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Group method of data handling to predict scour depth around vertical piles under regular waves

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Abstract This paper presents a new application of the Group Method Of Data Handling (GMDH), to predict pile scour depth exposed to waves. The GMDH network was developed using the Levenberg–Marquardt (LM) method in the training stage for scour prediction. Scour depth due to regular waves was modeled as a function of five dimensionless parameters, including pile Reynolds number, grain Reynolds number, sediment number, Keulegan–Carpenter number, and shields parameter. The testing results of the GMDH-LM were compared with those obtained using the Adaptive Neuro-Fuzzy Inference System (ANFIS), Radial Basis Function-Neural Network (RBF-NN), and empirical equations. In particular, the GMDH-LM provided the most accurate prediction of scour depth compared to other models. Also, the Keulegan–Carpenter number has been determined as the most effective parameter on scour depth through a sensitivity analysis. The GMDH-LM was utilized successfully to investigate the influence of the pile cross section and Keulegan–Carpenter number on scour depth.

1. Introduction

When a pile is placed in an erodible bed, scour phenomena will take place around it, due to the action of waves and currents. The pile scour is a complex process, in which the main element is the horseshoe vortex. This flow structure is formed around the piles just above the bed surface. This process is an important factor related to the stability of marine structures. Development of the scour process around hydraulic structures may reduce its stability, hence, leading to failure [1] (Figure 1). Wave and current-induced scour around piles has been widely investigated by several researchers. Local scour experiments around hydraulic structures exposed to wave or oscillatory flow have attracted wide attention, due to their applications to offshore coastal structures, bridges, and sea templates [1–12]. Extensive experiments have been carried out around small and large pile scour depths under waves, and combined waves and current. Also, studies have been conducted to analyze the influence of circular and square piles on the scour. Researchers have proposed empirical equations through their experiments [10–12]. For instance, Dey et al. [1] proposed easy and economical methods to reduce piles scour depth under waves and currents. They applied splitter pales, and the average reduction of scour depth was 61.6%.

Recently, various artificial intelligent approaches, such as Artificial Neural Networks (ANNs), Adaptive Neuro-Fuzzy Inference Systems (ANFIS), data mining, Linear Genetic Programming (LGP), and Genetic Programming (GP), were applied to predict scour depth around hydraulic structures [13–17]. Recently, GMDH networks were used to predict scour depth around bridge piers, and the GMDH performance produced a better estimation of scour depth than that of empirical equations [18–24].

In fact, the GMDH network is known as a system of identification methods implemented in various fields of engineering science. GMDH networks model and forecast the behavior of unknown or very complicated systems, based on given input–output data pairs [18,19,25–27].

The paper follows two main aims. First, the GMDH network is developed using an LM algorithm to predict scour depth

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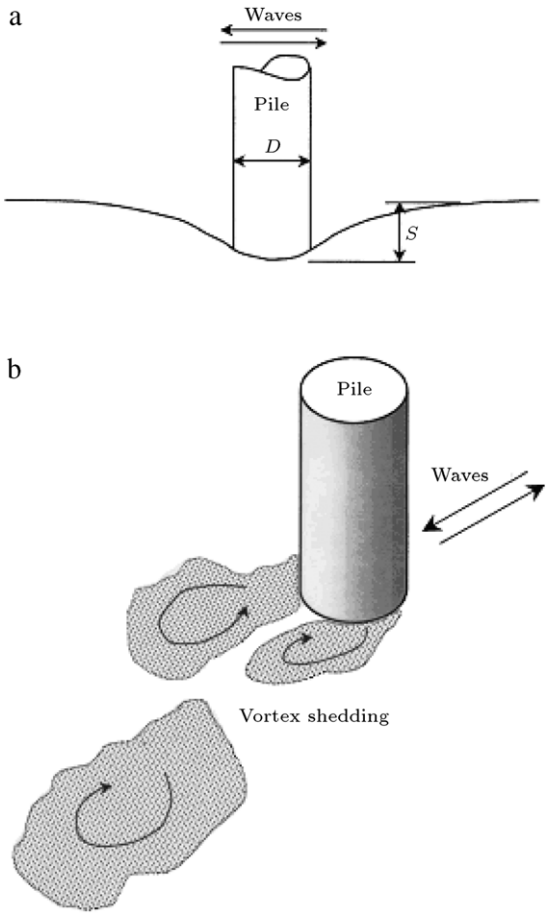

Figure 1: (a) Schematic diagram of a scour hole at a vertical circular pile under wave. (b) Vortex shedding is the dominant mechanism for scour at piles under waves.

around a vertical pile under regular waves. Also, the testing results of the GMDH-LM are compared with those obtained using the ANFIS model, RBF-NN, and empirical equations. Second, limitations of data variations on the GMDH-LM are studied.

