Maneuvering Target Tracking in Dense Clutter Based on Particle Filtering

YANG Xiaojun\textsuperscript{a,*}, XING Keyi\textsuperscript{b}, FENG Xingle\textsuperscript{a}

\textsuperscript{a}School of Information Engineering, Chang'an University, Xi'an 710064, China
\textsuperscript{b}The National Key Laboratory for Manufacturing System Engineering, Xi'an Jiaotong University, Xi'an 710049, China

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Abstract

An improved particle filtering (IPF) is presented to perform maneuvering target tracking in dense clutter. The proposed filter uses several efficient variance reduction methods to combat particle degeneracy, low mode prior probabilities and measurement-origin uncertainty. Within the framework of a hybrid state estimation, each particle samples a discrete mode from its posterior distribution and the continuous state variables are approximated by a multivariate Gaussian mixture that is updated by an unscented Kalman filtering (UKF). The uncertainty of measurement origin is solved by Monte Carlo probabilistic data association method where the distribution of interest is approximated by particle filtering and UKF. Correct data association and precise behavior mode detection are successfully achieved by the proposed method in the environment with heavy clutter and very low mode prior probability. The performance of the proposed filter is examined and compared by Monte Carlo simulation over typical target scenario for various clutter densities. The simulation results show the effectiveness of the proposed filter.

Keywords: particle filtering; Monte Carlo methods; Kalman filter; probability data association; target tracking; nonlinear filtering

1. Introduction

The aim of target tracking is to estimate the state of target sequentially from a set of noisy observations which are related to the target state through a known transformation. For a linear system with additive Gaussian noise, the Kalman filtering (KF) is the optimal estimator. However, if the model used by the filter does not match the actual system, the solution will tend to diverge. A similar difficulty arises for maneuvering target tracking mainly due to the sudden maneuvers and switching between different modes of behavior. Multiple model approach or hybrid estimation\cite{1,3}, which consists of a set of discrete modes and a set of continuous variables, is an efficient modeling for maneuvering target tracking. However, the optimal solution of maneuvering target tracking is impractical due to the exponential growth of computation. Many sub-optimal algorithms, such as interactive multiple model (IMM) and generalized pesudo-Bayesian (GPB)\cite{1,2}, exist in literature which is mainly based on Gaussian mixture of fixed number models.

For general case of nonlinear and non-Gaussian systems, some approximate Bayesian sequential estimation algorithms have been proposed. More promising are the classes of simulation-based numerical techniques known as particle filtering (PF) in which the posterior distribution of the system state is represented by a set of random samples with associated weights\cite{6-9}. PF has been successfully used in a number of target tracking applications, where the multiple model...
particle filtering (MMPF) has been proposed for tracking a maneuvering target\textsuperscript{[10-15]}. However, the performance of PF is influenced by various factors\textsuperscript{[15]}, such as importance density function, resampling scheme and the dimension of state space from which the particles are sampled. Although some alternative PFs have been proposed to improve the estimation performance, such as auxiliary particle filtering (APF)\textsuperscript{[16]} and Rao-Blackwellised particle filtering (RBPF)\textsuperscript{[17-18]}, for optimal estimation of nonlinear maneuvering target tracking, the unknown onset time and very low prior transition probability of maneuver worsen the situation.

Another difficulty in the application of the PF framework to maneuvering target tracking in clutter is due to the uncertainty of measurement origin. The probabilistic data association (PDA) approach\textsuperscript{[19]} is an efficient Bayesian technique for dealing with the measurement-origin uncertainty. Recently, Monte Carlo probability data association (MC-PDA) strategies\textsuperscript{[20-22]} have been proposed to combine the PDA with particle techniques to accommodate general nonlinear non-Gaussian models and uncertain measurement-origin. In these strategies the data association problem is addressed directly in the context of particle filtering. But their applications are restricted mainly to a simple non-maneuvering target. In a heavily cluttered environment, precise behavior mode identification and state estimation for maneuvering target are more challenging.

In this paper, we propose an improved particle filtering (IPF) for maneuvering target tracking, where we combine the unscented Kalman filter (UKF)\textsuperscript{[23-24]} and RBPF to design optimal importance function in order to reduce the variance of estimation and expedite computations. We also incorporate the MC-PDA into our proposed PF to address nonlinear target maneuvers, missing object detection, and extraneous observations. The proposed algorithm is closely related to RBPF and equally efficient. Being opposite to previous work, our approach combines the advantage of PF and the efficiency of existing data association. The combination is an organic integration. The filtering of the continuous and discrete state is separated by Rao-Blackwellization technique. The uncertainty of measurement origin is solved by MC-PDA, which applies the idea of PDA directly to the sample sets. The continuous state variables are approximated by Gaussian mixture distribution. The discrete mode is estimated by optimal PF and the maneuver with low prior probability is detected effectively.

