Available online at www.sciencedirect.com
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An International Journal
computers & mathematics with applications
ELSEVIER Computers and Mathematics with Applications 49 (2005) 1037–1044
www.elsevier.com/locate/camwa

The Existence of Positive Periodic Solutions of a Class of Lotka-Volterra Type Impulsive Systems with Infinitely Distributed Delay

MEILI LI
Department of Applied Mathematics
Xi'an Jiaotong University, Xi'an, 710049, P.R. China

YONGRUI DUAN
School of Economics and Management
Tongji University, Shanghai, 200092, P.R. China

WEIPING ZHANG
Research Institute of Micro/Nanometer Science and Technology
Shanghai Jiaotong University, Shanghai, 200030, P.R. China

MIANSEN WANG
Department of Applied Mathematics,
Xi'an Jiaotong University, Xi'an, 710049, P.R. China

(Received January 2004; revised and accepted November 2004)

Abstract—In this paper, the existence of positive periodic solutions of a class of periodic Lotka-Volterra type impulsive systems with distributed delays is studied. By using the continuation theorem of coincidence degree theory, a set of easily verifiable sufficient conditions are obtained, which improve and generalize some existing results.

Keywords—Infinitely distributed delay, Coincidence degree, Positive periodic solution, Impulse.

1. INTRODUCTION

It is well known that, in the periodic environment, the population dynamics of two competing species can be described by the famous nonautonomous Lotka-Volterra competing system,

$$\begin{aligned} x_1'(t) &= x_1(t)\left[a_1(t) - a_{11}(t)x_1(t) - a_{12}(t)x_2(t)\right], \\ x_2'(t) &= x_2(t)\left[a_2(t) - a_{21}(t)x_1(t) - a_{22}(t)x_2(t)\right], \end{aligned} \tag{1.1}$$

where $x_i(t)$ is the population density of species i; $a_i(t)$ is the rate of cell proliferation of species i per hour; $a_{ij}(t)$ is the rate of intraspecific competition if $i = j$, and the rate of interspecific competition if $i \neq j$, $i, j = 1, 2$. Because the environment usually varies continuously with certain period (e.g., seasonal effects of weather conditions, food supplies, temperature, mating habits,

The authors are very grateful to the referees for their careful reading of the manuscript and a number of excellent suggestions, which improve the representation of the present paper.

 Typeset by $\mathcal{A}_{\mathcal{M}}\mathcal{S}$-TEX
doi:10.1016/j.camwa.2004.11.009

etc.), and the assumption of periodicity of the parameters a_i and a_{ij} is a way of incorporating the periodicity of the environment, we assume a_i and a_{ij} are ω-periodic.

In recent years, there has been much research about the existence and global asymptotic stability of periodic solutions, where the main technique is the Schauder fixed-point theorem or a V-function. Furthermore, if the delay is considered when investigating the reciprocity between two species, equation (1.1) will be extended to a nonautonomous Lotka-Volterra system with delays. In [1,2], Liapunov functions, monotone flow theory and the Horn asymptotic fixed-point theorem are used to study the existence and global asymptotic stability of periodic solutions. In [3], the following nonautonomous Lotka-Volterra system with delays is studied by applying coincidence degree theory,

$$\begin{aligned} x_1'(t) &= x_1(t)\left[a_1(t) - a_{11}(t)x_1(t-\tau_{11}(t)) - a_{12}(t)x_2(t-\tau_{12}(t))\right], \\ x_2'(t) &= x_2(t)\left[a_2(t) - a_{21}(t)x_1(t-\tau_{21}(t)) - a_{22}(t)x_2(t-\tau_{22}(t))\right], \end{aligned} \tag{1.2}$$

and some sufficient conditions for the existence of positive periodic solutions are given.