2. Data presentation

Experimental and field investigations of scour prediction have been carried out under clear-water and live-bed conditions of flow. In the present study, used data sets were collected from Dey et al. [1] (41 data sets), Sumer et al. [28] (43 data sets), and Sumer et al. [10] (9 data sets) to train and test the models. According to the denoted experiments, the function of scour depth can be expressed as follows:

$$S = f(d_{50}, g, U_m, U_{fm}, T, \upsilon, D, \rho), \tag{1}$$

where S, d_{50}, g, U_m, U_{fm}, T, υ, D, and ρ are scour depth, median particle diameter, acceleration due to gravity, maximum value of outer oscillatory velocity in the wave, maximum value of undisturbed shear velocity in the wave period, wave period, dynamic viscosity, pile diameter, and mass density of water, respectively.

Base on the preceding investigations, dimensionless parameters performed more accurate scour prediction than dimensional parameters [14–17]. The following dimensionless parameters have been obtained using the Buckingham theorem:

$$S/D = f(Re, Re_d, N_s, KC, \theta), \tag{2}$$

where Re, Re_d, N_s, KC, and θ are Reynolds number for pile, grain Reynolds number, sediment number, Keulegan–Carpenter number, and shields parameter. These dimensionless parameters are presented as follows:

$$Re = U_m D/\upsilon, \tag{3}$$

$$Re_d = U_{fm} d_{50}/\upsilon, \tag{4}$$

$$N_s = U_m/\sqrt{g(G_s - 1) \cdot d_{50}}, \tag{5}$$

$$KC = U_m T/D, \tag{6}$$

$$\theta = U_{fm}^2/g(G_s - 1) \cdot d_{50}. \tag{7}$$

Occasionally, the range of effective parameters on scour depth is a limiting factor in the development of artificial intelligence techniques [14,16]. For instance, Sumer et al. [11] investigated the influence of pile cross section on the wave scour. They carried out 54 pile scour experiments, in which scour depth was not observed for seven runs. Also, Sumer and Fredsoe [12] performed 27 experiments for flow conditions, in which, waves and current have been combined. In this study, their experimental data sets are not used for the development of these soft computing tools, because experimental conditions are significantly distinctive from that of Sumer et al. [28] and Dey et al. [1] data sets.

It can be concluded that the selection of data sets depends widely on the purpose of the research and the availability of effective parameters on a phenomena. The dimensionless parameters of Eq. (2) were used as input and output parameters in the development of models. The ranges of data sets are presented in Table 1. In the present study, about 75% data sets (72 sets) were selected randomly for the training stage, whereas the remaining 25% (21 sets) were used for the testing stage.

Table 1: Ranges of original data set.

Parameter	Range
d_{50} (m)	0.0006–0.0002
D (m)	0.01–0.08
T (s)	1.2–3.84
U_m (m/s)	0.128–0.388
U_{fm} (m/s)	0.0143–0.0249
S (m)	0–0.0335
g (m/s^2)	9.806
ρ (kg/m^3)	1000
μ (Pa · s)	0.000001

3. Description of models

In this section, development of the GMDH network, ANFIS model, and RBF-NN will be discussed separately.

3.1. Descriptions of the GMDH network

The GMDH network is a learning machine based on the principle of heuristic self-organizing, proposed by Ivakhnenko in the 1960s. Also, it is a series of operations, such as seeding, rearing, crossbreeding, and the selection and rejection of seeds, corresponding to the determination of the input variables, structure and parameters of the model, and the selection of the model by the principle of termination [29,30].

The GMDH network is a very flexible structure and it can be hybridized using evolutionary and iterative algorithms, such as the genetic algorithm, genetic programming, particle swarm optimization, and back propagation [18,19,27,31–34]. Previous research established that hybridizations were successful in finding solutions of problems in different fields of engineering.

By means of the GMDH network, a model can be represented as a set of neurons, different pairs of which, in each layer, are connected through a quadratic and triquadratic polynomial; thus producing new neurons in the next layer. Such representation can be used in modeling, from map inputs to outputs. Formal definition of the system identification problem is to find a function, $\hat{f}$, that can be approximately used instead of an actual function, f, in order to predict the output, $\hat{y}$, for a given input vector $X = (x_1, x_2, x_3, \ldots, x_n)$, as close as possible to its actual output, y. Therefore, given n observations of multi-input-single-output data pairs, so that:

$$y_i = f(x_{i1}, x_{i2}, x_{i3}, \ldots, x_{in}) \quad (i = 1, 2, \ldots, M), \tag{8}$$

it is now possible to train the GMDH network to predict the output values, $\hat{y}_i$, for any given input vector $X = (x_{i1}, x_{i2}, x_{i3}, \ldots, x_{in})$, that is:

$$\hat{y}_i = \hat{f}(x_{i1}, x_{i2}, x_{i3}, \ldots, x_{in}) \quad (i = 1, 2, \ldots M). \tag{9}$$

In order to solve this problem, the GMDH builds the general relationship between output and input variables, in the form of a mathematical description, which is also called a reference.

