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Effective degrees of freedom of the quark–gluon plasma

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Abstract

The effective degrees of freedom of the quark–gluon plasma are studied in the temperature range $\sim (1\text{–}2)T_c$. We show that including light bosonic states one can reproduce the pressure and energy density of the quark–gluon plasma obtained by lattice simulations. The number of the bosonic states required is at most of the order of 20, consistent with the number of light mesonic states and in disagreement with a recently proposed picture of the quark–gluon plasma as a system populated with exotic bound states. We also constrain the quark quasiparticle chiral invariant mass to be $\lesssim 300$ MeV. Some remarks regarding the role of the gluon condensation and the baryon number-strangeness correlation are also presented.

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Quantum chromodynamics (QCD) predicts that at extremely high temperatures matter consists of a gas of weakly interacting quarks and gluons, the quark–gluon plasma (QGP). However at moderate temperatures $T = (1\text{–}2)T_c$, ($T_c \simeq 170$ MeV deconfinement temperature) it is less clear what the dynamical phase is.

The experimental data obtained at the Relativistic Heavy Ion Collider (RHIC), with the measurement of the p_t spectra and the related indications on the radial and elliptic flow (see [1] for a review), clearly suggest that at moderate temperatures the produced system is in a strongly interacting phase (sQGP) and there are remnant of the confining interaction up to temperatures $\simeq 2T_c$, in agreement with lattice (lQCD) results. Actually, lattice calculations of the pressure and energy density of the system do not reach the Stefan–Boltzmann values for a weakly interacting quark–gluon plasma even at very large temperatures $\simeq 5T_c$ [2,3].

This surprising picture calls for understanding the relevant degrees of freedoms to describe such a phase and, in this respect, several models have been proposed, where deconfinement and chiral symmetry restoration occur at a lower temperature than the $\bar{q}q$ dissociation temperature and "resonance" states may play an important dynamical role [4–9].

However, recent lattice and phenomenological analyses [10] have shown that the emerging degrees of freedom are quark and gluon quasiparticles and this result has to be compatible with the survival of $\bar{q}q$ states [2,11] as obtained by the analysis of mesonic spectral functions above T_c.

In this Letter we address these puzzling aspects by performing a phenomenological analysis of the pressure and of the energy density of the system taking into account the presence of quark and gluon quasiparticles as well as of $\bar{q}q$ states in the temperature range $(1\text{–}2)T_c$.

Let us consider a system of quark, antiquark, gluons and correlated particle states.

The gluonic sector contains quasiparticle contributions as well as nonperturbative condensation effects. The ratio, $c_e(T)$, between the chromo-electric condensate evaluated at finite temperature and at $T = 0$ strongly decreases for temperature $T > 1.2T_c$, whereas the same ratio for the magnetic condensate is $\simeq 1$ up to larger temperatures [12]. Since at $T = 0$ the chromo-electric and chromo-magnetic parts are equal, one can write the

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gluon condensate as

$$\left\langle \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^{\mu\nu}_a \right\rangle_T = \frac{1}{2} \left\langle \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^{\mu\nu}_a \right\rangle_0 \left[1 + c_e(T)\right], \tag{1}$$

where the ratio $c_e(T)$ can be approximated by the unquenched data of Ref. [12] and we take $\langle \alpha_s G^2/\pi \rangle_0 \simeq 0.01\ \text{GeV}^4$, consistently with QCD sum rules [13].

The gluon condensate, i.e. a macroscopically populated state with zero momentum, does not essentially contribute to the pressure but is crucial for the evaluation of the energy density [14]. On the other hand, the gluonic sector contains also gluon quasiparticles which contribute to the pressure and to the energy density and that we shall treat in a phenomenological way (see below).

Concerning the fermionic sector, we assume that the number of quark/antiquark degrees of freedom is fixed, $D_q = D_{\bar{q}} = 18$. Recently a fermionic system with a dispersion relations of the general form

$$\omega_{\bar{q}}(k) = \omega_q(k) = \sqrt{k^2 + m^2} + \Sigma_R, \tag{2}$$

has been studied in [6,15] where m and the self-energy Σ_R have been evaluated taking into account the interaction of the quasiparticles with the medium. For the relevant momenta, of the order of the thermal momentum, we consider that m/k is small [6] and treat the chiral invariant term Σ_R as a constant parameter M by using the dispersion relation

$$\omega_{\bar{q}}(k) = \omega_q(k) = k + M. \tag{3}$$

Therefore, at this level of approximation, we neglect the dynamical phenomena related to frequencies with $\omega/T \ll 1$, such as viscosity.

