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Image Segmentation Based on Level Set Method

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Abstract

Level set method can be effectively used to solve topology problems during the evolution of curves while the previous algorithms cannot deal with them. In recent years, there are many image segmentation algorithms based on level set method. For different applications of image processing, people have put forward the corresponding solutions, and a large number of researchers also continue to improve and enhance the efficiency and effectiveness of these algorithms. In this article, according to the development of the image segmentation methods based on level set, an overview is given for readers of different backgrounds in this field to use, and their characteristics are discussed.

key words—Image segmentation, Level set method, C-V Model

Introduction

Snake[1] is parameterized curve or surface which iter-atively evolves toward the desired locations according to the energy minimization criterion. The limitations of such kind of contour models are well known. For example, they are sen-sitive to initial conditions and should be placed usually near to the boundary of objects of interest. Besides, due to the explicit parameterization of the model, they cannot cope with sig-nificant protrusions and topological changes.

Although the people have made various improvements to the basic model, the shortcomings of Snake are still not over-come fundamentally[2-8].An algorithm to overcome these diff-iculties was first introduced by Osher and Sethian[9-10].

Borrowing ideas from hydromechanics, Osher and Set-hian put forward an algorithm named level set in 1988. Level set is a numerical solution for processing topological changes of contours. This algorithm has been widely used in the field of image processing and has made great progress in image segmentation especially. In this article, an overview on the developments for image segmentation based on level set

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method is given for readers of different backgrounds in this field to use, and their characteristics are discussed.

Level set method

Many of the PDEs used in image processing are based on moving curves and surfaces with curvature-based velocities. In this area, the level set method was very influential and useful. The basic idea is to represent the curves or surfaces as the zero level set of a higher dimensional hyper-surface. This technique not only provides more accurate numerical implementations but also handle topological change very easily.

Basically, it means that the closed curves in a two-dimensional surface are regarded as a continuous surface of a three-dimensional space. The definition of a smoothing function $\phi(x,y,t)$ stands for the surface while the set of definitions $\phi(x,y,t)=0$ for the curves. Thus, the evolution of a curve can be transformed into the evolution of a three-dimensional Level Set function. Given a Level Set function $\phi(x,y,t=0)$, whose zero level set corresponds to curve. With the curve as the boundary, the whole surface can be divided into an internal region and an external region of the curve. Define a Signed Distance Function (SDF) on the surface:

$$\phi(x,y,t=0)=d \tag{1}$$

Where, the value of d is the shortest distance between the point of x on the surface and the curve. In the whole evolutional process of the curve, its points will fit into the following formula:

$$\phi(x,y,t)=0 \tag{2}$$

The common movement formula of Level Set is:

$$\phi_t + F|\nabla\phi| = 0 \tag{3}$$

F is the speed function, which is a function related to evolving surface characteristics (e.g. curvature, normal direction, etc.) and image characteristics (e.g. gray, gradient). When applied into image segmentation, the design of F depends on the information of image and the ideal value is zero on the edge of the target (i.e. the bigger value of the gray gradient).

Level set method, due to its stability and irrelevancy with topology, displays a great advantage in solve the problems of corner point producing, curve breaking and combing, etc. Therefore, it is used in a wide range [11-12]. However, there are several disadvantages to this approach. Since the edge-stopping function depends on the image gradient, only objects with edges defined by gradient can be segmented. Another disadvantage is that in practice, the edge-stopping function is never exactly zero at the edges, and so the curve may eventually pass through object boundaries.

In implementing the level set method, it is numerically necessary to keep the evolving level set function close to a signed distance function [13,14]. Re-initialization, a technique for periodically re-initializing the level set function to a signed distance function during the evolution, has been extensively used as a numerical remedy for maintaining stable curve evolution and ensuring usable results. So many methods were put forward to implement re-initialization of level set function. But these methods are basically through solving a partial differential equation to achieve re-initialization during the iterative process of the level set function.