The problem is now determining the GMDH network, so that the square of difference between the actual and the predicted output is minimized, that is:

$$\sum_{i=1}^{M} \left[\hat{f}(x_{i1}, x_{i2}, x_{i3}, \ldots., x_{in}) - y_i\right]^2 \to \min. \tag{10}$$

The general connection between input and output variables can be expressed by a complicated discrete form of the Volterra function, a series in the form of:

$$y = w_0 + \sum_{i=1}^{n} w_i x_i + \sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij} x_i x_j + \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n} w_{ijk} x_i x_j x_k + \cdots, \tag{11}$$

which is known as the Kolmogorov–Gabor polynomial [29,30,35,36]. In the present study, the quadratic polynomial of the GMDH network is used, which is written as:

$$\text{Quadratic} : \hat{y} = G(x_i, x_j) = w_0 + w_1 x_i + w_2 x_j + w_3 x_i x_j + w_4 x_i^2 + w_5 x_j^2. \tag{12}$$

This network of connected neurons builds the general mathematical relation of input and output variables given in Eq. (11). The weighting coefficients of Eq. (12) are calculated using regression techniques, so that the difference between actual output, y, and the calculated one, $\hat{y}$, for each pair of x_i and x_j, as input variables, is minimized [35,36]. Indeed, it can be seen that a tree of polynomials is constructed using the quadratic form given in Eq. (12), whose weighting coefficients can be obtained by the least-squares sense. In this way, the weighting coefficients of quadratic function G_i are obtained to optimally fit the output in the whole set of input–output data pairs, that is:

$$E = \frac{\sum_{i=1}^{M} (y_i - G_i())^2}{M} \to \min. \tag{13}$$

In the basic form of the GMDH algorithm, all the possibilities of two independent variables out of total n input variables are taken, in order to construct the regression polynomial in the form of Eq. (11) that best fit the dependent observations $(y_i, i = 1, 2, \ldots, M)$ in a least-square sense. Consequently, $C_n^2 = n(n-1)/2$ neurons of quadratic polynomial will be built up in the first layer of the feed forward network from observations, $\{(y_i, x_{ip}, x_{iq}); (i = 1, 2, \ldots M)\}$, for different $p, q \in \{1, 2, \ldots .n\}$. In other words, it is now possible to construct M data triples, $\{(y_i, x_{ip}, x_{iq}); (i = 1, 2, \ldots M)\}$, from observation, using such $p, q \in \{1, 2, \ldots .n\}$, in the form of:

$$\begin{bmatrix} x_{1p} & x_{1q} & y_1 \\ x_{2p} & x_{2q} & y_2 \\ x_{mp} & x_{mq} & y_m \end{bmatrix}. \tag{14}$$

Using the quadratic sub-expression in the form of Eq. (11) for each row of M data triples, the following matrix equation can be readily obtained as:

$$AW = Y, \tag{15}$$

where W is the vector of unknown weighting coefficients of the quadratic polynomial in Eq. (15):

$$W = \{w_0, w_1, w_2, w_3, w_4, w_5\}^T. \tag{16}$$

The superscript, T, representing the transpose of the matrix:

$$Y = \{y_1, y_2, y_3, \ldots, y_M\}^T, \tag{17}$$

is the vector of the observation values of outputs. It can be readily seen that:

$$A = \begin{bmatrix} 1 & x_{1p} & x_{1q} & x_{1p}x_{1q} & x_{1p}^2 & x_{1q}^2 \\ 1 & x_{2p} & x_{2q} & x_{2p}x_{2q} & x_{2p}^2 & x_{2q}^2 \\ 1 & x_{mp} & x_{mq} & x_{mp}x_{mq} & x_{mp}^2 & x_{mq}^2 \end{bmatrix}. \tag{18}$$

The least-squares technique from multiple-regression analysis leads to the solution of the normal equations in the form of:

$$W = (A^{TA})^{-1} A^T Y, \tag{19}$$

which determines the vector of the best weighting coefficients of quadratic Eq. (11) for the whole set of M data triples.

