ELSEVIER
Available online at www.sciencedirect.com
ScienceDirect
Physics Letters B 649 (2007) 417–421
PHYSICS LETTERS B
www.elsevier.com/locate/physletb

A fresh look at neutral meson mixing

M.A. Gomshi Nobary *, B. Mojaveri

Department of Physics, Faculty of Science, Razi University, Kermanshah, Iran

Received 15 February 2007; received in revised form 25 March 2007; accepted 7 April 2007

Available online 19 April 2007

Editor: T. Yanagida

Abstract

In this work we show that the existence of a complete biorthonormal set of eigenvectors of the effective Hamiltonian governing the time evolution of neutral meson system is a necessary condition for diagonalizability of such a Hamiltonian. We also study the possibility of probing the *CPT* invariance by observing the time dependence of cascade decays of type $P^\circ(\bar{P}^\circ) \to \{M_a, M_b\}X \to fX$ by employing such basis and exactly determine the *CPT* violation parameter by comparing the time dependence of the cascade decays of tagged P° and tagged $\bar{P}^\circ$.

PACS: 11.30.Er; 13.20.Eb; 12.15.Ff

Keywords: CP violation; Cascade decays; Biorthonormal basis

1. Introduction

In the Wigner–Weisskopf (W–W) approximation [1] the effective Hamiltonian which describes the P°–$\bar{P}^\circ$ system is not Hermitian. Therefore the eigenkets of this Hamiltonian are indistinguishable (unless the Hamiltonian is normal, $[\hat{H}, \hat{H}^\dagger] = 0$). The reason is that for such a system the orthogonality and completeness relations could not be written in terms of its eigenkets. In the presence of T violation in the P°–$\bar{P}^\circ$ system, we are dealing with a non-Hermitian Hamiltonian which is not normal. Therefore we can use the principles of non-Hermitian quantum mechanics and reconsider the definition of diagonalizability of an operator. We use the biorthonormal basis for this propose. Here we emphasise that when a non-Hermitian and non-normal operator is encountered, use of a complete biorthonormal basis is in order which reduces to orthogonal basis as soon as the operator is considered Hermitian and normal. Therefore we conclude that we may use such a set of basis to describe the time evolution of neutral mesons in the presence of T violation.

As mentioned, in the presence of T violation the eigenkets of a non-Hermitian Hamiltonian does not satisfy the completeness and orthogonality relations. Therefore the eigenkets of such a Hamiltonian are not distinguishable. Due to this fact in writing down the transition amplitudes for cascade decays $P^\circ(\bar{P}^\circ) \to \{M_a, M_b\}X \to fX$, we use the biorthonormal basis when the intermediate states are eigenstates of $\hat{H}$.

We prove that the existence of a complete biorthonormal set of eigenvectors of $\hat{H}$ is necessary condition for diagonalizability of the effective Hamiltonian governing the time evolution of the neutral meson systems and write down the spectral form of the Hamiltonian operator with this basis in Section 2. In Section 3 we discuss the time evolution of neutral meson system and introduce the T and *CPT* violation complex parameters and obtain the time evolution of flavor eigenkets. Finally in the last section we study the possibility of probing *CPT* invariance by observation of the time dependence of the cascade decays of type $P^\circ(\bar{P}^\circ) \to \{M_a, M_b\}X \to fX$ by using the biorthonormal basis and introduce new ratios of decay amplitudes and exactly determine the *CPT* violation parameter by comparing the time dependence of the cascade decays of tagged P° and tagged $\bar{P}^\circ$.

* Corresponding author.
E-mail address: mnobary@razi.ac.ir (M.A. Gomshi Nobary).