In 2005, Dr Li chunming introduced a new variational formulation that forced the level set function to be close to a signed distance function, and therefore completely eliminated the need of the costly re-initialization procedure[15]. The energy function consists of an internal energy term and an external energy term, respectively. The internal energy term $P(\phi)$ penalizes the deviation of the level set function from a signed distance function, whereas the external energy term $\varepsilon(\phi)$ drives the motion of the zero level set to the desired image features such as object boundaries. The resulting evolution of the level set function is the gradient flow that minimizes the overall energy functional. The energy function is:

$$E(\phi) = \mu P(\phi) + \varepsilon_{g,\lambda,v}(\phi) = \mu\int_{\Omega}\frac{1}{2}(|\nabla\phi|-1)^2 dxdy + \lambda\int_{\Omega} g\delta(\phi)|\nabla\phi| dxdy + v\int_{\Omega} gH(\phi)dxdy \quad (4)$$

Where $\mu > 0$ is a parameter controlling the effect of penalizing the deviation of ϕ from a signed distance function, and g is the edge indicator function defined by

$$g = \frac{1}{1+|\nabla G_{\sigma} * I|^2} \quad (5)$$

Let I is an image, and G_{σ} is the Gaussian kernel with standard deviation σ.

It is well known that a signed distance function must satisfy a desirable property of $|\nabla\phi| = 1$. So $P(\phi)$ reflected the deviation between level set function and the signed distance function exactly. Meanwhile, due to the penalizing effect of the internal energy, the evolving function ϕ will be naturally and automatically maintained as an approximate signed distance function during the evolution. Therefore the re-initialization procedure is completely eliminated in the proposed formulation.

The formulation proposed by Dr Li has three main advantages over the traditional level set formulations. First, a significantly larger time step can be used for numerically solving the evolution, and therefore speeds up the curve evolution. Second, the level set function could be initialized as functions that are computationally more efficient to generate than the signed distance function. Third, the proposed level set evolution can be implemented using simple finite difference scheme, instead of complex upwind scheme as in traditional level set formulations..

Narrow band method

The original level set method proposed by Osher and Sethian is relatively simple and easy programming. However, it updated all the level sets, not just the zero level set. Therefore the computing time is unbearable and the computational efficiency is not high. To solve the problem above, Adalsteinsson and Sethian introduced a method named narrow band method[16],which confine computation to a narrow band around the interface of interest. The narrow band was of arbitrary size. As the front moved and reached the edge of the narrow band, the calculation was stopped, and a new initial level set function corresponding to the signed distance function was re-built. This was known as re-initialization.

Because the distance of each evolution of the curve is very small and does not suddenly jump to a location far from the place, it is required to update the level set function of a very narrow band near zero level set. The basic idea is that an adaptive narrow-band is conducted around the contour. During each evolution, only function values of the grid points within the narrow band are updated. To prevent the curve points across the narrow band in the process of curve evolution, the inner and outer boundaries of narrow band are needed to store. When the curve points are close to the inner and outer boundaries, the narrow band is needed to re-initialize. As width of the narrow band is generally narrow, narrow points need to update in the narrow band are not much. Therefore, the calculation for updating level set function is greatly reduced.

However, the points of zero level set function may exceed the scope of narrow band after several iterations. So, the main problems of narrow band are dynamic update of the narrow band points and re-initialization of the distance function.

C-V model

Models described above are based on boundary information for image segmentation. Although these models superior to the traditional segmentation based on edge detection, they are also on the basis of edge detection theory. So these models have same limitation as traditional segmentation methods have. That is to say they are sensitive to noise. So, a new energy functional for homogeneity-based segmentation derived from the work of Mumford and Shah was proposed by Chan and Vese[17]. C-V model is not based on an edge-function, like in the classical active contour models, to stop the evolving curve on the desired boundary. We do not need to smooth the initial image, even if it is very noisy and in this way, the locations of boundaries are very well detected. Also, C-V model can detect objects whose boundaries are not necessarily defined by gradient or with very smooth boundaries. The model automatically detects interior contours, starting with only one initial curve. The initial curve does not necessarily start around the objects to be detected.