3.1.1. Development of the GMDH network using Levenberg–Marquardt algorithm

The Levenberg–Marquardt (LM) algorithm is an iterative technique that locates the minimum of a function that is expressed as the sum of the squares of nonlinear functions. Details of the Levenberg–Marquardt (LM) algorithm are presented completely in the literature [37–39]. In this section, the learning method of the improved GMDH network is explained in brief. As one example, the following case is considered. In Figure 2, x_K and z_s are the input and intermediate variables, respectively. W_{ts} denotes the weight vector. Furthermore, X_{ts} is the input vector for the neurons (t = number of layers and s = number of neurons in layer). These variables are defined as follows:

$$E = \frac{(y - \hat{y})^2}{2}, \tag{20}$$

$$\hat{y} = W_{21}^T \cdot X_{21}, \tag{21}$$

$$z_1 = W_{11}^T \cdot X_{11}, \tag{22}$$

$$z_2 = W_{12}^T \cdot X_{12}, \tag{23}$$

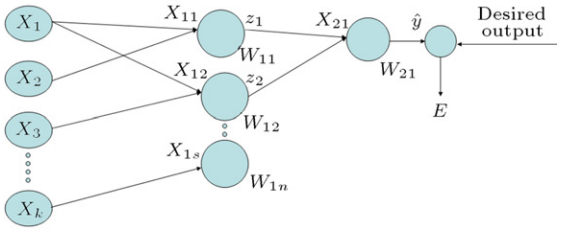

Figure 2: Structure of the GMDH network.

$$W_{21} = \left\{w_{21}^0, w_{21}^1, w_{21}^2, w_{21}^3, w_{21}^4, w_{21}^5\right\}^T, \tag{24}$$

$$W_{11} = \left\{w_{11}^0, w_{11}^1, w_{11}^2, w_{11}^3, w_{11}^4, w_{11}^5\right\}^T, \tag{25}$$

$$W_{12} = \left\{w_{12}^0, w_{12}^1, w_{12}^2, w_{12}^3, w_{12}^4, w_{12}^5\right\}^T, \tag{26}$$

$$X_{21} = \left\{1, z_1, z_2, z_1z_2, z_1^2, z_2^2\right\}^T, \tag{27}$$

$$X_{11} = \left\{1, x_1, x_2, x_1x_2, x_1^2, x_2^2\right\}^T, \tag{28}$$

$$X_{12} = \left\{1, x_1, x_3, x_1x_3, x_1^2, x_3^2\right\}^T. \tag{29}$$

The authors have defined the Jacobian matrix as the partial differentiation taken for the error function, based on the chain rule, that is:

$$[J_{2s}] = \left\{\frac{\partial E}{\partial W_{2s}}\right\}^T = \frac{\partial E}{\partial \hat{y}}\left\{\frac{\partial z_s}{\partial w_{2s}^0}, \frac{\partial z_s}{\partial w_{2s}^1}, \frac{\partial z_s}{\partial w_{2s}^2}, \frac{\partial z_s}{\partial w_{2s}^3}, \frac{\partial z_s}{\partial w_{2s}^4}, \frac{\partial z_s}{\partial w_{2s}^5}\right\}^T, \tag{30}$$

$$[J_{1s}] = \left\{\frac{\partial E}{\partial W_{1s}}\right\}^T = \frac{\partial E}{\partial \hat{y}}\left\{\begin{array}{l}\frac{\partial \hat{y}}{\partial X_{2s}}\cdot\frac{\partial X_{2s}}{\partial z_s}\cdot\frac{\partial z_s}{\partial w_{1s}^0}, \frac{\partial \hat{y}}{\partial X_{2s}}\cdot\frac{\partial X_{2s}}{\partial z_s}\cdot\frac{\partial z_s}{\partial w_{1s}^1},\\ \cdot\frac{\partial \hat{y}}{\partial X_{2s}}\cdot\frac{\partial X_{2s}}{\partial z_s}\cdot\frac{\partial z_s}{\partial w_{1s}^2}, \frac{\partial \hat{y}}{\partial X_{2s}}\cdot\frac{\partial X_{2s}}{\partial z_s}\cdot\frac{\partial z_s}{\partial w_{1s}^3},\\ \frac{\partial \hat{y}}{\partial X_{2s}}\cdot\frac{\partial X_{2s}}{\partial z_s}\cdot\frac{\partial z_s}{\partial w_{1s}^4}, \frac{\partial \hat{y}}{\partial X_{2s}}\cdot\frac{\partial X_{2s}}{\partial z_s}\cdot\frac{\partial z_s}{\partial w_{1s}^5}\end{array}\right\}^T. \tag{31}$$

Thus, the learning laws are obtained as follows:

$$W_{2s}^{new} = W_{2s}^{old} + \left[J_{2s}^T J_{2s} + \mu \cdot I\right] J_{2s}^T E, \tag{32}$$

$$W_{1s}^{new} = W_{1s}^{old} + \left[J_{1s}^T J_{1s} + \mu \cdot I\right] J_{1s}^T E, \tag{33}$$

where μ is learning rate between 0 and 1.