We know that the birth of many species is not continuous, but occurs at fixed time intervals (some wild animals have seasonal births). In the long run, the birth of these species can be considered as an impulse to the system. To describe this phenomenon exactly, we proposed the following periodic two-species Lotka-Volterra competition impulsive system with infinitely distributed delays, which is a generalization of (1.2),

$$\begin{aligned} x_1'(t) &= x_1(t)\left[-d_1(t) - x_1(t) - \sum_{j=1}^{2} a_{1j}(t)\int_{-\infty}^{0} k_{1j}(s)x_j(t+s)\,ds\right], \quad t\ge 0, \quad t\ne t_k, \\ x_2'(t) &= x_2(t)\left[-d_2(t) - x_2(t) - \sum_{j=1}^{2} a_{2j}(t)\int_{-\infty}^{0} k_{2j}(s)x_j(t+s)\,ds\right], \quad t\ge 0, \quad t\ne t_k \\ x_1\left(t_k^+\right) - x_1(t_k) &= b_{1k}x_1(t_k), x_2\left(t_k^+\right) - x_2(t_k) = b_{2k}x_2(t_k), \quad k=1,2,\dots, \end{aligned} \tag{1.3}$$

where $0 = t_0 < t_1 < t_2 < \cdots < t_k < \cdots$ are fixed impulsive points with $\lim_{k\to\infty} t_k = \infty$.

In this paper, we will investigate the existence of periodic solutions of (1.3) by using coincidence degree theory.

First, we give the following assumptions.

(A_1) $b_{ik} > 0$ is the birth rate of x_i at t_k, and there exists $q \in N$, such that $t_{k+q} = t_k + \omega$, $b_{i(k+q)} = b_{ik}$.

(A_2) $d_i(t)$ is the death rate of x_i at time t, $d_i(t+\omega) = d_i(t)$, $i = 1, 2$.

(A_3) $k_{ij} \in C((-\infty, 0], [0, +\infty))$ and $\int_{-\infty}^{0} k_{ij}(s)\,ds = 1$, $i, j = 1, 2$.

(A_4) $a_{ij} \in C(R, [0,\infty))$, $i, j = 1, 2$ are continuous ω-periodic functions with $\int_0^\omega a_{ij}(t)\,dt > 0$.

Without loss of generality, here, and in the following, we assume that

$$[0,\omega] \cap \{t_k\} = \{t_1, t_2, \dots, t_m\},$$

so, $q = m$.

By the definition of x_i, we have $x_i(0) > 0$. In view of

$$\begin{aligned} x_i(t) &= x_i(0)\exp\left\{\int_0^t \left[-d_i(t) - x_i(t) - \sum_{j=1}^{2} a_{ij}(t)\int_{-\infty}^{0} k_{ij}(s)x_j(t+s)\,ds\right] dt\right\}, \quad t\in[0,t_1], \\ x_i(t) &= x_i\left(t_k^+\right)\exp\left\{\int_{t_k}^t \left[-d_i(t) - x_i(t) - \sum_{j=1}^{2} a_{ij}(t)\int_{-\infty}^{0} k_{ij}(s)x_j(t+s)\,ds\right] dt\right\}, \quad t\in(t_k,t_{k+1}], \\ x_i\left(t_k^+\right) &= (1+b_{ik})\,x_i(t_k), \quad k=1,2,\dots, \quad i=1,2, \end{aligned}$$

the solution of (1.3) is positive.

Let $u_i(t) = \ln x_i(t)$, $i = 1, 2$, then equation (1.3) is transformed into

$$u_1'(t) = -d_1(t) - e^{u_1(t)} - \sum_{j=1}^{2} a_{1j}(t) \int_{-\infty}^{0} k_{1j}(s)\, e^{u_j(t+s)}\, ds,$$
$$u_2'(t) = -d_2(t) - e^{u_2(t)} - \sum_{j=1}^{2} a_{2j}(t) \int_{-\infty}^{0} k_{2j}(s)\, e^{u_j(t+s)}\, ds, \tag{1.4}$$
$$u_i\left(t_k^+\right) - u_i\left(t_k\right) = \ln\left(1 + b_{ik}\right), \qquad i = 1, 2.$$

So, the existence of periodic solutions of (1.3) is equivalent to that of (1.4).