0370-2693 © 2007 Elsevier B.V. Open access under CC BY license.
doi:10.1016/j.physletb.2007.04.025

2. Diagonalizability and the complete set of biorthonormal basis

A linear operator $\hat{H}$ acting in a separable Hilbert space and having a discrete spectrum is diagonalizable if and only if there are eigenvectors $|\psi_n\rangle$ of $\hat{H}$ and $|\phi_n\rangle$ of $\hat{H}^\dagger$ that form a complete set of biorthonormal basis of $\{\psi_n, \phi_n\}$, i.e. they satisfy

$$\hat{H}|\psi_n\rangle = E_n|\psi_n\rangle, \qquad \hat{H}^\dagger|\phi_n\rangle = E_n^*|\phi_n\rangle, \tag{1}$$

and

$$\langle\psi_m|\phi_n\rangle = \delta_{mn}, \qquad \sum_n |\psi_n\rangle\langle\phi_n| = \sum_n |\phi_n\rangle\langle\psi_n| = 1, \tag{2}$$

where n is the spectral label and † and * denote the adjoint and complex-conjugate respectively as usual. Moreover δ_{mn} is the Kronecker delta function and 1 represents the identity operator.

Nowhere in this definition it is assumed that the operator is normal, i.e., $[\hat{H}, \hat{H}^\dagger] = 0$. A normal operator, in finite dimensions with no extra conditions and in infinite dimensions with appropriate extra conditions, admits a diagonal matrix representation in some orthogonal basis. This is usually called *diagonalizability* by a unitary transformation. In view of (1) and (2) the spectral form of $\hat{H}$ and $\hat{H}^\dagger$ may be written in the following form

$$\hat{H} = \sum_n E_n|\psi_n\rangle\langle\phi_n|, \qquad \hat{H}^\dagger = \sum_n E_n^*|\phi_n\rangle\langle\psi_n|. \tag{3}$$

In order to see the equivalence of the existence of a complete biorthonormal set of eigenvectors of $\hat{H}$ and its diagonalizability, we note that by definition a diagonalizable Hamiltonian $\hat{H}$ satisfies $\hat{A}^{-1}\hat{H}\hat{A} = \hat{H}_\circ$ for an invertible linear operator $\hat{A}$ and a diagonal linear operator $\hat{H}_\circ$, i.e., there is an orthogonal basis $\{|n\rangle\}$ in the Hilbert space and complex numbers E_n such that $\hat{H}_\circ = \sum_n E_n|n\rangle\langle n|$. Then letting $|\psi_n\rangle := \hat{A}|n\rangle$ and $|\phi_n\rangle := (\hat{A}^{-1})^\dagger|n\rangle$, we can easily check that $\{|\psi_n\rangle, |\phi_n\rangle\}$ is a complete biorthonormal system for $\hat{H}$. The converse is also true, for if such a system exists we may set $\hat{A} := \sum_n |\psi_n\rangle\langle n|$ for some orthogonal basis $\{|n\rangle\}$ and by using Eq. (2) check that $\hat{A}^{-1} = \sum_n |n\rangle\langle\phi_n|$ and $\hat{A}^{-1}\hat{H}\hat{A} = \hat{H}_\circ$, i.e., $\hat{H}$ is diagonalizable.

As long as T is invariant (no violation), the effective Hamiltonian is normal. In such a case the orthonormality relations between the basis of $\hat{H}$ are valid and moreover that the eigenkets of $\hat{H}$ are discriminant and the biorthonormal basis turns into orthonormal basis automatically and $|\psi_n\rangle$'s are the same as $|\phi_n\rangle$'s.

3. The time evolution of neutral meson system

In the Wigner–Weisskopf (W–W) approximation, which we shall use throughout, a beam of oscillating and decaying neutral meson system is described in its rest frame by a two component wave function

$$|\psi(t)\rangle = \psi_1(t)|P^\circ\rangle + \psi_2(t)|\bar{P}^\circ\rangle, \tag{4}$$

where t is the proper time and $|P^\circ\rangle$ stands for K, D, B_d or B_s. The wave function evolves according to a Schrödinger like equation

$$i\frac{d}{dt}\begin{pmatrix}\psi_1(t)\\ \psi_2(t)\end{pmatrix} = \begin{pmatrix}H_{11} & H_{12}\\ H_{21} & H_{22}\end{pmatrix}\begin{pmatrix}\psi_1(t)\\ \psi_2(t)\end{pmatrix}. \tag{5}$$