The basic idea is to look for a particular partition of a given image into two regions, one representing the objects to be detected and one representing the background. Consider a simple case where the image $I(x,y)$ is formed by two regions of piecewise constant intensity. Denote the intensity values by c_1 and c_2. Furthermore, assume that the object to be detected has a region whose boundary is C_0 and intensity c_1. Then inside(C_0), the intensity of $I(x,y)$ is approximately c_1, whereas outside(C_0) the intensity of $I(x,y)$ is approximately c_2. Then consider the fitting term:

$$\begin{aligned} F(c_1,c_{2,}C) = {} & \mu \cdot Length(C) + \nu \cdot Area(inside(C)) \\ & + \lambda_1 \int_{inside(C)} \left| I(x,y) - c_1 \right|^2 dxdy \\ & + \lambda_2 \int_{outside(C)} \left| I(x,y) - c_2 \right|^2 dxdy \end{aligned} \tag{6}$$

where C is a curve, and the constants c_1, c_2 are the averages of $I(x,y)$ inside and outside of C respectively. Length(C) is the length of the closed border and Area(inside(C)) is the area of the interior of C. $\mu,\nu \geq 0$, λ_1, λ_2 are fixed parameters. When the closed border C does not lie on the border of the two homogeneous regions, F(C) cannot have the minimum value. Only when the contour lies on the border of two homogeneous regions can F(C) have the minimum value. Finally, the fitting energy is minimized if C=C_0. Accordingly, the simplified Mumford-Shah Model takes advantage of the entire information of the image to result in the best image segmentation.

C-V model is not based on an edge-function to stop the evolving curve on the desired boundary, so it can detect objects whose boundaries are not necessarily defined by gradient or those with very smooth boundaries. But C-V model is only carried out on the premise that image has only two types of homogeneous regions and has limitation for multi-region segmentation. C-V model using one level set function is not able to obtain sub-objects in the object because one level set can only represent one object and one background via its sign. Although Chen and Vese proposed a multi-level set C-V model[18], the model is numerically necessary to use the Euler-Lagrange equation to solve the problem of functional minimization. So it requires very high numerical stability. And its each iteration requires the calculation of all image data and the computing time is unbearable.

To overcome the shortcoming for the C-V model, a large number of researchers had conducted studies deeply and made improvements accordingly. For example, Danhua-Xu and others proposed a regional division method makes segmentation of an image into segmentation of a number of images[19]. The method enhanced the image segmentation capabilities for rich level gray images and then solved the problem of image segmentation for separating different gray target. The basic idea is the image field is divided into N different sub-regions and images of each sub-region u_0 have been divided into N sub-region u_{0i} accordingly. A closed boundary C_i divided each sub-region u_{0i} into two homogeneous regions: target (internal of C_i) and background (external of C_i) of sub-region. The energy function of each sub-region is:

$$E_i(c_{1i},c_{2i},C_i) = E_{inside(C_i)} + E_{outside(C_i)} =$$

$$\mu_{1i} \int_{inside(C_i)} (u_{0i} - c_{1i})^2 dxdy + \mu_{2i} \int_{outside(C_i)} (u_{0i} - c_{2i})^2 dxdy$$

(7)

At this point, the energy function of the whole image is sum of the external energy and internal energy of various sub-regions.

$$E(u_1,u_2,\cdots,u_N;c_{11},c_{12},\cdots,c_{1N},c_{21},c_{22},\cdots,c_{2N};C_1,C_2,\cdots,C_N)=\sum_{i=1}^{N}E_i$$
$$=\sum_{i=1}^{N}\mu_{1i}\int_{inside(C_i)}(u_{0i}-c_{1i})^2\,dxdy+\sum_{i=1}^{N}\mu_{2i}\int_{outside(C_i)}(u_{0i}-c_{2i})^2\,dxdy \quad (8)$$

To keep $|\nabla\phi|=1$ of distance functions of the implicit surface function of each sub-region, Xu and others borrowed ideas from reference [15].They added penalty term in the energy function and eliminated the need for re-initialization of distance function.

Conclusion

Curve evolution, which is very difficult to solve previously, was effectively handled by level set method. At present, there are so many image segmentation methods based on level set. For different applications of image processing, people have put forward the corresponding solutions, and a large number of researchers also continue to improve and enhance the efficiency and effectiveness of these algorithms. Now, level set method has become an important method for image segmentation.

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