Scientia Iranica D (2011) 18 (3), 611–616

Sharif University of Technology
Scientia Iranica
Transactions D: Computer Science & Engineering and Electrical Engineering
www.sciencedirect.com

Invited paper

Zadeh–MacFarlane–Jamshidi theorems on decoupling of a fuzzy rule base

C.W. de Silva

Industrial Automation Laboratory, Department of Mechanical Engineering, The University of British Columbia, Vancouver, BC, Canada

Received 11 August 2010; revised 28 October 2010; accepted 8 February 2011

KEYWORDS
L.A. Zadeh;
A.G.J. MacFarlane;
M. Jamshidi;
Fuzzy rule decoupling;
Single-context fuzzy decision making;
Fuzzy rule-base equivalence.

Abstract This paper outlines the inspiration received by the author from the Zadeh–MacFarlane–Jamshidi trio in his pursuit concerning the theory and Application of fuzzy logic. Beginning with Zadeh's pioneering work, a hierarchical control system was developed, in collaboration with MacFarlane, for application in robotic manipulators. Subsequently, the work was extended to an analytical basis for controller tuning using fuzzy decision making. On the prompting of Jamshidi to address the issue of knowledge-base simplification, theorems were developed related to decoupling a fuzzy rule base. These developments provided a theoretical basis for applying single-context decision making to a problem governed by the knowledge base of coupled fuzzy rules. The developed theorems establish an analytical equivalence between the decisions made from a coupled set of fuzzy rules and an uncoupled set of fuzzy rules concerning the same problem domain. These developments have been applied to supervisory control of an industrial fish cutting machine. The paper presents the pertinent theory and illustrative examples.

1. Introduction

In the mid 1960's, Professor L.A. Zadeh developed the concept of fuzzy sets and the use of fuzzy logic in approximate reasoning. Even though this author had been exposed to the basic notions of fuzzy logic and the unsubstantiated anecdotal reports on Zadeh-Kalman rivalry, it was only in the mid 1980s that he embarked on some serious work concerning the subject. In fact, it was under the coercion of one of his graduate students from Korea, who insisted on working in self-organizing fuzzy control, that he carefully read a paper by Zadeh [1], and then studied the book by Dubois and Prade [2] to learn the basics of fuzzy logic. At that crucial juncture of his career, amidst teaching, research, student supervision, committee meetings, business travel and research proposal writing, the author did not even have many opportunities for quality time with his family. An opportunity for dedicated work in fuzzy logic arose only in 1987 when he visited Cambridge University under a Senior Fulbright Fellowship. Fortunately, his host at Cambridge, Professor A.G.J. MacFarlane-an authority in traditional control-enthusiastically agreed to collaborate with him on fuzzy control. So began a serious career in the subject.

Collaboration with Professor MacFarlane resulted in the development of a hierarchical control system with fuzzy tuning for robotic manipulators [3,4]. This work was subsequently extended to an analytical basis for fuzzy tuning of servo controllers [5]. The analytical difficulties and the computational burden of using a coupled rule base for making tuning decisions were found to be paramount concerns, however. Consider, for example, a fuzzy rule base containing n condition variables, one action variable; each condition variable having m fuzzy states (its fuzzy resolution), and each membership function discretized by N points for digital computing. It has been shown [5], for example, that the formation of the coupled rule base requires nm^nN^n min operations and m^nN^n max operations, while making a fuzzy inference from the rule base requires nN^{n+1} min operations and $N(N^n - 1)$ max operations. If only one condition variable and one action variable are present in a rule, these computations will be reduced to approximately nmN^2 min operations and $n(m-1)N^2$ max operations.

Generally, some accuracy is lost by assuming that the rules in a knowledge base are uncoupled when they are coupled in reality. In view of this, it is important to examine the conditions under which this assumption can be made without sacrificing

E-mail address: desilva@mech.ubc.ca.