The matrix $\hat{H}$ is usually written as $\hat{H} = \hat{M} - i\hat{\Gamma}/2$, where $\hat{M} = \hat{M}^\dagger$, and $\hat{\Gamma} = \hat{\Gamma}^\dagger$ are 2×2 matrices called the mass and the decay matrices [1–7]. Decomposition of $\hat{H}$ reads

$$\hat{H} = |P^\circ\rangle H_{11}\langle P^\circ| + |P^\circ\rangle H_{12}\langle\bar{P}^\circ| + |\bar{P}^\circ\rangle H_{21}\langle P^\circ| + |\bar{P}^\circ\rangle H_{22}\langle\bar{P}^\circ|. \tag{6}$$

The flavor basis $\{|P^\circ\rangle, |\bar{P}^\circ\rangle\}$ satisfies orthogonality and completeness relations

$$\langle P^\circ|\bar{P}^\circ\rangle = \langle\bar{P}^\circ|P^\circ\rangle = 0, \qquad \langle P^\circ|P^\circ\rangle = \langle\bar{P}^\circ|\bar{P}^\circ\rangle = 1,$$
$$|P^\circ\rangle\langle P^\circ| + |\bar{P}^\circ\rangle\langle\bar{P}^\circ| = 1. \tag{7}$$

It is readily shown that $\hat{H}$ is not Hermitian. If $[\hat{H}, \hat{H}^\dagger] \neq 0$, then the orthogonality and completeness relations for eigenstates of $\hat{H}$ are not satisfied, i.e., the eigenstates of $\hat{H}$ are not discriminant states. Therefore we cannot diagonalize $\hat{H}$ or write its spectral form though its basis. To do this job we make the benefit of biorthonormal basis. Such basis could be set up for neutral meson system. Indeed the Hamiltonian is diagonalizable only with such a set of basis.

According to Section 2 the eigenvalues of $\hat{H}$ are denoted by $\mu_a = m_a - i\Gamma_a/2$ and $\mu_b = m_b - i\Gamma_b/2$ corresponding to the eigenvectors $|P_a\rangle$ and $|P_b\rangle$ respectively. So that

$$\hat{H}|P_a\rangle = \mu_a|P_a\rangle,$$
$$\hat{H}|P_b\rangle = \mu_b|P_b\rangle. \tag{8}$$

We also denote the eigenvalues of $\hat{H}^\dagger$ by $\mu_a^* = m_a + i\Gamma_a/2$ and $\mu_b^* = m_b + i\Gamma_b/2$ corresponding to the eigenvectors $|\tilde{P}_a\rangle$ and $|\tilde{P}_b\rangle$ respectively. So that

$$\hat{H}^\dagger|\tilde{P}_a\rangle = \mu_a^*|\tilde{P}_a\rangle,$$
$$\hat{H}^\dagger|\tilde{P}_b\rangle = \mu_b^*|\tilde{P}_b\rangle. \tag{9}$$

It is not difficult to check that the set $\{|P_n\rangle, |\tilde{P}_n\rangle\}$, $n = a, b$, is a complete biorthonormal system for $\hat{H}$ such that

$$\langle P_a|\tilde{P}_b\rangle = \langle\tilde{P}_a|P_b\rangle = 0, \qquad \langle P_a|\tilde{P}_a\rangle = \langle\tilde{P}_b|P_b\rangle = 1, \tag{10}$$

$$|P_a\rangle\langle\tilde{P}_a| + |P_b\rangle\langle\tilde{P}_b| = |\tilde{P}_a\rangle\langle P_a| + |\tilde{P}_b\rangle\langle P_b| = 1. \tag{11}$$