When increasing the layer, the update rules corresponding to each layer are derived based on the same idea. The initial layer is simply the input layer. The first layer is created by computing regressions of the input variables and then choosing the best ones. The second layer is created by computing regressions of the values in the first layer, along with the input variables. This means that the algorithm essentially builds polynomials of polynomials. Again, only the best are chosen by Eq. (20). This mechanism will be continued until a prespecified selection criterion is met. In the output layer, errors of the training network, estimated by Eq. (20), and the new weighting coefficients, are calculated using Eqs. (30)–(33). Again, the output of each neuron is estimated from the first layer to the output layer. This process is called feed forward, and the correction of weighting coefficients of a network is called backward pass. This mechanism is to be continued until errors in the training network (E) are minimized.

The value of the learning rate, μ, was adjusted to 0.01 for training the GMDH-LM. Furthermore, from adjusting the weighting coefficients using Levenberg–Marquardt, the corresponding quadratic polynomial neurons were presented as follows:

$$(S/D)_2^1 = -0.1734 - 0.0266N_S + 0.0488\theta + 0.0129(N_S)(\theta) + 0.0182(N_S)^2 + 0.0406(\theta)^2, \tag{34}$$

$$(S/D)_8^1 = -0.0785 + 0.0213KC + 1.1734 \times 10^{-5}Re + 3.3563 \times 10^{-7}(KC)(Re) - 1.1385 \times 10^{-4}(KC)^2 + 4.0551 \times 10^{-7}(Re)^2, \tag{35}$$

$$(S/D)_9^1 = -0.0306 - 0.0419KC + 6.3156Re_d + 1.8427(KC)(Re_d) + 7.577 \times 10^{-4}(KC)^2 - 226.2307(Re_d)^2, \tag{36}$$

$$(S/D)_1^2 = -0.012 + 0.1195(S/D)_2^1 + 0.09(S/D)_5^1 + 0.0142(S/D)_2^1(S/D)_5^1 + 0.0074((S/D)_2^1)^2 + 0.0067((S/D)_5^1)^2 \tag{37}$$

$$(S/D)_2^2 = 0.194 + 0.0068(S/D)_2^1 - 3.064 \times 10^{-4}(S/D)_8^1 + 7.0109 \times 10^{-5}(S/D)_2^1(S/D)_8^1 - 1.2048 \times 10^{-5}((S/D)_2^1)^2 - 1.6134 \times 10^{-6}((S/D)_8^1)^2, \tag{38}$$

$$(S/D)_1^3 = -0.0266 + 0.9618(S/D)_1^2 + 0.165(S/D)_2^2 - 0.0643(S/D)_1^2(S/D)_2^2 - 0.0405((S/D)_1^2)^2 - 0.0074((S/D)_2^2)^2. \tag{39}$$

Superscripts and subscripts of S/D from Eqs. (34)–(39) indicate the number of layers and number of neurons in the layer, respectively.

3.2. Adaptive neuro-fuzzy inference system

ANFIS, first introduced by Jang [40], is an approximator that is capable of approximating any real continuous function on a compact set to any degree of accuracy. The basic structure of fuzzy modeling, commonly known as the Fuzzy Inference System (FIS), is based on knowledge that can be inferred from available data or verbal information [41,42]. In this study, a Fuzzy Inference System (FIS) was generated for the scour modeling. It has five inputs and one output. The ANFIS model was trained using a hybrid algorithm, which is a combination of gradient descent and the least squares method. In this way, the best value of the learning rate in back propagation, μ, was adjusted to 0.01 for the ANFIS model. Also, it was generated using 4 rules for scour depth prediction (Table 2).

3.3. Artificial neural network

In this study, RBF-NN was used to predict scour depth. An important property of RBF neural networks is that a high-dimensional-space nonlinear-problem can be easily broken down through a set of combinations of radial basis functions [43]. The input layer is composed of n input neurons. The hidden layer consists of j locally tuned units and each unit has a radial basis function acting like a hidden neuron. In the present study, the proposed RBF-NN has 5 neurons in the input layer, 5 neurons in the hidden layer, and 1 neuron in the output layer. Also, this network was trained using 0.7 spread value.

Table 2: Rules of ANFIS model.