The structure of the in-medium correlated states as a function of temperature is not easily evaluated. These states may describe $\bar{q}q$ states as well as more exotic states [5]. Close to T_c, it should be reasonable to consider that the number of correlated state degrees of freedom D_b is of the order of 10, which corresponds to the pseudoscalar nonet. However in our analysis we will treat D_b as a parameter indicating that an effective number of bosonic states is present. In the following we will neglect, as a first approximation, the effect on the thermodynamics quantities of the width of the bosonic states. Therefore we employ the dispersion law $\omega_b = \sqrt{k^2 + 4M^2}$. Considering a different value of the mass of the mesons in the range M–$3M$ changes our results of less than 25%.

Finally, the interaction of the gluonic sector with fermions and correlated states is described, in a mean field-like treatment, by M and D_b which will be evaluated employing unquenched lattice data of pressure and energy density.

Our expressions of pressure and energy density are given by the sum of the contributions of quarks, antiquarks, bosonic pairs and gluons:

$$p(T) = T \sum_{i=q,\bar{q},b} D_i \int \frac{d^3k}{(2\pi)^3} \log\left(1 \pm e^{-\omega_i/T}\right)^{\pm 1} + p_g, \tag{4}$$

$$\epsilon(T) = \sum_{i=q,\bar{q},b} D_i \int \frac{d^3k}{(2\pi)^3} \frac{\omega_i}{e^{\omega_i/T} \pm 1} + \epsilon_g + \epsilon_{\text{con}}(T), \tag{5}$$

where the sign $+$ $(-)$ refers to fermions (bosons), $p_g(T)$ and $\epsilon_g(T)$ are respectively the contributions to the pressure and energy density due to gluon quasiparticles and $\epsilon_{\text{con}}(T)$ is the gluonic condensate given in Eq. (1).

In order to evaluate M and D_b we perform a simultaneous fit of the lattice data of pressure and energy density of 3 flavors quark matter with quark masses $m = 0.4T$ of Refs. [2,3] as a function of the temperature employing Eqs. (1), (4) and (5). For each value of the temperature we consider the central value of pressure and energy density of the lattice data. Considering a different value within the statistical error bars determines a variation in our results of less than 10%.

Due to the temperature dependence of the parameters $M(T)$ and $D_b(T)$ and to the introduction of the gluon condensate, the thermodynamics consistency must be carefully checked. The relation between pressure and energy density is given by

$$T \frac{dp}{dT} = p + \epsilon + C_r, \tag{6}$$

where the correction C_r depends on the temperature and is given by

$$\begin{aligned} C_r = {} & -T \frac{2D_q}{2\pi^2} \int dk\, k^2 \frac{1}{1 + \exp(\omega_q/T)} \frac{dM}{dT} \\ & - T \frac{D_b}{2\pi^2} \int dk\, k^2 \frac{1}{-1 + \exp(\omega_b/T)} \frac{4M}{\omega_b} \frac{dM}{dT} \\ & + \frac{T}{D_b} \frac{dD_b}{dT} p_b - \epsilon_{\text{con}}, \end{aligned} \tag{7}$$

where p_b is the contribution to the pressure of the in-medium correlated states. The thermodynamics consistency requires that

$$C_r = 0, \tag{8}$$

which implies that D_b satisfies the differential equation

$$A \frac{dD_b}{dT} + B D_b + C = 0, \tag{9}$$

where

$$A = -\frac{1}{2\pi^2} \int dk\, k^2 \log\left[1 - \exp(-\omega_b/T)\right], \tag{10}$$

$$B = -\frac{1}{2\pi^2} \int dk\, k^2 \frac{1}{-1 + \exp(\omega_b/T)} \frac{4M}{\omega_b} \frac{dM}{dT}, \tag{11}$$

$$C = -\frac{2D_q}{2\pi^2} \int dk\, k^2 \frac{1}{1 + \exp(\omega_q/T)} \frac{dM}{dT} - \frac{\epsilon_{\text{con}}}{T}. \tag{12}$$

Substituting the values of $D_b(T)$ and of $M(T)$, obtained fitting the lattice data of energy and pressure, in the previous equations, represents a self-consistent check of our result. We find that the differential equation (9) is satisfied with a good accuracy and the corresponding correction C_r is of the same order of the error in the lattice data.