Let Φ denotes the set of Lebesgue measurable functions $\phi : (-\infty, 0] \to R$.

DEFINITION 1.1. *For $\phi_1, \phi_2 \in \Phi$, a function $u = (u_1, u_2)^\top \in ((-\infty, \infty), R^2)$ is said to be a solution of (1.4) on $[0, \infty)$ satisfying the initial condition,*

$$u_i(s) = \phi_i(s), \qquad s \in (-\infty, 0], \quad \phi_i(0) > 0, \quad i = 1, 2,$$

if the following conditions are satisfied.

(i) *$u(t)$ is continuous on each interval (t_{k-1}, t_k), $k = 1, 2, \dots$.*
(ii) *For any t_k, $k = 1, 2, \dots$, $u(t_k^+)$, $u(t_k^-)$ exist and $u(t_k^-) = u(t_k)$.*
(iii) *$u(t)$ satisfies (1.4) almost everywhere in $[0, \infty)$ and at impulsive points t_k situated in $(0, \infty)$, may have a discontinuity of the first kind.*

2. THE EXISTENCE OF POSITIVE PERIODIC SOLUTIONS

In this section, we will investigate the existence of positive periodic solutions of (1.3). For convenience, we first summarize a few concepts and results in [4], which will be used in this section. Our existence results are based on the coincidence degree theory in [4].

Let X, Z be normed vector spaces, $L : \operatorname{dom} L \subset X \to Z$ be a linear mapping, and $N : X \to Z$ be a continuous mapping.

L is said to be a Fredholm mapping of index zero, if $\dim \operatorname{Ker} L = \operatorname{codim} \operatorname{Im} L < +\infty$ and $\operatorname{Im} L$ is closed in Z.

If L is a Fredhlom mapping of index zero, then there exist continuous projectors $P : X \to X$ and $Q : Z \to Z$, such that $\operatorname{Im} P = \operatorname{Ker} L$, $\operatorname{Ker} Q = \operatorname{Im} L = \operatorname{Im}(I - Q)$. It follows that $L \mid \operatorname{dom} L \cap \operatorname{Ker} P : (I - P)X \to \operatorname{Im} L$ is invertible. We denote the inverse of that map by K_P.

The mapping N is said to be L-compact on $\bar{\Omega}$, if Ω is an open bounded subset of X, $QN(\bar{\Omega})$ is bounded and $K_P(I - Q)N : \bar{\Omega} \to X$ is compact.

Since $\operatorname{Im} Q$ is isomorphic to $\operatorname{Ker} L$, there exists isomorphism $J : \operatorname{Im} Q \to \operatorname{Ker} L$.

In the proof of our existence theorem below, we will use the continuation theorem advanced in [4].

LEMMA 2.1. CONTINUATION THEOREM. *Let L be a Fredholm mapping of index zero and N be L-compact on $\bar{\Omega}$, if*

(a) *for each $\lambda \in (0, 1)$, every solution x of $Lx = \lambda Nx$ satisfies $x \notin \partial\Omega$, and*
(b) *for each $x \in \operatorname{Ker} L \cap \partial\Omega$, $\deg_B\{JQN, \Omega \cap \operatorname{Ker} L, 0\} \neq 0$, when $QNx \neq 0$, where $\deg B$ denotes the Brouwer degree, then the equation $Lx = Nx$ has at least one solution in $\operatorname{dom} L \cap \bar{\Omega}$.*

To prove main conclusion by means of the continuation theorem, we need to introduce some function spaces.

Suppose $J \subset R$ be any interval. Define $PC[J, R^2] = \{u : J \to R^2, u(t)$ is continuous for $t \in J$, $t \neq t_k$, and $u(t_k^+)$, $u(t_k^-)$ exist and $u(t_k) = u(t_k^-)\}$.