Dimensionless Parameters
1.If (Re is Re mf1) and (N_s is N_s mf1) and (θ is θ mf1) and (KC is KC mf1) and (Re_d is Re_d mf1) then (S/D is S/D mf1)
2. If (Re is Re mf2) and (N_s is N_s mf2) and (θ is θ mf2) and (KC is KC mf2) and (Re_d is Re_d mf2) then (S/D is S/D mf2)
3.If (Re is Re mf3) and (N_s is N_s mf3) and (θ is θ mf3) and (KC is KC mf3) and (Re_d is Re_d mf3) then (S/D is S/D mf3)
4. If (Re is Re mf4) and (N_s is N_s mf4) and (θ is θ mf4) and (KC is KC mf4) and (Re_d is Re_d mf4) then (S/D is S/D mf4)

Table 3: Statistical results of the testing stage for proposed technique.

Method	R	RMSE	MAE
ANFIS	0.954	0.0789	0.362
GMDH-LM	**0.983**	**0.0697**	**0.33**
RBF-NN	0.883	0.097	0.739
Empirical Equation	0.93	0.087	0.429

4. Result and discussion

The best performances of the GMDH network, RBF-NN, and ANFIS model for testing stages are validated in terms of the common statistical R (correlation coefficient), MAE (Mean Absolute of Error), and RMSE (Root Mean Square Error):

$$\mathrm{R} = \frac{\sum_{i=1}^{M}(Y_{i(Actual)} - \overline{Y}_{(Actual)})(Y_{i(Model)} - \overline{Y}_{(Model)})}{\sqrt{\sum_{i=1}^{M}(Y_{i(Actual)} - \overline{Y}_{(Actual)})^2 . \sum_{i=1}^{M}(Y_{i(Model)} - \overline{Y}_{(Model)})^2}}, \quad (40)$$

$$\mathrm{MAE} = \frac{1}{M}\left[\sum_{i=1}^{M}\left|\frac{Y_{i(Actual)} - Y_{i(Model)}}{Y_{i(Actual)}}\right|\right], \quad (41)$$

$$\mathrm{RMSE} = \left[\frac{\sum_{i=1}^{M}(Y_{i(model)} - Y_{i(Actual)})^2}{M}\right]^{1/2}, \quad (42)$$

where $Y_{i(model)}$ is the predicted values (network output), $\overline{Y}_{(Model)}$ is the average of predicted values, $Y_{i(Actual)}$ is the observed values (target), $\overline{Y}_{(Actual)}$ is the average of observed values, and M is the total of events. Sumer et al. [10] proposed the following empirical equation using experimental data for predicting the scour depth around a vertical pile due to oscillatory flow:

$$S/D = 1.3(1 - \exp(-0.03(KC - 6))). \quad (43)$$

This equation was suggested for live-bed conditions and $KC \geq 6$.

The statistical results of the proposed artificial intelligence approaches for training and testing stages are presented in Table 3. From Table 3, the performance of the GMDH network provided a scour depth prediction with a lower error ($R = 0.983$, $RMSE = 0.0697$, and $MAE = 0.33$) than those obtained using the ANFIS model and RBF-NN. The testing results indicated that the ANFIS model predicted scour depth with a relatively lower error ($RMSE = 0.0789$ and $MAE = 0.362$) and higher correlation coefficient ($R = 0.954$) in comparison with the RBF-NN and Eq. (43).

Eq. (43) can be said to fail in scour depth prediction with relatively high error ($RMSE = 0.0087$ and $MAE = 0.429$), compared to the GMDH-LM and ANFIS model.

The scatter plot between predicted and observed scour depth values for testing stages has been illustrated in Figure 3.

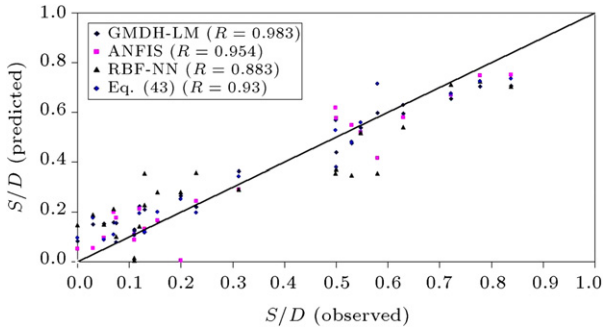

Figure 3: Scatter plot of observed values versus predicted values for the testing stage.

Table 4: Results of sensitivity analysis for independent dimensionless parameters.

