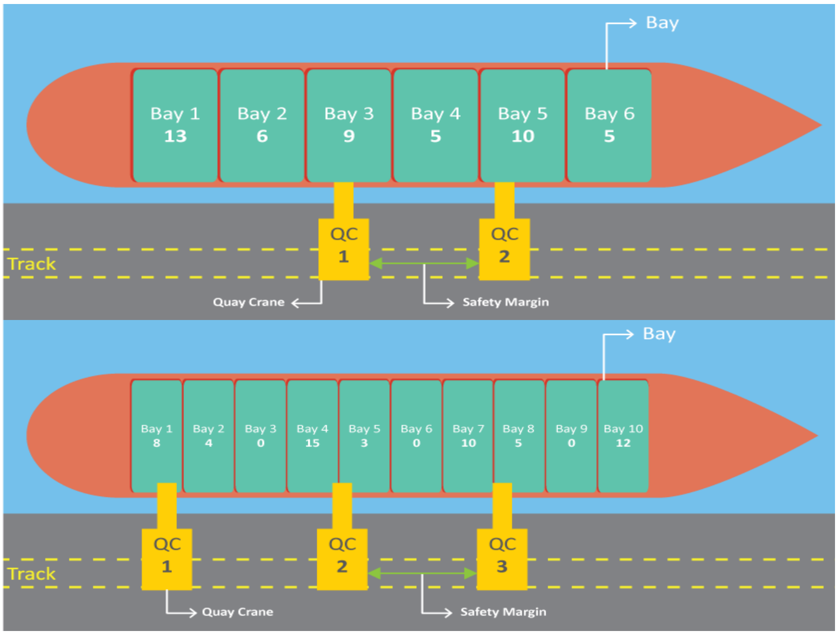


Figure 2. Container load distribution for the small instance (upper image) and large instance (lower image)

## 5. Conclusion and future work

In this paper, we have developed a branch-and-price algorithm that was used to solve a QCSP currently existing in the literature. It was first tested on a small instance and results were compared to those of commercial software and then later used to solve a large instance that commercial software failed to solve to optimality. For the large instance, the branch-and-price algorithm solved the problem in a matter of seconds giving an optimal answer.

In the future, we will relax the assumption that the cranes have no moving time as this tends to make the model more realistic. Other modifications to the code will include it generating its own feasible solution in the beginning (if one exists).

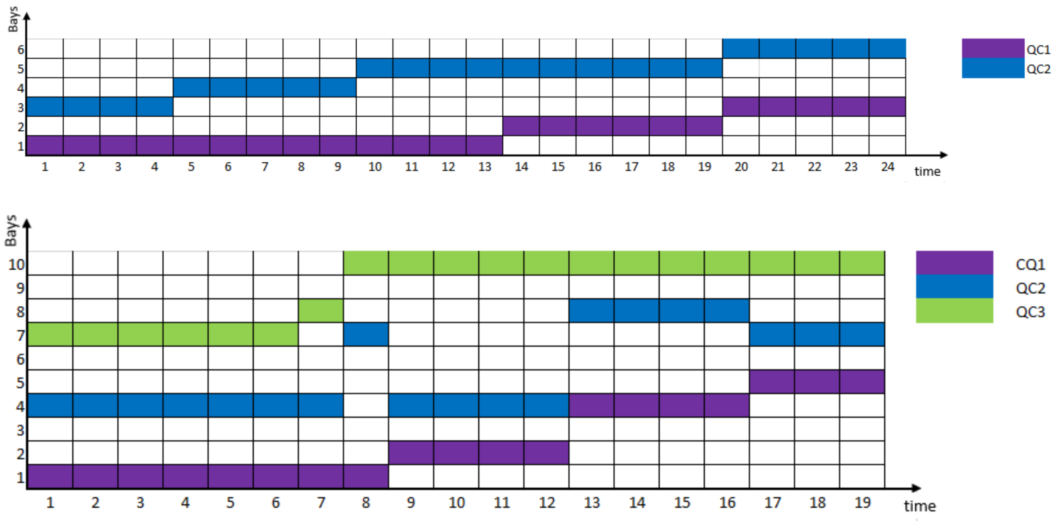


Figure 3. The small instance (upper) requires 24 time units to completely unload the vessel while the large instance (lower) requires 19 time units to completely unload the vessel. The y-axis shows the location of each quay crane at each time unit.

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