2. Problem Formulation

The behavior of a maneuvering target is efficiently described in terms of the hybrid stochastic systems\textsuperscript{[4-5]}. The target state vector $\mathbf{x}_{t} \in \mathbb{R}^{n_{t}}$ (where $n_{t}$ is the dimension of state vector), which includes the target position and velocity in Cartesian coordinates, is assumed to evolve according to

$$ x_{i} = f(x_{i-1}, r_{i}, v_{i-1}) $$

with the initial state $x_{0}$ being distributed as $p(x_{0})$. The function $f$ modifies the target state according to a dynamic model whose parameters evolve with time according to a finite state Markov chain $r$. The process noise $v_{i} \sim N(0, Q_{v})$ is a white random process, where $N(0, Q_{v})$ is the Gaussian distribution with the mean 0 and the covariance matrix $Q_{v}$. The target maneuvers are introduced according to $r_{i}$, which is a discrete-time, time-homogeneous and $s$-state first-order Markov chain with transition probabilities $p_{ij} = p(r_{i+1} = j | r_{i} = i)$ for any $i, j \in S$, where state space $S = \{1, 2, \cdots, s\}$, and the transition probability matrix $\pi = [\pi_{ij}]$. The initial probability distribution is denoted as $p_{0} = p_{r} \{r_{0}=i\}$ for $i \in S$ such that $p_{0} \geq 0$. Neither the continuous-state process $x_{i}$ nor the discrete state process $r_{i}$ is observed.

The target is observed using a sensor according to the following observation function

$$ y_{i} = h(x_{i}, r_{i}, w_{i}) $$

where $y_{i} \in \mathbb{R}^{y}$ is the noisy measurement process. The measurement noise $w_{i} \sim N(0, R_{w})$ is a white random process, where $N(0, R_{w})$ is the Gaussian distribution with the mean 0 and the covariance matrix $R_{w}$. The process noise $v_{i}$ and the measurement noise $w_{i}$ initial target state $x_{0}$ and initial maneuvering mode $r_{0}$ are mutually independent.

In addition to the object-originated measurement, the sensor may produce some extraneous observations under the influence of random noise, clutter, false alarms, and countermeasures. The measurement set of sensor at time $t$ is denoted as $y_{i}^{m} = \{y_{1}^{m}, y_{2}^{m}, \cdots, y_{m}^{m}\}$, where $m_{i}$ is the number of measurements. The measurement set contains clutter measurements and, if detected, a target measurement. The number of clutter points in a given area of the measurement space is assumed to be Poisson distributed with density $\lambda$. It is assumed that the clutter points are distributed uniformly in the measurement space. At each scan a validation gate is used to remove from consideration measurements which are distant from the predicted target location. The volume of the validation gate is denoted as $V_{i}$. At each scan the probability that the target is detected is denoted as $p_{t}$ and the probability that the target will appear in the validation gate given that it has been detected is denoted as $p_{G}$.

In the presence of such a measurement-origin uncertainty, the problem of maneuvering target tracking is to obtain the estimate of continuous state $x_{i}$ and mode state $r_{i}$, based on the information contained in the cumulative measurement set $y_{i}^{m} = \{y_{1}, y_{2}, \cdots, y_{m}\}$. Most generally, in the Bayesian framework, the aim of the optimal estimation is to construct the posterior probability density functions (PDFs) $p(x_{i} | y_{1:i})$ and
3.1 PF

Given measurements \( y_{1:t} \), the estimates of \( x \) and \( r_t \) rely on the posterior probability density \( p(x_t, r_t | y_{1:t}) \), especially the filtering distribution \( p(x_t | r_t, y_{1:t}) \). Assuming that the posterior density \( p(x_{t-1}, r_{t-1} | y_{1:t-1}) \) at time \( t-1 \) is available, the posterior probability density of \( x_t \) and \( r_t \) can be found using the Bayesian rule

\[
p(x_t, r_t | y_{1:t}) = \frac{p(y_{1:t} | x_t, r_t) p(x_t, r_t | y_{1:t-1})}{p(y_{1:t} | r_{t-1})}
\]

(3)

However, it may not be possible to obtain an analytical solution for Eq.(3) and numerical approximation would be required. The PF[6-7] samples the particles \( \{ (x_t^{(i)}, r_t^{(i)}) \}_{i=1}^{N} \) from a simple sampling importance density function \( \pi(x_t, r_t | y_{1:t}, x_{t-1}^{(i)}, r_{t-1}^{(i)}) \) and assigns each particle weight

\[
\omega_t^{(i)} \propto \frac{p(x_t, r_t | y_{1:t})}{\pi(x_t, r_t | y_{1:t}, x_{t-1}^{(i)}, r_{t-1}^{(i)})}
\]

(4)

The posterior probability distribution can be approximated as

\[
\hat{p}_N(x_{t-1}, r_{t-1} | y_{1:t}) = \sum_{i=1}^{N} \omega_t^{(i)} \delta_{x_{t-1}^{(i)}, r_{t-1}^{(i)}}(x_{t-1}, r_{t-1})
\]

(5)

where \( N \) is the number of particles.