Model	R	RMSE
$S/D = f(Re, Re_d, KC, \theta)$	0.959	0.085
$S/D = f(Re_d, N_s, KC, \theta)$	0.888	0.124
$S/D = f(Re_d, N_s, KC, \theta)$	0.941	0.0965
$S/D = f(Re, Re_d, N_s, KC)$	0.938	0.107
$S/D = f(Re, Re_d, N_s, \theta)$	0.74	0.184

Through the qualitative comparisons, Figure 3 indicates that most data points are concentrated on the best fit line. It should be noted that some of the predicted values are to be out of trend. For scour prediction of the zero values, the GMDH-LM and ANFIS model produced S/D with a relatively lower error than that of the RBF-NN and Eq. (43). For scour prediction of 0.11 and 0.2 values, the ANFIS model and RBF-NN predicted the S/D with quite higher error, compared to the GMDH-LM.

Also, it can be seen that the performance of the RBF-NN is lacking a generalization capacity. The generalization capacity is a common problem which has been described in preceding investigations [17]. Furthermore, the main advantage of the GMDH is that only six weighting coefficients are available in each neuron. The GMDH network has been developed using 6 quadratic polynomials neurons through trial and error processes, in order to reduce the calculations of each neuron [18,19].

5. Sensitivity analysis

To assign the relative significance of each of the effective parameters on scour depth, the GMDH-LM was applied to carry out a sensitivity analysis. The analysis was conducted in the absence of every parameter in dimensionless parameters. Results of the sensitivity analysis were given in Table 4. From Table 4, it is found that the Keulegan–Carpenter number, KC, ($R = 0.74$, $RMSE = 0.184$), is the most effective parameter on the scour depth. The other effective parameters on normalized scour depth, S/D, are seen to be ranked (from higher to lower) in the order: Re, θ, Re_d, and N_s. To develop the results of the

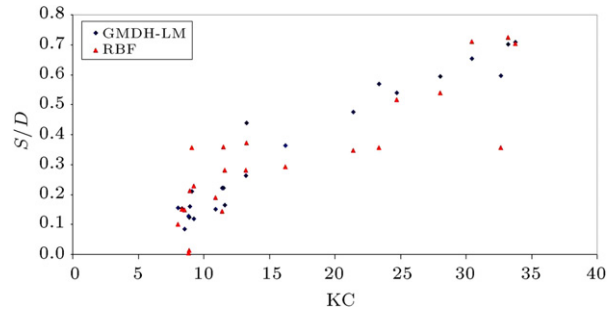

Figure 4: Variations of S/D versus KC using the GMDH-LM and RBF-NN.

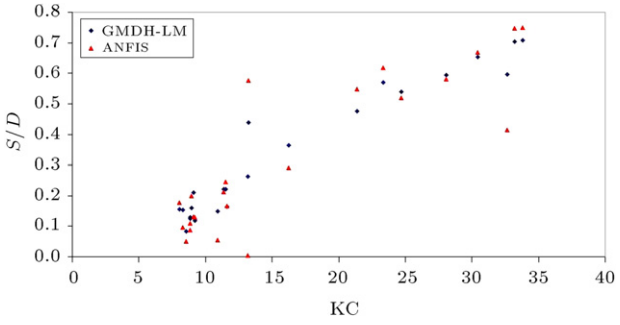

Figure 5: Variations of S/D versus KC using the GMDH-LM and ANFIS model.

sensitivity analysis, a parametric analysis was conducted using the GMDH-LM, RBF-NN, and ANFIS model. Variations of S/D versus KC were plotted in Figures 4 and 5. From Figure 4, it was found that the scour depth predicted using the ANFIS model is to be out of trend for the KC values of 13.2 and 32.64. Also, Figure 5 indicated quite a higher error of scour depth prediction for 32.64 of KC. As seen, the GMDH-LM is the most consistent with the underlying physical processes, while it was found that the other methods are too sensitive to variations of KC values.

The scour depth under regular waves was predicted using data mining and linear genetic programming [14,16]. The fundamental restriction of these approaches is the range of applicability that is restricted to the range of effective parameters utilized for training and testing stages. Hence, the efficiency of the GMDH-LM was investigated to cover these restrictions using experimental data sets that were not used in the training and testing of the models. The influence of cross section pile and the Keulegan–Carpenter number on scour depth have been considered. In this way, 44 data sets from the Sumer et al. [11] experiments were selected to study the influence of pile cross section on the scour depth. The live-bed experiments were carried out around a square pile with two arrangements of 90° and 45°. Schematic arrangements of square piles are illustrated in Figure 6. Ranges of data sets are presented in Table 5. Evaluation of the GMDH-LM for the square pile with a 45° arrangement produced the lower error of scour depth prediction, compared to that of the 90° arrangement. Results of the performances were compared with those obtained using empirical equations. Sumer et al. [11] presented two empirical equations for the square piles as follows:

$$S/D = 2(1 - \exp(-0.015(KC - 11))) \quad \text{For } KC \geq 11, \qquad (44)$$

$$S/D = 2(1 - \exp(-0.019(KC - 3))) \quad \text{For } KC \geq 3. \qquad (45)$$

Eqs. (44) and (45) were validated for the square pile with 90° and 45° orientations, respectively.