$PC^1[J, R^2] = \{u \in PC[J, R^2], u(t)$ is continuous differential, for $t \in J$, $t \neq t_k$, and $u'(t_k^+)$, $u'(t_k^-)$ exist and $u'(t_k) = u'(t_k^-)\}$.

Obviously, $PC[J, R^2]$ is a Banach space with the norm $\|u\|_{PC} = \sup_{t\in J} \|u(t)\|$, and $PC'[J, R^2]$ is also a Banach space with the norm $\|u\|_{PC'} = \max\{\|u\|_{PC}, \|u'\|_{PC}\}$, where $\|\cdot\|$ is any norm of R^2.

LEMMA 2.2. *$H \subset PC[J, R^2]$ is relatively compact if and only if the functions in H are uniformly bounded on J and equicontinuous on $(t_{k-1}, t_k]$, $k = 1, 2, \dots, K$, for any fixed $K > 1$.*

PROOF. It is easy to prove by the Ascoli-Arzela theorem.

For convenience, let

$$\overline{d}_i = \int_0^\omega d_i(t)\,dt, \qquad \Delta_i \overset{\text{def}}{=} \frac{\sum_{k=1}^m \ln(1+b_{ik})}{\omega} - \overline{d}_i.$$

In the following, we will give the main result of this paper.

THEOREM 2.1. *Assume (A_1)–(A_4) hold, then if*

$$\Delta_1 \bar{a}_{22} > \Delta_2 \bar{a}_{12} \left[\prod_{k=1}^m (1+b_{2k})\right], \qquad \Delta_2 \bar{a}_{11} > \Delta_1 \bar{a}_{21} \left[\prod_{k=1}^m (1+b_{1k})\right].$$

Equation (1.3) has at least one positive ω-periodic solution, where

$$\bar{a}_{ij} = \frac{1}{\omega}\int_0^\omega a_{ij}(t)\,dt, \qquad i, j = 1, 2.$$

PROOF. As stated in Section 1, we only need to prove that (1.4) has at least one ω-periodic solution.

Let

$$X = \left\{u(t) = (u_1(t), u_2(t))^\top \in PC\left(R, R^2\right) \mid u(t+\omega) = u(t)\right\}, \qquad Z = X \times R^{2m}.$$

For $u \in X$, take $\|u\|_{PC} = \sup_{t\in[0,\omega]}\{\|u(t)\|\}$, where $\|\cdot\|$ is any convenient norm on R^2, and for $z \in Z$, take $\| z \|_Z = \| u \|_{PC} + \| y \|$, where $u \in X$, $y \in R^{2m}$, and $\|\cdot\|$ is any convenient norm on R^{2m}, then X, Z are both Banach spaces with the norm $\|\cdot\|_{PC}$ and $\|\cdot\|_Z$, respectively.

Let

$$\operatorname{dom} L = \left\{u(t) = (u_1(t), u_2(t))^\top \in X : (u_1(t), u_2(t))^\top \in PC^1\left(R, R^2\right)\right\},$$
$$L : \operatorname{dom} L \to Z, \qquad u \to (u', \Delta u(t_1), \dots, \Delta u(t_m)), \qquad N : X \to Z,$$
$$Nu = \left(\begin{pmatrix} -d_1(t) - e^{u_1(t)} - \sum_{j=1}^2 a_{1j}(t)\int_{-\infty}^0 k_{1j}(s)\,e^{u_j(t+s)}\,ds \\ -d_2(t) - e^{u_2(t)} - \sum_{j=1}^2 a_{2j}(t)\int_{-\infty}^0 k_{2j}(s)\,e^{u_j(t+s)}\,ds \end{pmatrix}, \begin{pmatrix} \ln(1+b_{11}) \\ \ln(1+b_{21}) \end{pmatrix}, \dots, \begin{pmatrix} \ln(1+b_{1m}) \\ \ln(1+b_{2m}) \end{pmatrix}\right),$$

where

$$\Delta u(t_k) = \begin{pmatrix} \Delta u_1(t_k) \\ \Delta u_2(t_k) \end{pmatrix} = \begin{pmatrix} u_1(t_k^+) - u_1(t_k) \\ u_2(t_k^+) - u_2(t_k) \end{pmatrix}, \qquad k = 1, 2, \dots, m.$$