1026-3098 © 2011 Sharif University of Technology. Production and hosting by Elsevier B.V. Open access under CC BY-NC-ND license.
Peer review under responsibility of Sharif University of Technology.

doi:10.1016/j.scient.2011.04.014

Production and hosting by Elsevier

the accuracy of decision making. The author discussed this issue with Professor M. Jamshidi during their first meeting at the *International Symposium on Robotics and Manufacturing* held in Vancouver, Canada, in 1990, and organized by Professor Jamshidi [6]. Professors Zadeh and Jamshidi were invited by the author for numerous return visits to Vancouver where they gave public talks and workshops, punctuated by group discussions. On the first occasion of Zadeh's visit to the University of British Columbia, the author booked the biggest lecture theatre in the Mechanical Engineering Building for the talk. To his dismay, as the talk was about to start, the theatre became fully packed by attendees, with a long line-up outside. Quickly, in consultation with Professor Zadeh, a second talk was scheduled for the next day, thereby averting an adverse reaction from the prospective audience. The author was careful to book the biggest lecture theatre in the University for Zadeh's talks during subsequent visits.

Visits from Professor Zadeh were both enjoyable and intellectually challenging and he was religiously accompanied by his lovely wife, Fay. Zadeh was found to be a simple and easygoing man with many admirable qualities, and he shared with us anecdotes about his early days at MIT and Columbia. Particularly inspiring was the story of how he discovered the theory of fuzzy sets during a visit to his parents' house in New York City. The author of this paper dedicates the following analytical work to the Zadeh–MacFarlane–Jamshidi trio, without whose inspiration his career in fuzzy logic would not have been possible.

2. Theory of rule-base decoupling

First, the general fuzzy decision making problem with a coupled rule base is formulated. Next, the method of single degree-of-freedom decision making, using a coupled rule base, is given. Then, the assumption of an uncoupled rule base is incorporated into the single-degree-of-freedom decision making problem. On that basis, sufficient conditions are established for rule-base decoupling in the problem of single degree-of-freedom decision making. Finally, these conditions are further relaxed.

2.1. Preliminaries

Some useful definitions in fuzzy logic are restated below [7], which form preliminaries for the problem statement and subsequent analytical development.

Definition 1. Consider a membership function $\mu(\underline{x},\underline{y}) : \Re^n \times \Re^p \rightarrow [0,1]$. Its projection in the $X_i \times Y_j$ subspace is denoted by $\text{Proj}\lfloor\mu(\underline{x},\underline{y})\rfloor(x_i,y_j) : \Re \times \Re \rightarrow [0,1]$ and is given by:

$$\text{Proj}[\mu(\underline{x},\underline{y})](x_i,y_j) \triangleq \sup_{\substack{\forall x_k \neq x_i \\ \forall y_\ell \neq y_j}} \mu\left(\underline{x},\underline{y}\right). \quad (1)$$

Example 1. Consider a two-variable membership function (a fuzzy relation in x and y), as shown in Figure 1. In the figure, this relation is projected onto the plane of the variable, y. The application of the *sup* operation to determine the projection is clear in this example.

Definition 2. Consider:

$\underline{x} \triangleq [x_1, x_2, \ldots, x_n]^T \in \Re^n$,

$\underline{y} \triangleq \left[y_1, y_2, \ldots, y_p\right]^T \in \Re^p$ and

$\mu\left(x_i,\underline{y}\right) : \Re \times \Re^p \rightarrow [0,1]$ with $1 \leq i \leq n$.

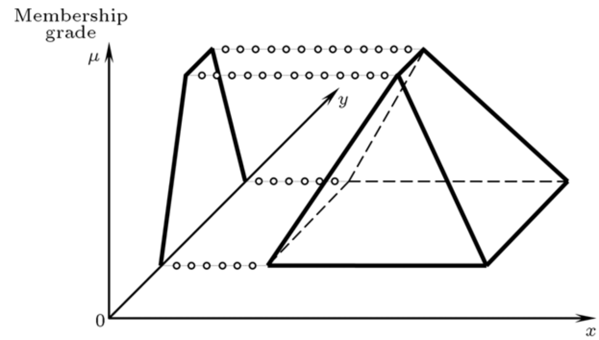

Figure 1: An example of fuzzy projection.

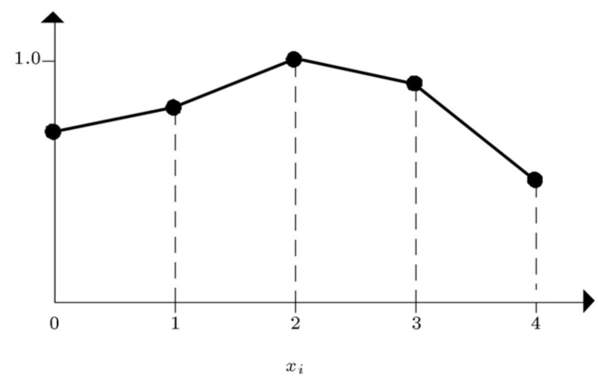

Figure 2a: A discrete membership function.