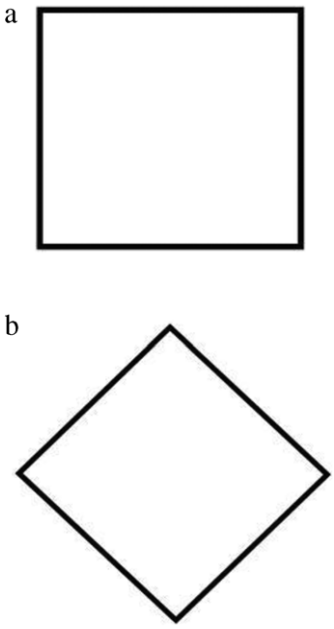

Figure 6: Square piles with (a) 0° orientation, and (b) 45° orientation.

Table 5: Ranges of original data sets for robustness of proposed GMDH-LM.

Parameter	Sumer et al. [10]	Sumer et al. [11]
	Range	Range
d_{50} (m)	0.00018–0.00058	0.00018
D (m)	0.01–0.1	0.009–0.141
T (s)	1.19–4.5	1.4–4.5
U_m (m/s)	0.112–0.533	0.17–0.469
U_{fm} (m/s)	0.013–0.025	0.016–0.025
S (m)	0–0.031	0–0.076
g (m/s^2)	9.806	9.806
ρ (kg/m^3)	1000	1000
μ (Pa · s)	0.000001	0.000001

Table 6: Accuracy of the proposed model for the square cross section.

Model	R	RMSE	MAE
GMDH-LM (with 45° orientation)	**0.9**	**0.18**	**0.108**
GMDH-LM (with 90° orientation)	**0.85**	**0.25**	**1.3**
Eq. (44)	0.93	0.253	0.43
Eq. (45)	0.3	0.35	1.68

The statistical results of the proposed techniques indicated that the GMDH-LM predicted scour depth with lower error ($RMSE = 0.25$ and MAPE $= 1.3$) and relatively higher accuracy ($R = 0.85$), compared to those performed using Eq. (44). For the square pile with 45° orientation, the GMDH-LM produced more a accurate prediction than that of Eq. (45). Meantime, the GMDH-LM provided quite a good prediction for the square pile with 45° orientation, compared to that of 90° orientation (Table 6).

Another robustness of the GMDH-LM is related to the influence of the Keulegan–Carpenter Number, KC, on the scour depth. In this way, 40 data sets from Sumer et al. [10] were used which have been classified into three groups, based on ranges of KC. Ranges of data sets are presented in Table 6. Results of performances indicated that the GMDH-LM has a higher

Table 7: The RMSE values of the *KC* ranges using proposed GMDH-LM and Eq. (27).

	$5 \leq KC < 10$	$10 \leq KC < 20$	$20 \leq KC < 50$
GMDH-LM	**0.0401**	**0.103**	**0.32**
Eq. (27)	0.064	0.112	0.8

accuracy of scour prediction in $5 \leq KC < 10$, compared to those of other ranges ($10 \leq KC < 20$ and $20 \leq KC < 50$). Pile scour due to regular waves is dependent on lee-wake vortices and the horseshoe vortex for $5 \leq KC < 10$ and $20 \leq KC < 50$. The statistical results of the GMDG-LM for the three ranges of *KC* are given in Table 7.

6. Conclusions

This paper presented a new approach in predicting scour depth around vertical piles due to waves. A new scheme of the GMDH network was proposed using the Levenberg–Marquardt method. In this study, the quadratic polynomial was selected as a transfer function to investigate the efficiency of the GMDH network for scour prediction. Also, the ANFIS model, RBF-NN, and empirical equations were utilized to indicate the capability of the scour depth prediction. Development of the models was carried out using dimensionless parameters. The testing results of the proposed soft computing tools indicated that the GMDH-LM has a more accurate prediction than the ANFIS model, RBF-NN, and empirical equations. Through a sensitivity analysis, the Keulegan–Carpenter number was determined as the most effective parameter on scour depth modeling. Performances of the GMDH-LM were proven flexible to variations of data set ranges. From the performances, it was found that the GMDH-LM predicted scour depth with lower error in $5 \leq KC < 10$, compared to other ranges. Another interesting point drawn is that the GMDH-LM covered well the limitations of data set ranges that had been faced by previous investigations. In this way, the GMDH-LM was applied successfully to investigate the influence of square pile on scour depth. In particular, the GMDH-LM produced the best realization of the inductive approach to predict the complexity of the scour process.

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