The PF keeps the distribution updated as new observations are made over time. The simplest algorithm takes importance probability density as transition prior probability density

\[
p(x_t, r_t | x_{t-1}, r_{t-1}) = p(x_t | x_{t-1}, r_t) p(r_t | r_{t-1})
\]

(6)

and the importance weight becomes the likelihood \( p(y_{1:t} | x_t, r_t) \). The resultant filter is referred to as the Bootstrap filter in accordance with the terminology used in the original work of Gordon[6].

Recent work on RBPF in Refs.[17]-[18] has focused on combining PF and KF for tracking linear multimodal systems. In these methods, rather than sampling a complete system state, the authors combines a PF that samples the discrete modes \( r_t \) with a KF for each discrete mode that propagates sufficient statistics for the continuous state \( x \). According to Ref.[18], the RBPF reduces the dimension of space in which the PF operates and thus improves the accuracy of estimation.

3.2 Optimal Importance Function Approximation

Consider the following factorization

\[
p(x_t, r_{t-1} | y_{1:t}) = \int p(x_{t-1}, r_t | y_{1:t}) p(x_t | r_{t-2}) p(r_{t-1} | r_{t-2}) dr_{t-2}
\]

(7)

where \( p(x_t, r_{t-1} | y_{1:t}) \) denotes the posterior probability distribution of the continuous state for the known discrete mode. In the case of conditional linear models in Eqs.(1)-(2), the distribution \( p(x_{t-1}, r_{t-1} | y_{1:t}) \) is Gaussian and the sufficient statistic can be analytically updated by KF[17-18].

In the case of conditional nonlinear models, the KF update is simply not possible. To approximate \( p(x_{t-1}, r_{t-1} | y_{1:t}) \), one may resort to the local linearization technique described as extended Kalman filter (EKF), but a better approximation is achieved with UKF (the detail of the algorithm is given in Refs.[23]-[24]). The resultant filter is similar to the unscented particle filter (UPF)[25] and those in Ref.[26], but has different configuration. Both EKF and UKF rely on approximations of the system, but they are of a different nature. UKF is an alternative to KF, which possesses many advantages. The UKF does not rely on linearization of the system function. The solution adopted by UKF is a second-order truncation of the posterior distribution. UKF approximates \( p(x_t, r_{t-1} | y_{1:t}) \) by Gaussian distribution

\[
\hat{p}_N(x_t, r_{t-1} | y_{1:t}) = N(\hat{x}_t, \hat{P}_t)
\]

(8)

expresses the filtered mean and covariance of continuous state conditional on \( r_t^{(i)} \), respectively.

Now assuming that we can use a weighted set of samples \( \{ \omega_t^{(i)}, x_t^{(i)} \}_{i=1}^{N} \) to represent the marginal posterior probability distribution

\[
\hat{p}_N(x_t | y_{1:t}) = \sum_{i=1}^{N} \omega_t^{(i)} \delta_{x_t^{(i)}}(x_t)
\]

(10)

The marginal probability density of continuous state \( x_0 \) can be approximated by Gaussian mixture

\[
\hat{p}_N(x_0 | y_{1:t}) = \int \hat{p}_N(x_t | y_{1:t}) p(x_t | y_{1:t}) dx_t = \sum_{i=1}^{N} \omega_t^{(i)} p(x_0, y_{1:t}, r_{t-1}^{(i)})
\]

(11)

RBPF combines this marginalization and sampling method. In RBPF, we only sample the discrete mode, and the continuous state \( x \) can be estimated analytically. Since the expressive power of every particle is higher, few particles will be needed to achieve the same approximation accuracy.

However, the main drawback associated with conventional PF[27-28] is that the variance of the importance weights increases with time, leading to particle degeneracy[3]. A possible strategy is to use the optimal importance probability den-
sity function
\[
\pi(r_t | r_{0:t-1}, Y_{t-1}) = p(r_t | r_{0:t-1}, Y_{t-1}) \tag{12}
\]
That minimizes the variance of the importance weights conditioned on the simulated particle trajectories and on the current observations (see Refs.[7]-[13]). By introducing the latest observation to the prediction of particles, we can improve the performance of PF. Applying the latest observation to the prediction of particles, we can improve the performance of PF. Applying the Bayesian rule
\[
p(r_t | r_{0:t-1}, Y_{t-1}) = \frac{p(Y_t | r_{t-1})p(r_t | r_{0:t-1})}{p(Y_t | r_{0:t-1})} \tag{13}
\]
the optimal importance probability density function can be expressed as
\[
p(r_t | r_{0:t-1}, Y_{t-1}) \propto p(Y_t | r_{t-1}, r_{0:t-1})p(r_t | r_{0:t-1}) \tag{14}
\]
In general, the maneuver detection and classification in maneuvering target tracking are very difficult for sampling based algorithms because of the unknown onset time and very low prior probability and transition probability of maneuver. This can lead to incorrect detection and classification of maneuver because there are little or no samples in some maneuver mode, thus affecting the accuracy of the tracking. However, very low prior probability of maneuver can be handled effectively here. Instead of sampling from transition prior probability distribution \(p(r_t | r_{0:t-1})\), we sample from the optimal importance probability distribution which is also the true posterior distribution of the mode \(r_t\). Thus the insufficiency of samples of maneuver mode can be alleviated.