It is clear that

$$\ker L = \left\{u \mid u \in X,\ u = h,\ h \in R^2\right\},$$
$$\operatorname{Im} L = \left\{z \mid z = (f, C_1, \dots, C_m) \in Z : \int_0^\omega f(s)\,ds + \sum_{k=1}^m C_k = 0\right\}.$$

So, $\operatorname{Im} L$ is closed in Z, and $\dim \operatorname{Ker} L = 2 = \operatorname{codim} \operatorname{Im} L$. Hence, L is a Fredholm mapping of index zero.

Set

$$Pu = \frac{1}{\omega}\int_0^{\omega} u(t)\, dt,$$

$$Qz = Q(f, C_1, \cdots, C_m) = \left(\frac{1}{\omega}\left[\int_0^{\omega} f(s)\, ds + \sum_{k=1}^{m} C_k\right], 0, 0, \ldots, 0\right).$$

It is easy to show that P and Q are continuous projectors, such that

$$\operatorname{Im} P = \operatorname{Ker} L, \qquad \operatorname{Im} L = \operatorname{Ker} Q = \operatorname{Im}(I - Q).$$

Furthermore, the generalized inverse (to L) $K_P : \operatorname{Im} L \to \operatorname{Ker} P \cap \operatorname{dom} L$ exists.

Set $z = (f, C_1, \cdots, C_m) \in \operatorname{Im} L$, then there exists $u \in X$ satisfying

$$u'(t) = f(t), \qquad t \neq t_k, \quad k = 1, 2, \ldots,$$
$$u(t_k^+) - u(t_k) = C_k,$$

that is

$$u(t) = \int_0^t f(s)\, ds + \sum_{t > t_k} C_k + u(0). \tag{2.1}$$

Because of $u(t) \in \operatorname{Ker} P$, we have $\int_0^{\omega} u(s)\, ds = 0$. So, from (2.1),

$$\int_0^{\omega}\int_0^t f(s)\, ds\, dt + \int_0^{\omega} \sum_{t > t_k} C_k\, dt + \omega u(0) = 0.$$

Then, from the last equation and (2.1),

$$u(t) = \int_0^t f(s)\, ds + \sum_{t > t_k} C_k - \frac{1}{\omega}\int_0^{\omega}\int_0^t f(s)\, ds\, dt - \frac{1}{\omega}\sum_{k=1}^{m} (\omega - t_k) C_k, \tag{2.2}$$

i.e.,

$$K_P z = \int_0^t f(s)\, ds + \sum_{t > t_k} C_k - \frac{1}{\omega}\int_0^{\omega}\int_0^t f(s)\, ds\, dt - \frac{1}{\omega}\sum_{k=1}^{m} (\omega - t_k) C_k. \tag{2.3}$$

$$QNu = \left(\begin{pmatrix} \dfrac{1}{\omega}\left\{\displaystyle\int_0^{\omega}\left[-d_1(t) - e^{u_1(t)} - \sum_{j=1}^{2} a_{1j}(t)\int_{-\infty}^{0} k_{1j}(s)\, e^{u_j(t+s)}\, ds\right] dt + \sum_{k=1}^{m} \ln(1 + b_{1k})\right\} \\ \dfrac{1}{\omega}\left\{\displaystyle\int_0^{\omega}\left[-d_2(t) - e^{u_2(t)} - \sum_{j=1}^{2} a_{2j}(t)\int_{-\infty}^{0} k_{2j}(s)\, e^{u_j(t+s)}\, ds\right] dt + \sum_{k=1}^{m} \ln(1 + b_{2k})\right\} \end{pmatrix}, 0, \ldots, 0\right),$$