The *cylindrical extension* of $\mu(x_i,\underline{y})$ over the entire space, $\Re^n \times \Re^p$, is given by:

$$\text{Cyl}\lfloor\mu\left(x_i,\underline{y}\right)\rfloor\left(\underline{x},\underline{y}\right) : \Re^n \times \Re^p \rightarrow [0,1] = \mu\left(x_i,\underline{y}\right),$$
$$\forall\underline{x},\underline{y};\quad 1 \leq i \leq n. \quad (2)$$

Example 2. A discrete fuzzy relation, $R(x_i, y_j)$, is given by the following membership function matrix, defined in $X \times Y$ where $X = [0, 1, 2, 3, 4]$ and $Y = [0, 1, 2, 3, 4]$:

$\mu_R(x_i, y_j) =$

	$y_0 = 0$	$y_1 = 1$	$y_2 = 2$	$y_3 = 3$	$y_4 = 4$
$x_0 = 0$	0.0	0.4	0.7	0.3	0.0
$x_1 = 1$	0.1	0.5	0.8	0.4	0.1
$x_2 = 2$	0.6	0.7	1.0	0.5	0.2
$x_3 = 3$	0.3	0.4	0.9	0.7	0.4
$x_4 = 4$	0.0	0.1	0.5	0.3	0.1

The following discrete fuzzy set is derived from $R(x_i, y_j)$:

$$A(x_i) = \underset{x_i}{\text{Projection}}\, R(x_i, y_j).$$

Then:

$$\mu_A(x_i) = \sup_{y_j}[\mu_R(x_i, y_j)],$$

which gives:

$$A(x_i) = \left[\frac{0.7}{0}, \frac{0.8}{1}, \frac{1.0}{2}, \frac{0.9}{3}, \frac{0.5}{4}\right].$$

This membership function is sketched in Figure 2a.

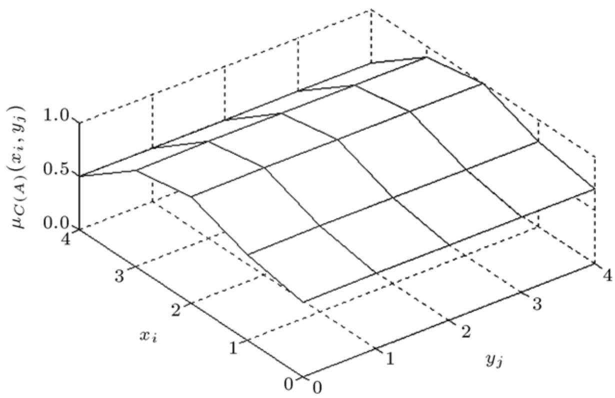

Figure 2b: The cylindrical extension of A.

Now consider the cylindrical extension of A in $X \times Y$, which is given by:

$$C_{X\times Y}(A) = \sum_{x_i, y_j} \frac{\mu_A(x_i)}{x_i, y_j}$$

$$= \begin{array}{c} \\ x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} \begin{array}{c} \begin{array}{ccccc} y_0 & y_1 & y_2 & y_3 & y_4 \end{array} \\ \begin{pmatrix} 0.7 & 0.7 & 0.7 & 0.7 & 0.7 \\ 0.8 & 0.8 & 0.8 & 0.8 & 0.8 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ 0.9 & 0.9 & 0.9 & 0.9 & 0.9 \\ 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \end{pmatrix} \end{array}.$$

This membership function is sketched in Figure 2b.

Now several properties are given [7].

Property 1. *Consider two independent universes, X and Y. Then the commutativity of the sup and min operations holds:*

$$\sup_{x,y} \min[(\mu(x), \mu(y))] = \min\left[\sup_x \mu(x), \sup_y \mu(y)\right],$$
$$\text{for } \forall x \in X, \forall y \in Y. \tag{3a}$$

This produces the property:

$$\sup_y \min[(\mu(x), \mu(y))] = \min\left[\mu(x), \sup_y \mu(y)\right]$$
$$= \min[\mu(x), hgt\,\mu(y)],$$
$$\text{for } \forall x \in X, \forall y \in Y, \tag{3b}$$

in which "hgt" is the height of a membership function which is its global peak value.