According to the definition given in the previous section, $\hat{H}$ could be diagonalized such that

$$\chi^{-1}\hat{H}\chi = \begin{pmatrix}\mu_a & 0\\ 0 & \mu_b\end{pmatrix}, \qquad \chi = \begin{pmatrix}p_a & q_a\\ p_b & -q_b\end{pmatrix}, \tag{12}$$

which means

$$|P_a\rangle = p_a|P^\circ\rangle + q_a|\bar{P}^\circ\rangle,$$
$$|P_b\rangle = p_b|P^\circ\rangle - q_b|\bar{P}^\circ\rangle, \tag{13}$$

and

$$|\tilde{P}_a\rangle = \frac{1}{p_a q_b + p_b q_a}\big[q_b|P^\circ\rangle + p_b|\bar{P}^\circ\rangle\big],$$
$$|\tilde{P}_b\rangle = \frac{1}{p_a q_b + p_b q_a}\big[q_a|P^\circ\rangle - p_a|\bar{P}^\circ\rangle\big]. \tag{14}$$

The signs in front of q_a and q_b in (13) and p_a and p_b in (14) are just a convention which may differ among different authors, or even from one neutral meson system to another within the same paper. Now we can write the spectral form of the Hamiltonian H,

$$\hat{H} = \mu_a|P_a\rangle\langle\tilde{P}_a| + \mu_b|P_b\rangle\langle\tilde{P}_b|. \tag{15}$$

The normalization conditions are

$$|p_a|^2 + |q_a|^2 = |p_b|^2 + |q_b|^2 = 1. \tag{16}$$

We find from Eqs. (8) and (13) that

$$\frac{q_a}{p_a} = \frac{\mu_a - H_{11}}{H_{12}} = \frac{H_{21}}{\mu_a - H_{22}}, \tag{17}$$
$$\frac{q_b}{p_b} = \frac{H_{11} - \mu_b}{H_{12}} = \frac{H_{21}}{H_{22} - \mu_b}. \tag{18}$$

By considering the effect of discrete symmetries on the matrix elements of $\hat{H}$ we have

CPT conservation $\to H_{11} = H_{22}$,

T conservation $\to |H_{12}| = |H_{21}|$,

CP conservation $\to H_{11} = H_{22}$ and $|H_{12}| = |H_{21}|$.

The above conditions suggest the dimensionless complex CP and CPT parameter as [2]

$$\theta \equiv \frac{\frac{q_a}{p_a} - \frac{q_b}{p_b}}{\frac{q_a}{p_a} + \frac{q_b}{p_b}} = \frac{H_{22} - H_{11}}{\mu_a - \mu_b}, \tag{19}$$

and the CP and T violation parameter as

$$\delta \equiv \frac{|\frac{p_b}{q_b}| - |\frac{q_a}{p_a}|}{|\frac{p_b}{q_b}| + |\frac{p_a}{q_a}|} = \frac{|H_{12}| - |H_{21}|}{|H_{12}| + |H_{21}|}. \tag{20}$$

It is convenient to introduce

$$\frac{q}{p} = \sqrt{\frac{q_a q_b}{p_a p_b}} = \sqrt{\frac{H_{21}}{H_{12}}}. \tag{21}$$

If CPT violation is absent from the mixing, then $q/p = q_a/p_a = q_b/p_b$ and $\sqrt{1-\theta^2} = 1$. In that case one only needs to use q/p.

The time evolution of the neutral meson system is easily obtained using the spectral form of the Hamiltonian $\hat{H}$,

$$e^{-i\hat{H}t} = e^{-i\mu_a t}|P_a\rangle\langle\tilde{P}_a| + e^{-i\mu_b t}|P_b\rangle\langle\tilde{P}_b|. \tag{22}$$

Using Eqs. (13), (19) and (22) one finds at time t for the states $|P^\circ\rangle$ and $|\bar{P}^\circ\rangle$ created at time $t = 0$,

$$\big|P^\circ(t)\big\rangle = \big[g_+(t) - \theta g_-(t)\big]|P^\circ\rangle + \frac{q}{p}\sqrt{1-\theta^2}\,g_-(t)|\bar{P}^\circ\rangle, \tag{23}$$

$$\big|\bar{P}^\circ(t)\big\rangle = \left[\frac{p}{q}\sqrt{1-\theta^2}\,g_-(t)\right]|P^\circ\rangle + \big[g_+(t) + \theta g_-(t)\big]|\bar{P}^\circ\rangle, \tag{24}$$