According to the results obtained in quenched lattice simulations, the contribution ϵ_g to the energy density is small with respect to the gluon condensate, moreover the effect of p_g on the pressure of the whole system (i.e. including fermions and correlated states) is expected to be small [2]. Therefore, as a first step of our analysis let us assume that $p_g = \epsilon_g = 0$.

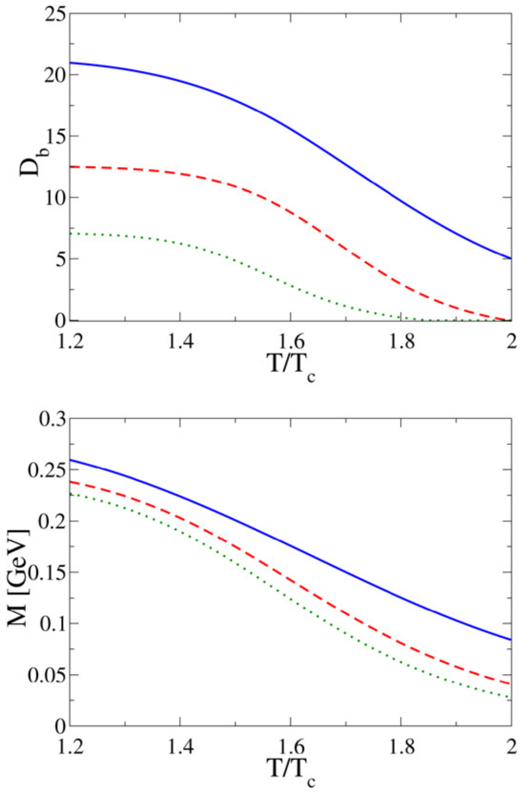

Fig. 1. (Color online.) Effective number of bosonic degrees of freedom (upper panel) and quasiparticle chiral mass (lower panel) as a function of the temperature for $T = (1.2\text{–}2)T_c$. Full (blue online) lines correspond to $p_g = \epsilon_g = 0$; dashed (red online) lines correspond to pressure and energy density evaluated with $m_g = 1.0$ GeV and dotted (green online) lines to $m_g = 0.9$ GeV.

The result of the combined, $p(T)$–$\epsilon(T)$, analysis, for $p_g = \epsilon_g = 0$, are shown in the plots of Fig. 1, full (blue online) lines.

From the upper panel of Fig. 1 one sees that even if the gluon quasiparticles are turned off, i.e. one is artificially increasing the number of correlated pairs, the effective number of bosonic degrees of freedom, in the range $T = (1.2\text{–}2)T_c$, is at most of the order of 20. Notice that such result is not consistent with the outcomes of Ref. [5] where the existence of a large number (of order 100) of exotic states in the deconfined phase is hypothesized.

The chiral invariant mass of the quark quasiparticles, shown in the lower panel of Fig. 1, turns out to be a decreasing function of the temperature suggesting that the mechanism which determines the chiral mass becomes less efficient as the temperature increase. It is interesting to note that the decreasing of the chiral mass as a function of the temperature determined with this approach is in qualitative agreement with the one determined in Ref. [6] with a different method.

Notice that the previously obtained values of D_b are an upper bound to the effective number of correlated degrees of freedom. Indeed they were obtained taking $p_g = \epsilon_g = 0$. If in Eqs. (4) and (5) one switches on gluons, that is if one includes the contributions of the gluon quasiparticles to the pressure and to the energy density, these terms reduce the weight of the fermions and of the correlated bosonic states. To check numerically this effect we have fitted the unquenched lattice data of Refs. [2,3] including in Eqs. (4) and (5) the gluonic pressure and energy density respectively given by

$$p_g = D_g \int \frac{d^3k}{(2\pi)^3} \log\left(1 - e^{-\omega_g/T}\right)^{-1}, \tag{13}$$

$$\epsilon_g = D_g \int \frac{d^3k}{(2\pi)^3} \frac{\omega_g}{e^{\omega_g/T} - 1}, \tag{14}$$

where $D_g = 24$ is the number of gluonic degrees of freedom[1] and the dispersion relation of gluons is given by $\omega_g = \sqrt{m_g^2 + k^2}$ with m_g the gluon quasiparticle mass. The results are reported in Fig. 1 which shows that increasing the values of p_g and ϵ_g, i.e. employing different values of m_g, one obtains that the values of D_b and M decrease. The dashed (red online) lines corresponds to $m_g = 1.0$ GeV; the dotted (green online) lines corresponds to $m_g = 0.9$ GeV. Employing values of m_g smaller than 0.9 GeV both D_b and M are reduced (the effect is larger on D_b). However for very small values of the mass of the gluons one cannot simultaneously reproduce the lattice data of energy density and pressure. For values of m_g larger than 1.0 GeV both D_b and M increase. Considering values of the mass of the gluons larger than ~ 1.5 GeV, the corresponding values of M and D_b cannot be distinguished from the full line which corresponds to $p_g = 0$ and $\epsilon_g = 0$. We will discuss the dependence of D_b and M on m_g with more details in a future paper.