After sampling the mode particle from optimal importance probability density function \(r_t^{(i)}\), the corresponding importance weight \(\omega_t^{(i)}\) is updated using the recursion\(^{(8,13)}\)
\[
\omega_t^{(i)} = \omega_t^{(i-1)}p(Y_t | r_{t-1}^{(i-1)}, r_{0:t-1}^{(i-1)}) \tag{15}
\]
where the predicted likelihood can be expanded as
\[
p(Y_t | r_{t-1}^{(i)}, r_{0:t-1}^{(i-1)}) = \sum_{r_{t-1}^{(i)}} p(Y_t | r_{t-1}^{(i)}, Y_{t-1}, r_{0:t-1}^{(i-1)})p(r_{t-1}^{(i)}) \tag{16}
\]
It should be noted that the term \(p(Y_t | r_{t-1}^{(i)}, r_{0:t-1}^{(i-1)}, r_t)\) in Eqs.(14)-(16) does not simplify to \(p(Y_t | r_t)\) because there is dependency on past values through \(r_{0:t-1}\). Eq.(16) shows that the importance weight of each particle \(r_t^{(i)}\) equals the sum of posterior probability of its successor modes.

Even when using the optimal importance distribution, there is still the problem of sample impoverishment. To circumvent it to a significant extent, we notice that the optimal importance weight does not depend on \(r_t\). It is possible to perform the selection step before the sequential importance sampling step. Similar to APF, we can select the most promising trajectories before extension at time \(t-1\) using information at time \(t\). Selection before sampling results in a richer sample set. The proposed PF is similar to UPF in that they both use a set of particles, each of which performs an UKF update at every time step. In UPF, the UKF approximation of the posterior is used as a proposal for the PF, but in the proposed filter, this approximation is used as the filter result.

3.3. MC-PDA

In the particle filtering proposed above, the optimal importance probability density function and importance weight rely on likelihood \(p(Y_t | r_{t-1}, r_{0:t-1})\) which is conditional on the assumption that the measurement to target association is known. Tracking algorithm requires the association of each measurement in turn with the target.

We apply PDA directly to each particle \(r_t^{(i)}\) of PF to estimate the discrete mode variable and combine PDA with UKF to estimate the continuous state variables. According to Ref.[15], the resultant method is called MC-PDA. The computation of likelihood \(p(Y_t | r_{t-1}, r_{0:t-1})\) by MC-PDA is derived in the Appendix.

The details of the proposed IPF algorithm for maneuvering target at the current time step proceed as follows:

(1) **Initialization**

For \(i=1,2,\cdots, N\), sample \(r_{0:t}^{(i)} \sim \rho_0\) and set
\[
\begin{align*}
\hat{x}_0^{(i)} &= E(x_0) \\
\hat{p}_0^{(i)} &= \text{cov}(x_0) \\
\omega_0^{(i)} &= 1/N \\
t &= 1
\end{align*}
\tag{17}
\]
At time \(t\), for each particle \(s_t^{(-i)} = (\hat{p}_t^{(-i)}, \hat{x}_t^{(-i)}, P_t^{(-i)})\).

(2) **Prediction**

For each possible successor mode \(r_t \in S\) (such that \(p(r_t | r_{0:t}^{(i)}) > 0\)), before actually sampling a discrete mode, do

1. Perform time update of UKF using state transition equation and measurement equation conditional on mode \(r_t\):
\[
\begin{align*}
\hat{x}_{t-1}^{(i \cdot)} &= E(x_t | Y_{t-1}, r_t) \\
\hat{P}_{t-1}^{(i \cdot)} &= \text{cov}(x_t | Y_{t-1}, r_t) \\
\hat{y}_{t-1}^{(i \cdot)} &= E(y_t | Y_{t-1}, r_t) \\
\hat{P}_{t-1}^{y, x} &= \text{cov}(y_t | Y_{t-1}, r_t)
\end{align*}
\tag{18}
\]
2. For measurements \(y = \{y_1, y_2, \cdots, y_m\}\), compute likelihood
\[
p(Y_t | Y_{t-1}, r_{0:t-1}^{(i)}, r_t) =
\]
\[
1 - P_D P_G + P_D P_G / \lambda \sum_{j=1}^{N} N(y_j'; \mathbf{\tilde{P}}_{y_j'; \mathbf{r_j'}}^{(n)})/1 - P_D P_G / \lambda V_{r_j'}
\] 