Property 2. *Consider a series of independent universes, X_i, $i = 1, 2, \ldots, n$, and the function:*

$$f(\underline{x}) = \min[\mu_1(x_1), \mu_2(x_2), \ldots, \mu_n(x_n)] : \Re^n \to [0, 1],$$
$$\underline{x} = [x_1, x_2, \ldots, x_n]^T.$$

Then by definition we have:

$$\mathrm{Proj}[f(x)](x_1, x_2) = \sup_{x_3,\ldots,x_n} \min[\mu_1(x_1), \mu_2(x_2), \ldots \mu_n(x_n)],$$

and from Property 1 we obtain:

$$\mathrm{Proj}[f(x)](x_1, x_2)$$
$$= \min\left[\mu_1(x_1), \mu_2(x_2), \sup_{x_3} \mu_3(x_3), \ldots, \sup_{x_n} \mu_n(x_n)\right]$$
$$= \min[\mu_1(x_1), \mu_2(x_2), hgt\ \mu_3(x_3), \ldots, hgt\,\mu_n(x_n)]. \tag{4}$$

Property 3. *Here use the fact that:*

$$\min[\mu(x), \mu(y), \alpha] = \min[\min[\mu(x), \mu(y)], \alpha],$$
$$\forall x \in X, \forall y \in Y.$$

Suppose that:

$$\sup_{x,y} \min[\mu(x), \mu(y)] \le \alpha.$$

Then in view of the commutativity property (Eq. (3a)), one has:

$$\min\left[\sup_x \mu(x), \sup_y \mu(y)\right] \le \alpha.$$

Since by definition:

$$\min[\mu(x), \mu(y)] \le \sup_{x,y} \min[\mu(x), \mu(y)],$$

one has:

$$\min[\mu(x), \mu(y)] \le \alpha.$$

Then:

$$\min[\min[\mu(x), \mu(y)], \alpha] = \min[\mu(x), \mu(y)].$$

Hence, if:

$$\min[\sup_x \mu(x), \sup_y \mu(y)] \le \alpha,$$

one has:

$$\min[\mu(x), \mu(y), \alpha] = \min[\mu(x), \mu(y)],$$
$$\text{for independent universes } X \text{ and } Y. \tag{5}$$

2.2. Coupled rule base

Consider a general, coupled, fuzzy rule base:

$$R_{\text{coupled}} : \mathrm{Else}_i \left(Y_1^i, Y_2^i, \ldots, Y_n^i\right) \to \left(U_1^i, U_2^i, \ldots, U_p^i\right), \tag{6}$$

where, Y_j^i denotes the fuzzy state of the jth context variable in the ith rule, and U_k^i denotes the fuzzy state of the kth context variable in the ith rule. The membership function of the rule base is given by:

$$\mu_R(\underline{y}, \underline{u}) = \max_i \min[\mu_{Y_1^i}(y_1), \mu_{Y_2^i}(y_2), \ldots, \mu_{Y_n^i}(y_n),$$
$$\mu_{U_1^i}(u_1), \mu_{U_2^i}(u_2), \ldots, \mu_{U_p^i}(u_p)]$$
$$= \max_i \min[\mathrm{Cyl}\mu_{Y_1^i}(\underline{y}, \underline{u}), \mathrm{Cyl}\mu_{Y_2^i}(\underline{y}, \underline{u}), \ldots,$$
$$\mathrm{Cyl}\mu_{Y_n^i}(\underline{y}, \underline{u}), \mathrm{Cyl}\mu_{U_1^i}(\underline{y}, \underline{u}),$$
$$\mathrm{Cyl}\mu_{U_2^i}(\underline{y}, \underline{u}), \ldots, \mathrm{Cyl}\mu_{U_p^i}(\underline{y}, \underline{u})]. \tag{7}$$

Rule interaction can take place due to a coupled rule base. As a result, the decision obtained from individual rules can be affected by the presence of other rules in the rule base.

Example 3. First, consider just one rule, $R_1 : A_1 \to C_1$, expressed in the discrete form where:

$A_1 = 0.7/a_1 + 0.6/a_2 + 0.1/a_3$, with cardinality 3,
$C_1 = 0.5/c_1 + 0.4/c_2$, with cardinality 2.