$$K_P(I - Q)Nu = \begin{pmatrix} \displaystyle\int_0^t\left[-d_1(\mu) - e^{u_1(\mu)} - \sum_{j=1}^{2} a_{1j}(\mu)\int_{-\infty}^{0} k_{1j}(s)\, e^{u_j(\mu+s)}\, ds\right] d\mu + \sum_{t > t_k} \ln(1 + b_{1k}) \\ \displaystyle\int_0^t\left[-d_2(\mu) - e^{u_1(\mu)} - \sum_{j=1}^{2} a_{2j}(\mu)\int_{-\infty}^{0} k_{2j}(s)\, e^{u_j(\mu+s)}\, ds\right] d\mu + \sum_{t > t_k} \ln(1 + b_{2k}) \end{pmatrix}$$

$$- \begin{pmatrix} \displaystyle\frac{1}{\omega}\int_0^{\omega}\int_0^{\mu}\left[-d_1(t) - e^{u_1(t)} - \sum_{j=1}^{2} a_{1j}(t)\int_{-\infty}^{0} k_{1j}(s)\, e^{u_j(t+s)}\, ds\right] dt\, d\mu + \frac{1}{\omega}\sum_{k=1}^{m} (\omega - t_k)\ln(1 + b_{1k}) \\ \displaystyle\frac{1}{\omega}\int_0^{\omega}\int_0^{\mu}\left[-d_2(t) - e^{u_2(t)} - \sum_{j=1}^{2} a_{2j}(t)\int_{-\infty}^{0} k_{2j}(s)\, e^{u_j(t+s)}\, ds\right] dt\, d\mu + \frac{1}{\omega}\sum_{k=1}^{m} (\omega - t_k)\ln(1 + b_{2k}) \end{pmatrix}$$

$$- \begin{pmatrix} \displaystyle\left(\frac{t}{\omega} - \frac{1}{2}\right)\left\{\int_0^{\omega}\left[-d_1(\mu) - e^{u_1(\mu)} - \sum_{j=1}^{2} a_{1j}(\mu)\int_{-\infty}^{0} k_{1j}(s)\, e^{u_j(\mu+s)}\, ds\right] d\mu + \sum_{k=1}^{m} \ln(1 + b_{1k})\right\} \\ \displaystyle\left(\frac{t}{\omega} - \frac{1}{2}\right)\left\{\int_0^{\omega}\left[-d_2(\mu) - e^{u_2(\mu)} - \sum_{j=1}^{2} a_{2j}(\mu)\int_{-\infty}^{0} k_{2j}(s)\, e^{u_j(\mu+s)}\, ds\right] d\mu + \sum_{k=1}^{m} \ln(1 + b_{2k})\right\} \end{pmatrix}.$$

Clearly, QN and $K_P(I-Q)N$ are continuous. By Lemma 2.2, we can easily show that $K_P(I-Q)N(\bar{\Omega})$ is relatively compact for any open bounded set $\Omega \subset X$. Moreover, $QN(\bar{\Omega})$ is bounded. Thus, N is L-compact on $\bar{\Omega}$ for any open bounded set $\Omega \subset X$.

In order to apply Lemma 2.1, we have to obtain an appropriate open bounded subset Ω. Corresponding to the operator equation $Lu = \lambda Nu$ with $\lambda \in (0,1)$, we have

$$\begin{aligned} u_1'(t) &= \lambda\left[-d_1(t) - e^{u_1(t)} - \sum_{j=1}^{2} a_{1j}(t)\int_{-\infty}^{0} k_{1j}(s)\, e^{u_j(t+s)}\,ds\right], \qquad t \neq t_k, \quad k = 1,2,\dots, \\ u_2'(t) &= \lambda\left[-d_2(t) - e^{u_2(t)} - \sum_{j=1}^{2} a_{2j}(t)\int_{-\infty}^{0} k_{2j}(s)\, e^{u_j(t+s)}\,ds\right], \qquad u_i\left(t_k^+\right) - u_i(t_k) = \lambda \ln(1+b_{ik}), \\ &\qquad i = 1,2, \\ u_i(0) &= u_i(\omega). \end{aligned} \tag{2.4}$$