(19)

③ Compute posterior probability distribution for mode \( r_t \)

\[
p(r_t | r_{0:t-1}^{(i)}, y_{1:t}) \propto p(y_t | y_{1:t-1}, r_{0:t-1}^{(i)}; r_t) p(r_t | r_{0:t-1}^{(i)})
\] 

(20)

④ Compute importance weight

\[
\hat{\omega}_t^{(i)} = \omega_t^{(i)} p(y_t | y_{1:t-1}, r_{0:t-1}^{(i)}) = \omega_t^{(i)} \sum_{r_{0:t-1}^{(i)}} p(r_t | r_{0:t-1}^{(i)}).
\] 

(21)

Normalize the importance weight

\[
\omega_t^{(i)} = \frac{\hat{\omega}_t^{(i)}}{N}
\] 

(22)

(3) Selection

For \( i = 1, 2, \cdots, N \), resample particle

\[
\mathbf{s}_t^{(i)} = \{ \mathbf{z}_t^{(i)}, \mathbf{x}_t^{(i)}, \mathbf{p}_t^{(i)} \}
\] 

(23)

and its subordinate particle

\[
\{ \mathbf{x}_t^{(i)}, \mathbf{p}_t^{(i)} \} = \{ \mathbf{x}_t^{(i)}, \mathbf{p}_t^{(i)} \}
\] 

(24)

where \( r_t \in S \), with respect to normalized important weight \( \omega_t^{(i)} \) to obtain \( N \) new particles

and the corresponding subordinate particle

\[
\{ \mathbf{x}_t^{(i)}, \mathbf{p}_t^{(i)} \} = \{ \mathbf{x}_t^{(i)}, \mathbf{p}_t^{(i)} \}
\] 

(25)

(4) Sequential importance sampling

For \( i = 1, 2, \cdots, N \), sample new successor mode \( r_t^{(i)} \sim p(r_t | r_{0:t-1}^{(i)}, y_{1:t}) \) from the normalized optimal importance probability density function.

(5) UKF measurement update

For each predicted particle

\[
\{ \mathbf{x}_t^{(i)}, \mathbf{p}_t^{(i)} \}
\] 

which is conditional on mode \( r_t^{(i)} \), repeat the following steps.

① For \( i = 1, 2, \cdots, N \), compute \( \mathbf{x}_t^{(i)}, \mathbf{p}_t^{(i)} \) using MC-PDA measurement update Eq.(A7) and Eq.(A11).

② For obtained \( N \) new particles

\[
\mathbf{s}_t^{(i)} = \{ \mathbf{z}_t^{(i)}, \mathbf{x}_t^{(i)}, \mathbf{p}_t^{(i)} \}
\] 

(26)

(6) Output

The estimate of continuous state

\[
\mathbf{\tilde{x}}_t = \sum_{i=1}^{N} \omega_t^{(i)} \mathbf{x}_t^{(i)}
\] 

(27)

The posterior probability of mode \( r_t \)

\[
\hat{p}(r_t = i | y_{1:t}) = \frac{1}{N} \sum_{j=1}^{N} \langle r_j^{(1)} \rangle, i, j \in \{1, 2, \cdots, N\}
\] 

(28)

where \( i \in S \) and the symbol \(||\) indicates the cardinality of a set.

4. Monte Carlo Simulation

The maneuvering target behavior is described by the following discrete-time multiple-model equation, planar constant turn motion model\(^{(1)}\):

\[
x_t = F(\omega)x_{t-1} + Gv_t
\] 

(29)

where

\[
F(\omega) = \begin{bmatrix}
1 & \frac{\sin(\omega T)}{\omega} & 0 & -\frac{1 - \cos(\omega T)}{\omega} \\
0 & \cos(\omega T) & 0 & -\sin(\omega T) \\
0 & 1 - \cos(\omega T) & 1 & \sin(\omega T) \\
0 & 0 & \omega & \omega
\end{bmatrix}
\] 

(30)

\[
G = \begin{bmatrix}
\frac{1}{2}T^2 & 0 & 0 \\
0 & T & 0 \\
0 & \frac{1}{2}T^2 & 0 \\
0 & 0 & T
\end{bmatrix}
\] 

(31)

\( \omega \) and \( T \) are the target turn rate and the sampling interval respectively. The state space vector \( x_t = [p_x, v_x, p_y, v_y]^T \) contains target positions \( p_x, p_y \) (m) and velocities \( v_x, v_y \) (m/s) in horizontal Cartesian coordinate frame. The set of models, describing multiple model configurations, includes one nearly constant velocity model \((\omega = 0)\) and two nearly coordinated turn models with known mean values \( \pm\omega \) for left and right turns, respectively\(^{(1)}\). An underlying Markovian chain with known initial and transition probabilities controls the model switching.