Table 1
Pressure and energy density for three different values of the temperature. P and E correspond to the values obtained in lattice simulations Refs. [2,3], whereas p and ϵ are the sum of quark, bosonic and gluonic contribution evaluated by Eqs. (4), (5), (13) and (14) with $m_g = 1.0$ GeV

T/T_c	P/T^4	p/T^4	E/T^4	ϵ/T^4
1.2	1.7	1.6	12.1	12.0
1.5	2.9	2.9	12.6	12.7
2.0	3.6	3.6	12.6	12.8

In order to check our results we have evaluated pressure and energy density summing the contributes of quarks, bosonic degrees of freedom and gluons and compared our results with the actual values obtained by lattice simulations [2,3]. The result of such a comparison is shown in Table 1. The agreement with lattice results in both cases is very good.

The main result of the present analysis of the unquenched lattice data of pressure and energy density is that if a quasiparticle description of the quark–gluon plasma holds in the temperature range $(1.2\text{–}2)T_c$, then the number of correlated states and of the quasiparticle masses is strongly constrained.

Few comments are now in order. The value of the mass of the quasiparticles that we obtain is smaller than the one obtained in Ref. [16] or in Ref. [17] where M is estimated to be $\sim (3\text{–}4)T$ from fits of lattice data. This difference essentially relies on the fact that we have considered the dispersion law of Eq. (3) with a chirally invariant mass, whereas in [16] and [17] the quasiparti-

[1] Here we are assuming that for $T \leqslant 2T_c$ gluons have a non-vanishing mass. However at larger temperatures the effective number of gluonic degrees of freedom will be reduced to 16. We will consider such effect in a forthcoming paper.

cle dispersion law has been parameterized as $\omega_q = \sqrt{k^2 + M^2}$. However in the chirally symmetric phase, the fermion quasiparticle mass cannot be a chiral breaking Dirac mass. Moreover in Ref. [16], in order to reproduce the lattice results, a (small) bag constant has been employed. In our case the contribution of the gluonic condensate and of the mesonic resonant states play a crucial role in determining the correct values of pressure and energy density.

Finally let us comment on the correlation between baryon number and strangeness (C_{BS}) as an indication of the effective dynamical degrees of freedom of the system. The analysis of lQCD results performed in [10] indicates that at $1.5T_c$ the BS correlation is very close to 1.

We can estimate, in an admittedly rough way, the BS correlation as

$$C_{\mathrm{BS}} \sim \frac{\frac{2}{3} D_q \langle n_q \rangle}{\frac{2}{3} D_q \langle n_q \rangle + \frac{4}{9} D_b \langle n_b \rangle}, \tag{15}$$

where $\langle n_b \rangle$ ($\langle n_q \rangle$) is the number density of bosonic (fermionic) states, the coefficient $D_q/3$ takes into account that in the chiral symmetric limit one third of fermions are strange (the factor two takes into account antiparticles), whereas the coefficient $4/9 D_b$ is an effective way to weight the number of strange bosons in D_b according to the meson nonet.

Employing the data of Fig. 1 at $T = 1.5T_c$ it turns out that the correlation is about 0.95 for $m_g = 1.0$ GeV. Using smaller values of m_g the correlation further increases. In any case the correlation turns out to be $\simeq 1$ at $T = 2.0T_c$.

In conclusion, according to the present work, the relevant degrees of freedom in QCD, for temperatures above T_c are q, $\bar{q}$, g quasiparticles and bosonic states. Contrary to recent claims, the effective number of degrees of freedom associated with the bosonic states is at most of order 20 suggesting that only light nonexotic states are present.

For $T \gtrsim 2T_c$ the contribution of mesonic bound states to pressure and energy density is vanishing small and only quasiparticles are relevant. On the other hand, gluon condensation and its persistence above T_c is a fundamental ingredient in the energy balance.

Further investigations are needed to clarify the underlying nonperturbative dynamics in terms of resonance scattering, chiral phase fluctuations and instantons.

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