Using the min operation to represent fuzzy implication (if–then), this rule base may be expressed as:

$$\mu_{R_1}(a_i, c_j) = \begin{bmatrix} 0.5 & 0.4 \\ 0.5 & 0.4 \\ 0.1 & 0.1 \end{bmatrix}.$$

Now, suppose that a fuzzy observation, $A_1' = 0.7/a_1 + 0.6/a_2 + 0.1/a_3$, is made. By applying the max–min composition, the corresponding inference is given by:

$$\mu_{C'}\left(c_j\right) = \max_{\text{row}} \min_{\text{column}} \begin{bmatrix} 0.7 \\ 0.6 \\ 0.1 \end{bmatrix} \begin{bmatrix} 0.5 & 0.4 \\ 0.5 & 0.4 \\ 0.1 & 0.1 \end{bmatrix}$$

$$= \max_{\text{row}} \begin{bmatrix} 0.5 & 0.4 \\ 0.5 & 0.4 \\ 0.1 & 0.1 \end{bmatrix} = \begin{bmatrix} 0.5 & 0.4 \end{bmatrix},$$

or:

$$C' = 0.5/c_1 + 0.4/c_2.$$

Next, suppose that a second rule, $R_2 : A_2 \rightarrow C_2$, is available with:

$$A_2 = 0.6/a_1 + 0.7/a_2 + 0.2/a_3,$$
$$C_2 = 0.4/c_1 + 0.5/c_2.$$

The corresponding rule membership function is:

$$\mu_{R_2}\left(a_i, c_j\right) = \begin{bmatrix} 0.4 & 0.5 \\ 0.4 & 0.5 \\ 0.2 & 0.2 \end{bmatrix}.$$

Then using "max" for the OR operation, the membership function of the combined rule base, $R = R_1 \vee R_2$, is given by:

$$\mu_{R}\left(a_i, c_j\right) = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \\ 0.2 & 0.2 \end{bmatrix}.$$

Now, for the same fuzzy observation as before:

$$A_1' = 0.7/a_1 + 0.6/a_2 + 0.1/a_3,$$

the corresponding inference is obtained as:

$$\mu_{C'}\left(c_j\right) = \max_{\text{row}} \min_{\text{column}} \begin{bmatrix} 0.7 \\ 0.6 \\ 0.1 \end{bmatrix} \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \\ 0.2 & 0.2 \end{bmatrix}$$

$$= \max_{\text{row}} \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \\ 0.1 & 0.1 \end{bmatrix} = \begin{bmatrix} 0.5 & 0.5 \end{bmatrix},$$

$$C' = 0.5/c_1 + 0.5/c_2,$$

which is different from the previous inference, as a result of rule interaction.

2.3. Single-condition decision making through a coupled rule base

In making single-condition inferences using the coupled rule base (Relation (6)), it is necessary to project Expression (7) onto the subspace of single degree-of-freedom decision making. Without loss of generality, consider the context variable, y_1, and the inference variable, u_1. We have:

$$\mathrm{Proj}\mu_R\left[\left(\underline{y}, \underline{u}\right)\right](y_1, u_1) = \sup_{y_2,\ldots,y_n,u_2,\ldots,u_p} \max_i \min [\mu_{Y_1^i}(y_1), \mu_{Y_2^i}(y_2), \ldots, \mu_{Y_n^i}(y_n), \mu_{U_1^i}(u_1), \mu_{U_2^i}(u_2), \ldots, \mu_{U_p^i}(u_p)]. \tag{8}$$

A sufficient condition for the rule-base subspace, given by Eq. (8), to be equivalent to a single rule relating y_1 and u_1, is given by the theorem below.

Theorem 1. *For Rule i in rule base R, which satisfies Eq. (7), define:*

$$\alpha_i = \min[hgt\,\mu_{Y_2^i}(y_2), \ldots, hgt\ \mu_{Y_n^i}(y_n), hgt\ \mu_{U_2^i}(u_2), \ldots, hgt\ \mu_{U_p^i}(u_p)]. \tag{9}$$

The fuzzy relation of the variable pair (y_1, u_1) is uncoupled in the rule base R, if:

$$\alpha_i \geq \min[hgt\ Y_1^i, hgt\ U_1^i], \quad \text{for } \forall i \in M \tag{10}$$

where Y_1^i and U_1^i are the fuzzy states of Y_1 and U_1, respectively, in the ith rule of R, and $M = \{1, 2, \ldots, m\}$ is the set of rule indices in R.