Integrating (2.4) from 0 to ω, we have

$$\int_0^{\omega}\left[-d_i(t) - e^{u_i(t)} - \sum_{j=1}^{2} a_{ij}(t)\int_{-\infty}^{0} k_{ij}(s)\, e^{u_j(t+s)}\,ds\right] dt + \sum_{k=1}^{m}\ln(1+b_{ik}) = 0 \qquad (i,j = 1,2;\ i \neq j),$$

that is,

$$\int_0^{\omega}\left[e^{u_i(t)} + \sum_{j=1}^{2} a_{ij}(t)\int_{-\infty}^{0} k_{ij}(s)\, e^{u_j(t+s)}\,ds\right] dt = \omega\Delta_i. \tag{2.5}$$

From (2.4) and (2.5), it follows that

$$\begin{aligned} \int_0^{\omega} |u_i'(t)|\,dt &\leq \bar{d}_i\omega + \int_0^{\omega}\left[e^{u_i(t)} + \sum_{j=1}^{2} a_{ij}(t)\int_{-\infty}^{0} k_{ij}(s)\, e^{u_j(t+s)}\,ds\right] dt \\ &= \bar{d}_i\omega + \omega\Delta_i \\ &= \sum_{k=1}^{m}\ln(1+b_{ik}). \end{aligned} \tag{2.6}$$

Since $u(t) \in X$, there exists $\xi_i \in [0,\omega]$, such that

$$u_i(\xi_i) = \min_{t\in[0,\omega]} u_i(t), \qquad i = 1,2. \tag{2.7}$$

From (2.5) and (2.7), we have

$$\omega\bar{a}_{ii}e^{u_i(\xi_i)} \leq \omega\Delta_i, \qquad i = 1,2.$$

Moreover,

$$u_i(\xi_i) \leq \ln\left\{\frac{\Delta_i}{\bar{a}_{ii}}\right\}, \qquad i = 1,2. \tag{2.8}$$

Then,

$$u_i(t) \leq u_i(\xi_i) + \int_0^{\omega} |u_i'(t)|\,dt \leq \ln\left\{\frac{\Delta_i}{\bar{a}_{ii}}\right\} + \sum_{k=1}^{m}\ln(1+b_{ik}) \stackrel{\text{def}}{=} M_i^+. \tag{2.9}$$

On the other hand, since $\sup_{t\in[0,\omega]}\{u_i(t)\}$ exists, there exists $\eta_i \in [0,\omega]$ satisfying

$$u_i\left(\eta_i^+\right) = \sup_{t\in[0,\omega]}\{u_i(t)\}, \qquad i = 1,2. \tag{2.10}$$

From (2.10), if $\eta_i \neq t_k$, then $u_i(\eta_i^+) = u_i(\eta_i)$; if $\eta_i = t_k$, then $u_i(\eta_i^+) = u_i(t_k^+)$. From (2.5) and (2.9), it follows that

$$\begin{aligned}\Delta_i \omega &= \int_0^\omega \left[e^{u_i(t)} + \sum_{j=1}^{2} a_{ij}(t) \int_{-\infty}^{0} k_{ij}(s)\, e^{u_j(t+s)}\, ds \right] dt \\ &\leq e^{u_i(\eta_i^+)} \omega + \sum_{j=1}^{2} \bar{a}_{ij} \omega e^{u_j(\eta_j^+)}, \qquad i = 1, 2.\end{aligned}$$

That is,

$$e^{u_i(\eta_i^+)} \geq \frac{\Delta_i - \bar{a}_{ij} e^{u_j(\eta_j^+)}}{\bar{a}_{ii} + 1}, \qquad i \neq j, \quad i, j = 1, 2. \tag{2.11}$$