where

$$g_\pm(t) = \frac{1}{2}\big(e^{-i\mu_a t} \pm e^{-i\mu_b t}\big). \tag{25}$$

4. Cascade decay and *CPT* violation

It was conjectured by Azimov [7] that additional tests of CPT invariance (violation) could be performed by looking for $B_d^\circ(\bar{B}_d^\circ) \to J/\psi K^\circ(\bar{K}^\circ) \to J/\psi f$ *cascade decay*, involving the neutral B_d and neutral kaon system in succession. This idea has been followed by Dass and Sarma [8]. In all these cases, an initial B° meson (B° stands for both B_d° and B_s°) can only decay to one of the kaon's flavor eigenstates. To the leading order in the Standard Model, the decays $B_d^\circ \to \bar{K}^\circ + X$ and $B_s^\circ \to K^\circ + X$ and respective CP conjugate decays are forbidden [9,10]. The possibility of probing CPT invariance by observing the time dependence of the cascade decays of the type $B_d^\circ(\bar{B}_d^\circ) \to J/\psi K^\circ/\bar{K}^\circ \to J/\psi f$ was investigated more recently [11] by considering that in new physics and in higher order, one cannot neglect the processes $B_d^\circ \to J/\psi \bar{K}^\circ$ and $\bar{B}_d^\circ \to J/\psi K^\circ$ when considering such a radical possibility as CPT violation.

In such cases there are two times and two CPT violation parameters. The time t for B_d° meson to oscillate before decaying into $J/\psi K^\circ$ and time t' in which K° oscillates before decaying into f. The CPT violation parameter θ in the (B_d°–$\bar{B}_d^\circ$) meson mixing and the CPT violation parameter θ' in the (K°–$\bar{K}^\circ$) meson mixing. It is possible to determine θ by comparing the t dependence of the cascade decays of tagged P° and tagged $\bar{P}^\circ$. Indeed, θ is computed in much the same way as from the time dependence of non-cascade decays [12,13]. The parameter θ' cannot be determined, because it always appears entangled with some undetermined ratios of decay amplitudes.

We study the possibility of probing CPT invariance by observation of the time dependence of the cascade decay of the type $B_d^\circ(\bar{B}_d^\circ) \to J/\psi\{K_L, K_S\} \to J/\psi f$ by employing the complete biorthonormal basis and introduce new ratios of decay amplitudes. In such cascade decays there are two times and one CPT violation parameter. The time t for the B_d° meson to oscillate before decaying into $J/\psi\{K_S, K_L\}$ and time t' in which the decay into final state $J/\psi f$ takes place. Since CP is violated, there is no final state that can be obtained only from K_S (or K_L) and not from K_L (or K_S). Therefore all calculations must involve the full transition chain

$$i \to J/\psi\{K_L, K_S\} \to J/\psi f. \tag{26}$$

We consider an experiment in which a tagged B_d° evolves for time t and decays into an intermediate state $J/\psi K_L$ or $J/\psi K_S$, which after time t' decays into the final state $J/\psi f$. The amplitude for this process is

$$e^{-i\mu_S t'}\langle f|\hat{T}|K_S\rangle\langle\tilde{K}_S J/\psi|\hat{T}\big|B_d^\circ(t)\big\rangle + e^{-i\mu_L t'}\langle f|\hat{T}|K_L\rangle\langle\tilde{K}_L J/\psi|\hat{T}\big|B_d^\circ(t)\big\rangle. \tag{27}$$