The range to the target and bearing is measured by radar. The nonlinear measurement function is specified as

\[
h(x_t) = \sqrt{p_x^2 + p_y^2 - \frac{(p_x - P_{\gamma})^2}{P_{\gamma}}}
\] 

(32)

with measurement noise standard deviation from range and bearing \( \sigma_r, \sigma_\gamma \) and sampling interval \( T = 1 \) s. The sensor produces measurements with detection probability \( p_D < 1 \). The number of false measurements is
modeled independently from scan to scan as a sequence of Poisson distributed random variable of a known parameter \( \lambda \), uniformly distributed in measurement space.

The initial target state \( x_0 \sim N [x_0, m_0, P_0] \), where \( m_0 = [150, 20, 150, 20]^T \) is set to the exact initial value of the target and \( P_0 = \text{diag}(50^2, 8^2, 50^2, 8^2) \). The total duration of simulation scenario is 80 s. The initial model states are generated according to the initial Markov chain probabilities. The turn rate sequence of target trajectory is depicted in Fig. 1. Mean turn rate values of 0.1 rad/s are assigned to the left and right turn models. The test scenario includes three modes of maneuver. The process noise standard deviations for each mode are as follows: \( \sigma_v^1 = 3, \sigma_v^2 = 3, \sigma_v^3 = 4 \). Initial probabilities of underlying Markovian chain are \( p_1 = 0.4, p_2 = 0.3, p_3 = 0.3 \), and transition probabilities of the underlying Markovian chain \( p_{m,n} = 0.9, p_{m,n} = 0.05 \) (\( m, n = 1, 2, 3, \text{ and } m \neq n \)). In order to assess and compare the qualities of the filter, the following quantitative measurements of performance are selected: position root mean square error (RMSE) (both coordinates combined), speed RMSE (magnitude of velocity vector), probability of correct mode identification, and the percentage of track loss. A loss of track is established when the absolute position error exceeds a threshold by three consecutive sampling intervals, even if later it returns to admissible values. The level of the threshold \( \psi = 10 \sigma \) was determined based on a simulation data analysis.

The performances of the proposed IPF are compared with Bootstrap multiple mode (BMM) particle filtering\([12,15]\) and IMM-PDA (IMM is performed using UKF)\([15]\) tracking algorithms over the following test scenario. The algorithms are examined over the following parameters: detection probability \( p_D = 0.99 \), clutter density \( \lambda = 0.001 \text{ m}^{-2} \), measurement noise standard deviation \( \sigma_v = 100, \sigma_s = 0.15 \), size of particles set \( N = 2000 \). Results presented below are obtained based on 100 Monte Carlo runs. Fig. 2 shows the target track and estimated tracks by IPF, BMM and IMM-PDA respectively. The time-plots of the position and speed RMSEs can be seen in Figs.3-4 respectively. The results show that for moderate clutter density, IPF considerably outperforms BMM and IMM-PDA. The RMSE of the BMM and IMM-PDA algorithms are higher than the errors of the IPF, especially during maneuvering phases. During the maneuvering motion modes, the accuracy of the IPF is about two to three times better in comparison with the BMM algorithm and four to five times better than IMM-PDA algorithm. The ability of IPF to track targets in dense clutter is noticed in all figures, while IMM-PDA algorithm break down. This significant performance improvement of the IPF algorithm is due to the near optimality of the filter, providing better adaptation to the true system behavior.
ning while there is a big lag in the mode identification by the BMM and IMM-PDA algorithms when the mode switches. Because the IPF algorithm samples discrete mode particle from its true posterior distribution, it can identify mode and detect the switching more accurately.

Next, we fix the sample size $N = 2000$ and vary the clutter density to examine the effects of clutter density. Track loss percentages are computed from 1000 realization over the following set of clutter densities: $\lambda = 0.000 \text{ m}^{-2}$, $0.001 \text{ m}^{-2}$, and detection probability $p_D = 0.99$. The percentages of track loss for different clutter densities are shown in Table 1. The misclassification rates of mode $r_i$ which are estimated based on $\hat{r}_i = \text{argmax}\{p(r_i = i)\}$ are presented in Table 2.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Percentage of track loss/</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda = 0.000 \text{ m}^{-2}$</td>
<td>$\lambda = 0.001 \text{ m}^{-2}$</td>
</tr>
<tr>
<td>IMM-PDA</td>
<td>13.00</td>
</tr>
<tr>
<td>BMM</td>
<td>8.00</td>
</tr>
<tr>
<td>IPF</td>
<td>2.00</td>
</tr>
</tbody>
</table>

Table 2 Rate of misclassification for different clutter densities

<table>
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<th>Algorithm</th>
<th>Rate of misclassification/</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda = 0.000 \text{ m}^{-2}$</td>
<td>$\lambda = 0.001 \text{ m}^{-2}$</td>
</tr>
<tr>
<td>IMM-PDA</td>
<td>1.6</td>
</tr>
<tr>
<td>BMM</td>
<td>1.7</td>
</tr>
<tr>
<td>IPF</td>
<td>1.2</td>
</tr>
</tbody>
</table>