By (2.9) and (2.11), we have

$$e^{u_i(\eta_i^+)} \geq \frac{\bar{a}_{jj}\Delta_i - \bar{a}_{ij}\Delta_j \left[\prod_{k=1}^{m} (1 + b_{jk}) \right]}{\bar{a}_{ii}\bar{a}_{jj} + \bar{a}_{jj}}, \qquad i \neq j, \quad i, j = 1, 2,$$

which implies that

$$u_i\left(\eta_i^+\right) \geq \ln \left\{ \frac{\bar{a}_{jj}\Delta_i - \bar{a}_{ij}\Delta_j \left[\prod_{k=1}^{m} (1 + b_{jk}) \right]}{\bar{a}_{ii}\bar{a}_{jj} + \bar{a}_{jj}} \right\} \stackrel{\text{def}}{=} M_i, \qquad i \neq j, \quad i, j = 1, 2. \tag{2.12}$$

By (2.6) and (2.12), we have

$$\begin{aligned}u_i(t) &\geq u_i\left(\eta_i^+\right) - \int_0^\omega \left|u_i'(t)\right| dt \\ &\geq M_i - \sum_{k=1}^{m} \ln(1 + b_{ik}) \stackrel{\text{def}}{=} M_i^-.\end{aligned} \tag{2.13}$$

Again, by (2.9) and (2.13),

$$\sup_{t \in [0,\omega]} |u_i(t)| < \max\left\{ \left|M_i^+\right|, \left|M_i^-\right| \right\} \stackrel{\text{def}}{=} H_i. \tag{2.14}$$

It is evident that, H_i is independent of the choice of λ. Moreover, it is not difficult to show by using the assumption of Theorem 2.1 that the system of algebraic equations,

$$e^{u_i} + \sum_{j=1}^{2} \bar{a}_{ij} e^{u_j} = \Delta_i, \qquad i = 1, 2 \tag{2.15}$$

has a unique solution $(u_1^*, u_2^*)^\top \in R^2$.

Let $H = \|(H_1, H_2)^\top\| + C$, where C is large enough so that the unique solution of (2.14) satisfies $\|(u_1^*, u_2^*)^\top\| < C$.

Let

$$\Omega = \{u(t) = (u_1, u_2)^\top \in X : \|u\|_{PC} < H\}.$$

It is clear that Ω satisfies Condition (a) in Lemma 2.1. When $x \in \operatorname{Ker} L \cap \partial\Omega = R^2 \cap \partial\Omega$, u is a constant vector in R^2 with $\|u\| = H$. Then,

$$QNu = \left(\begin{pmatrix} \Delta_1 - e^{u_1} - \sum_{j=1}^{2} \bar{a}_{1j} e^{u_j} \\ \Delta_2 - e^{u_2} - \sum_{j=1}^{2} \bar{a}_{2j} e^{u_j} \end{pmatrix}, \; 0, \ldots, 0 \right) \neq 0.$$

Take $J : \operatorname{Im} Q \to X, (d, 0, \cdots, 0) \to d$, then if $u \in \operatorname{Ker} L \cap \partial\Omega$, we have

$$JQNu = \begin{pmatrix} \Delta_1 - e^{u_1} - \sum_{j=1}^{2} \bar{a}_{1j} e^{u_j} \\ \Delta_2 - e^{u_2} - \sum_{j=1}^{2} \bar{a}_{2j} e^{u_j} \end{pmatrix}.$$

Furthermore, in view of the assumptions in Theorem 2.1, it is easy to prove that

$$\deg \{JQNu, \Omega \cap \operatorname{Ker} L, 0\} \neq 0,$$

We have now proved that Ω satisfies all the conditions in Lemma 2.1. Hence by Lemma 2.1, (1.4) has at least one ω-periodic solution $u^*(t)$ in $\bar{\Omega}$. So, $x^*(t) = (x_1^*(t), x_2^*(t))^\top$ with $x_i^*(t) = \exp\{u_i^*(t)\}$ is a positive ω-periodic solution of (1.3). The proof of Theorem 2.1 is complete.

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