By using Eqs. (23) and (24) (time evolution of flavor eigenstates) it is easy to show that the above expression casts into the following

$$\frac{1}{2}\langle f|\hat{T}|K_S\rangle e^{-i\mu_S t'}\Big[\big((1-\theta)e^{-i\mu_a t} + (1+\theta)e^{-i\mu_b t}\big)\langle\tilde{K}_S J/\psi|\hat{T}\big|B_d^\circ\rangle + \frac{q}{p}\sqrt{1-\theta^2}\big(e^{-i\mu_a t} - e^{-i\mu_b t}\big)\langle\tilde{K}_S J/\psi|\hat{T}\big|\bar{B}_d^\circ\rangle\Big] + \frac{1}{2}\langle f|\hat{T}|K_L\rangle e^{-i\mu_L t'}\Big[\big((1-\theta)e^{-i\mu_a t} + (1+\theta)e^{-i\mu_b t}\big)\langle\tilde{K}_L J/\psi|\hat{T}\big|B_d^\circ\rangle + \frac{q}{p}\sqrt{1-\theta^2}\big(e^{-i\mu_a t} - e^{-i\mu_b t}\big)\langle\tilde{K}_L J/\psi|\hat{T}\big|\bar{B}_d^\circ\rangle\Big]. \tag{28}$$

Now we introduce the four parameters

$$\lambda = -\frac{q}{p}\frac{\langle\tilde{K}_S J/\psi|\hat{T}|\bar{B}^\circ\rangle}{\langle\tilde{K}_S J/\psi|\hat{T}|B^\circ\rangle}, \tag{29}$$

$$y = -\frac{q}{p}\frac{\langle\tilde{K}_L J/\psi|\hat{T}|B^\circ\rangle}{\langle\tilde{K}_S J/\psi|\hat{T}|B^\circ\rangle}, \tag{30}$$

$$\bar{y} = -\frac{\langle\tilde{K}_L J/\psi|\hat{T}|\bar{B}^\circ\rangle}{\langle\tilde{K}_S J/\psi|\hat{T}|B^\circ\rangle}, \tag{31}$$

and

$$\eta_f = \frac{\langle f|\hat{T}|K_L\rangle}{\langle f|\hat{T}|K_S\rangle}. \tag{32}$$

Therefore we may write the amplitude as

$$e^{-i\mu_S t'}\langle f|\hat{T}|K_S\rangle\langle\tilde{K}_S J/\psi|\hat{T}\big|B_d^\circ(t)\big\rangle + e^{-i\mu_L t'}\langle f|\hat{T}|K_L\rangle\langle\tilde{K}_L J/\psi|\hat{T}\big|B_d^\circ(t)\big\rangle \propto e^{-i\mu_S t'}\big[e^{-i\mu_a t} + Re^{-i\mu_b t}\big] + Q\eta_f e^{-i\mu_L t'}\big[e^{-i\mu_a t} + Se^{-i\mu_b t}\big], \tag{33}$$

where

$$R = \frac{(1+\theta)+\lambda\sqrt{1-\theta^2}}{(1-\theta)-\lambda\sqrt{1-\theta^2}}, \qquad S = \frac{\bar{y}(1+\theta)-y\sqrt{1-\theta^2}}{\bar{y}(1-\theta)+y\sqrt{1-\theta^2}}. \tag{34}$$

We next consider the analogous experiment with an initial $\bar{B}_d^\circ$. The amplitude is

$$e^{-i\mu_S t'}\langle f|\hat{T}|K_S\rangle\langle\tilde{K}_S J/\psi|\hat{T}\big|\bar{B}_d^\circ(t)\big\rangle + e^{-i\mu_L t'}\langle f|\hat{T}|K_L\rangle\langle\tilde{K}_L J/\psi|\hat{T}\big|\bar{B}_d^\circ(t)\big\rangle \propto e^{-i\mu_S t'}\big[e^{-i\mu_a t} + \bar{R}e^{-i\mu_b t}\big] + Q\eta_f e^{-i\mu_L t'}\big[e^{-i\mu_a t} + \bar{S}e^{-i\mu_b t}\big], \tag{35}$$

where

$$\bar{R} = \frac{\lambda(1-\theta)+\sqrt{1-\theta^2}}{\lambda(1+\theta)-\sqrt{1-\theta^2}}, \qquad \bar{S} = \frac{y(1-\theta)-\bar{y}\sqrt{1-\theta^2}}{y(1+\theta)+\bar{y}\sqrt{1-\theta^2}}. \tag{36}$$

It is easily checked that

$$\bar{R} = -\frac{1-\theta}{1+\theta}R, \qquad \bar{S} = -\frac{1-\theta}{1+\theta}S. \tag{37}$$