Proof. Directly follows from Properties 2 and 3.
The sufficient condition in Theorem 1 may be relaxed further. To this end, several further definitions are given next. Without loss of generality, consider a condition fuzzy variable, Y_1, an inference fuzzy variable, U_1, and a set of rule indices denoted by $M = \{1, 2, \ldots, m\}$ for a rule base, R, given by Eq. (7), which relates various fuzzy states of Y_1, U_1, and other variables. □

Definition 3. The isolated joint membership function of Y_1 and U_1 in the ith rule is given by:

$$\mu_i(y_1, u_1) = \min[\mu_{Y_1^i}(y_1), \mu_{U_1^i}(u_1)]. \tag{11}$$

Definition 4. The α_i cut of $\mu_i(y_1, u_1)$ is the crisp membership function:

$$\begin{aligned} \mu_{\alpha i}(y_1, u_1) &= 1 \quad \text{for } \mu_i(y_1, u_1) \geq \alpha_i \\ &= 0 \quad \text{elsewhere.} \end{aligned} \tag{12}$$

Definition 5. The coupling function for the condition-inference pair (Y_1, U_1) in the ith rule is given by:

$$\beta_i(y_1, u_1) = \min[\mu_i(y_1, u_1), \mu_{\alpha_i}(y_1, u_1)], \tag{13}$$

in which threshold α_i is given by the least height of the fuzzy variables in the ith rule, excluding Y_1 and U_1, as expressed in Eq. (9).

Definition 6. The coupling subset of rules $M_c \subseteq M$ for the pair (Y_1, U_1) is given by:

$$M_c = \{i : \sup_{y_1,u_1} \beta_i(y_1, u_1) > 0\}. \tag{14}$$

Corollary 1. *Under the conditions of Theorem 1, the coupling function, $\beta_i(y_1, u_1) = 0$ for $\forall i \in M$.*

Theorem 2. *The fuzzy relation of (Y_1, U_1) is uncoupled in the rule base R if and only if for $\forall i \in M_c$, there exist some $\ell \in M$ such that:*

$$\mu_\ell(y_1, u_1) \geq \beta_i(y_1, u_1) \quad \text{with } \ell \neq i. \tag{15}$$

Proof. By construction. If part: Assume that the "$\geq$" condition holds and show that the rule base is uncoupled, as in Theorem 1. Only if part: Assume that the "$\geq$" condition does not hold and show that at least one rule is coupled.
In the analytical developments presented in this section, just one inference variable may be assumed (i.e. $p = 1$) without loss of generality. Furthermore, any T-norm may be used in place of the min operation and any S-norm [6] may be used in place of the max operation. □

3. Illustrative example

In this section, an example is given to illustrate the main analytical concepts presented in the paper. Three other examples were given previously to illustrate the underlying fundamentals.

Consider a fuzzy rule base R with its ith rule R_i containing the two contexts, A_i and B_i, and one action, C_i, as given by:

$$R_i : (A_i, B_i) \to C_i, \qquad R = \cup_i R_i. \tag{16}$$

Suppose that the membership functions of the pertinent fuzzy states take a Gaussian shape expressed as:

$$\mu_{A_i}(x) = b_i \exp -(x - a_i)^2, \tag{17}$$

$$\mu_{B_i}(y) = q_i \exp -(y - p_i)^2, \tag{18}$$

$$\mu_{C_i}(c) = s_i \exp -(c - r_i)^2. \tag{19}$$

Then the membership function of the overall rule base, R, is:

$$\begin{aligned}\mu_R(x, y, c) = \max_i \min[&b_i \exp -(x - a_i)^2,\\ &q_i \exp -(y - p_i)^2,\\ &s_i \exp -(c - r_i)^2].\end{aligned} \tag{20}$$

For making single-condition-single-action decisions, $A \to C$, using Definition 1 and Eq. (8), we have the projection:

$$\mathrm{Proj}[\mu_R(x, y, c)](x, c) = \sup_y \max_i \min[b_i \exp -(x - a_i)^2,$$

$$q_i \exp -(y - p_i)^2, s_i \exp -(c - r_i)^2]. \tag{21}$$

This, in view of Property 1 and the fact that:

$$\sup_y q_i \exp -(y - p_i)^2 = q_i,$$

gives:

$$\mathrm{Proj}[\mu_R(x, y, c)](x, c)$$

$$= \max_i \min \quad [b_i \exp -(x - a_i)^2, q_i, s_i \exp -(c - r_i)^2]. \tag{22}$$