As would be expected, all algorithms exhibit a decrease in performance as clutter density increases. However the degradation in performance is less for IPF than that for BMM and IMM-PDA. For clutter density $\lambda = 0.000 \text{ m}^{-2}$ up to $\lambda = 0.001 \text{ m}^{-2}$, there are little differences in the performances of the three filters with a slight superiority of the IPF. But for the heavy clutter densities $\lambda = 0.005 \text{ m}^{-2}$, IPF and BMM considerably outperform IMM-PDA, which loses all tracks in the examined target scenarios. For all clutter densities, the IPF is able to correctly identify the current behavior mode of target.

We now fix the sample size $N = 2000$ and clutter density $\lambda = 0.001 \text{ m}^{-2}$, and vary the sensor detection probability which covers a wide range to examine the effects of target detection probability. Track loss percentages are computed from 1000 realization over the following set of detection probabilities: $p_D = 0.80$, 0.90, 0.95, 0.99. The percentages of track loss for different detection probabilities are shown in Table 3.

Table 3 Percentage of track loss for different detection probabilities

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>$p_D = 0.80$</th>
<th>$p_D = 0.90$</th>
<th>$p_D = 0.95$</th>
<th>$p_D = 0.99$</th>
</tr>
</thead>
<tbody>
<tr>
<td>IMM-PDA</td>
<td>13.00</td>
<td>56.00</td>
<td>92.00</td>
<td>92.00</td>
</tr>
<tr>
<td>BMM</td>
<td>8.00</td>
<td>27.00</td>
<td>56.00</td>
<td>56.00</td>
</tr>
<tr>
<td>IPF</td>
<td>2.00</td>
<td>9.00</td>
<td>21.00</td>
<td>21.00</td>
</tr>
</tbody>
</table>

Once again, a similar tendency is observed. The three algorithms exhibit a monotonic decrease in performance as the detection probability reduces. This is the result of an increase in the effective clutter density with the decrease of detection probability. The performances of IMM-PDA and BMM degrade rapidly. The IPF significantly outperforms the other two algorithms in that the IPF can maintain the track of target.
in an environment with lower detection probability.

The improved tracking capabilities of the IPF algorithm are at the cost of a considerable computational burden which is common for most simulation methods. The average time for a measurement update depends on the sample size $N$ and the clutter density $\lambda$. In a simulation platform with CPU Pentium IV, 2.7 G, EMS memory 512 M and MATLAB 6.5, the executions are performed for three algorithms. We present in Table 4 the CPU time for a measurement update of the three algorithms for different sample sizes and clutter densities. It was estimated that for $N = 2000$ the IPF algorithm requires about 50 times more computation time than the IMM-PDA.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>$N = 2000$</th>
<th>$N = 3000$</th>
</tr>
</thead>
<tbody>
<tr>
<td>IMM-PDA</td>
<td>0.015</td>
<td>0.018</td>
</tr>
<tr>
<td>BMM</td>
<td>0.470</td>
<td>0.530</td>
</tr>
<tr>
<td>IPF</td>
<td>0.760</td>
<td>0.820</td>
</tr>
</tbody>
</table>

Table 4  CPU time for different sample sizes and clutter densities

5. Conclusions

In this paper, we present an on-line simulation-based algorithm to perform maneuvering target tracking in clutter. The results show a considerable improvement. Based on several efficient variance reduction methods, an IPF is designed to combat particle degeneracy, low probabilities of mode transitions, and data association. The filtering of the continuous state and discrete mode is separated by Rao-Blackwellization technique to reduce the dimension of the sample space. The uncertainty of measurement origin is solved by an MC-PDA method. The continuous state is approximated with a Gaussian distribution by UKF, and the discrete mode is estimated by optimal PF. The problems of correct data association and precise behavior mode estimation in an environment with heavy clutter and low mode prior probability are successfully solved.

The performance of the proposed algorithm has been examined and compared by Monte Carlo simulations over typical target scenario for various clutter densities and detection probabilities. The IPF algorithm has shown a better adaptation capability and considerably improved performance in a heavily cluttered environment. Although the performance improvement is achieved at the cost of a considerable computational load, the inherent parallelism of this algorithm and recently proposed parallel distributed resampling architectures of PF can facilitate the algorithm’s practical implementation. The application of our methodology to complex multiple maneuvering target tracking scenarios is in progress.

References

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Biography:

YANG Xiaojun  Born in 1971, he received his B.S. degree in applied mathematics from Sichuan University, M.S. degree in operation and control theory from Xidian University, and Ph.D. degree in control theory and control engineering from Northwestern Polytechnical University respectively. He is now an associate professor in Chang’an University. His main research interests include adaptive control, signal and information processing, estimation theory, adaptive filtering, target tracking, sensor networks and information fusion. E-mail: xjyang@chd.edu.cn

Appendix

Derivation of MC-PDA

Define θi | (j=1,2,⋯,mi), and θi | denotes the hypothesis that measurement yi | is due to the target and the remaining measurements are due to clutter; θ0 | denotes the hypothesis that all measurements are due to clutter.