If we use the first order approximation for small parameters, i.e., the approximation of neglecting all products of θ and λ, then we find

$$R \simeq (1+\lambda)(2\theta+\lambda+1), \tag{38}$$

$$S \simeq \left(1-\frac{y}{\bar{y}}\right)\left(2\theta-\frac{y}{\bar{y}}+1\right). \tag{39}$$

One concludes from Eq. (37) that the *CPT* violation in B mixing (the parameter θ) can in principle be determined either from the comparison of R and $\bar{R}$, or from the comparison of S and $\bar{S}$. Indeed, $\bar{R} \neq -R$ and $\bar{S} \neq -S$ unequivocally indicate the presence of *CPT* violation in the mixing of the B-meson. One can measure the *CPT* violation in B-meson mixing by observation of the time dependence of the tagged cascade decays.

5. Conclusion

In this work we introduced the system of complete set of biorthonormal basis as a necessary condition for diagonalizability of non-Hermitian Hamiltonian (regardless of being normal or not normal). We also noticed that in the presence of T violation, since $\hat{H}$ is not normal, the mass eigenstates do not satisfy completeness and orthonormal relations and therefore are indistinguishable and non-physical.

We also studied the possibility of *CPT* invariance in the cascade model of the type $P^\circ(\bar{P}^\circ) \to \{P_a, P_b\}X \to fX$ by observation of the time dependence of the process. In this case there are two time parameters of t and t' and one *CPT* violation parameter. We showed that it is possible to determine the θ parameter by comparison of time dependence of the cascade models for B° and $\bar{B}^\circ$.

We conclude by remarking that in the cascade model of $P^\circ(\bar{P}^\circ) \to X(M^\circ/\bar{M}^\circ) \to Xf$ there are two *CPT* violation parameters (θ and θ') where θ is the *CPT* violation parameter in P°–$\bar{P}^\circ$ and θ' is the *CPT* violation parameter in M°–$\bar{M}^\circ$ system. In this case the parameter θ' in indeterminable due to entanglement with other parameters [12]. However using the biorthonormal system of basis for $P^\circ(\bar{P}^\circ) \to \{P_a, P_b\}X \to fX$, there is only one *CPT* violation parameter which is quite determinable.

References

[1] V.F. Weisskopf, E.T. Wigner, Z. Phys. 63 (1930);
V.F. Weisskopf, E.T. Wigner, Z. Phys. 54 (1930);
V.F. Weisskopf, E.T. Wigner, Z. Phys. 65 (1930) 18.
[2] G.C. Branco, L. Lavoura, J.P. Silva, CP Violation, Cambridge Univ. Press, 1999.
[3] T.D. Lee, C.S. Wu, in: M.K. Gallard, M. Nikolic (Eds.), Weak interactions, Annu. Rev. Nucl. Sci. 16 (1966) 471.
[4] J.W. Gronin, Rev. Mod. Phys. 53 (1981) 373.
[5] L. Wolfenstein (Ed.), CP Violation, Current Sources and Comments, vol. 5, North-Holland, 1989.
[6] Y. Nir, hep-ph/0109090.

[7] Ya.I. Azimov, JETP Lett. 50 (1989) 447;
Ya.I. Azimov, Phys. Rev. D 42 (1990) 3705.

[8] G.V. Dass, K.V.L. Sarma, Int. J. Mod. Phys. A 7 (1992) 6081;
G.V. Dass, K.V.L. Sarma, Int. J. Mod. Phys. A 8 (1993) 1183(E).

[9] Ya.I. Azimov, Pis'ma Zh. Eksp. Teor. Fiz. 50 (1989) 413.

[10] C.C. Meca, J.P. Silva, Phys. Rev. Lett. 81 (1998) 1377.

[11] L. Lavoura, Phys. Rev. D 62 (2000) 056002.

[12] L. Lavoura, Ann. Phys. (N.Y.) 207 (1991) 428.

[13] L. Lavoura, J.P. Silva, Phys. Rev. D 60 (1999) 056003.