Resorting to the idea of probability data association (PDA) approximation, after extending over the association hypotheses the likelihood can be found as

\[ p(y_i | y_{121}, r_i, \theta_i^j) = \sum_{t=0}^{m_i} p(y_i | y_{121}, r_0, r_i, \theta_i^j). \]

According to the hypotheses of the clutter distribution, the components of the summation (A1) can be found as

\[ p(\theta_i^j | y_{121}, r_0, r_i) = \begin{cases} \frac{P_D P_G m_i + (1 - P_D P_G) \Delta V_i}{(1 - P_D P_G) \Delta V_i} & (i = 1, 2, \cdots, m_i) \\ \frac{P_D P_G m_i + (1 - P_D P_G) \Delta V_i}{(1 - P_D P_G) \Delta V_i} & (i = 0) \end{cases} \]

Conditional on mode \( r_{0t} \), the likelihood \( p(y_i | y_{121}, r_0, r_i, \theta_i^j) \) in Eq.(A2) can be computed analytically by UKF as follows:

\[ p(y_i | y_{121}, r_0, r_i, \theta_i^j) = p^{-1}_G N(y_i | \bar{y}_{i,j}^{(j)}, P_{ij}^{(j)}) \]

for \( i=1,2,\cdots,m_i \), where the mean and the covariance of predicted measurement

\[ \bar{y}_{ij}^{(j)} = E(y_i | r_i, y_{121}) \]

\[ P_{ij}^{(j)} = \text{cov}(y_i | r_i, y_{121}) \]

can be approximated by the sigma points of UKF.

Substituting Eqs.(A2)-(A3) into Eq.(A1) gives

\[ p(y_i | y_{121}, r_0, r_i, \theta_i^j) = \frac{1 - P_D P_G + P_D P_G \Delta V_i}{1 - P_D P_G + m_i P_D P_G / (\Delta V_i)} \]

By the combination of PDA and UKF, conditional on mode \( r_{0t} \), the estimate of continuous state \( x_t \) can be derived as follows.

Let \( \beta_i = p(\theta_i | y_{121}, r_i), i=0,1, \cdots, m_i \), denote the posterior probability of association hypotheses conditional on \( r_i \). We can now show that

\[ x_t^{(e)} = E(x_t | y_{121}) = \sum_{i=0}^{m_i} E(x_t | y_{121}, r_i) \beta_i \]

\[ p(\theta_i | y_{121}, r_i) = \sum_{l=0}^{m_i} x_{l}^{(e)} \beta_i \]

where the one-step prediction mean and covariance of \( x_t \) conditional on \( r_i \) are as follows:

\[ x_{t+1}^{(e)} = E(x_t | r_i, y_{121}) \]

\[ P_{t+1}^{(e)} = \text{cov}(x_t | r_i, y_{121}) \]
the filtered mean and covariance of \( x_t \) conditional on \( r_t \) are as follows:

\[
\begin{align*}
\bar{x}_t^c &= \bar{x}_t^{(r)} + K_t (y_t' - \bar{y}_t^{(r)}) & (i = 1, 2, \cdots, m_r) \\
\bar{x}_t^0 &= \bar{x}_t^{(r)}
\end{align*}
\]

where \( K_t \) is the UKF gain. These quantities can be computed analytically by the sigma points of UKF.

Substituting these quantities into Eq.(A7) gives the mean of \( x_t \) conditional on mode \( r_t \):

\[
\bar{x}_t^{(r)} = \bar{x}_t^{(r)} + K_t v_t
\]  

(A10)

The covariance of \( x_t \) conditional on mode \( r_t \) is found as

\[
P_t^{(r)} = P_{t|t-1} - K_t P_{\phi|\gamma} K_t^T
\]

\[
P_t = K_t [\sum_{i=1}^{m_r} \beta_t^i v_t' (v_t')^T - v_t v_t^T] K_t^T
\]

\[
v_t' = y_t' - \bar{y}_{t|t-1}
\]

\[
\beta_t^i = \frac{1}{b + \sum_{j=1}^{m_r} \epsilon_j} (i = 1, 2, \cdots, m_r)
\]

\[
\beta_t^0 = \frac{b}{b + \sum_{j=1}^{m_r} \epsilon_j}
\]

where

\[
e_t = \exp\{ -\frac{1}{2} (v_t')^T P_{\phi|\gamma} v_t' \}
\]

\[
b = \lambda [2 \pi P_{\phi|\gamma}]^{-\frac{1}{2}} (1 - P_D P_C) / P_D
\]  

(A12)