Now:

$$\mathrm{hgt}\ \mu_{A_i}(x) = \mathrm{hgt}\ b_i \exp -(x - a_i)^2 = b_i,$$

$$\mathrm{hgt}\ \mu_{C_i}(c) = \mathrm{hgt}\ s_i \exp -(c - r_i)^2 = s_i.$$

Also, in using Theorem 1, the parameter defined by Eq. (9) is:

$$\alpha_i = q_i.$$

Hence, the sufficient condition in Theorem 1, as given by Relation (10) is:

$$q_i \geq \min[b_i, s_i], \quad \text{for } \forall i \in M. \tag{23}$$

This condition assures the complete validity of the uncoupled decision making.

4. Concluding remarks

Fuzzy logic provides an approximate yet practical means of representing knowledge regarding a system (e.g. describing the behavior of the system) that is too complex or ill-defined, and not easy to tackle using precise mathematical tools. The approach also provides a means of making inferences using that knowledge, which can be used in making correct decisions regarding the system and for carrying out appropriate actions. In particular, human-originated knowledge can be effectively handled using fuzzy logic. As the complexity of a system increases, the ability to develop precise analytical models of the system diminishes until a threshold is reached, beyond which analytical modeling becomes intractable. Under such circumstances, precise model-based decision making is not practical. Fuzzy knowledge-based decision making is particularly suitable then.

The early work of fuzzy sets and fuzzy logic was pioneered by Zadeh in the 1960s. Subsequent developments and industrial applications in Europe and other regions in the 1970s, and widespread commercial application in consumer appliances, ground transportation, and various industrial products, primarily in Japan in the 1980s have established this field with the respect it rightfully deserves. The October 19, 1987 issue of *Nikkei Industrial News* wrote: "Toshiba has developed an AI system which controls machinery and tools using Fuzzy Logic. It has control rules, simulation and valuation. Toshiba will add an Expert System function to it and accomplish synthetic AI. Toshiba is going to turn it into practical uses in the field of industrial products, traffic control and nuclear energy." This news item is somewhat ironic and significant because around the same time at the Information Engineering Division of Cambridge University, a similar application of fuzzy logic was developed [3,4]. This work was subsequently extended at the Industrial Automation Laboratory of the University of British Columbia, Canada, where the applications were centered on the fish processing industry [8]. Many engineers, scientists, researchers, and other professionals throughout the world have made significant contributions to bringing the field of fuzzy logic into maturity, both in research and practical applications. These contributions are too numerous to mention here.

Fuzzy-rule-based decision making that incorporates a single condition (context) and a single action (inference), has significant computational advantages and simplicity in comparison to that incorporating coupled rules having many condition variables. As a contribution inspired by the Zadeh–MacFarlane–Jamshidi trio, this paper presented a theoretical basis for applying single-context decision making to a problem governed by a knowledge base of coupled fuzzy rules. To this end, two theorems which provide necessary and sufficient conditions were established, together with underlying analytical details. Several examples were given to illustrate various concepts presented in the paper.

Acknowledgments

This work has been supported through funding from the Natural Sciences and Engineering Research Council (NSERC) of Canada, Tier 1 Canada Research Chair, Canada Foundation for Innovation, and British Columbia Knowledge Development Fund. The support and encouragement provided by Professor Mo Jamshidi towards the author's professional development are gratefully appreciated.

References

[1] Zadeh, L.A. "Outline of a new approach to the analysis of complex systems and decision processes", *IEEE Trans. Systems, Man, and Cybernetics*, 3(1), pp. 28–44 (1973).
[2] Dubois, D. and Prade, H., *Fuzzy Sets and Systems*, Academic Press, Orlando, FL (1980).
[3] De Silva, C.W. and MacFarlane, A.G.J., *Knowledge-Based Control with Application to Robots*, Springer-Verlag, Berlin, Germany (1989).
[4] De Silva, C.W. and MacFarlane, A.G.J. "Knowledge-based control approach for robotic manipulators", *International Journal of Control*, 50(1), pp. 249–273 (1989).
[5] De Silva, C.W. "An analytical framework for knowledge-based tuning of servo controller", *Engineering Applications of Artificial Intelligence*, 4(3), pp. 177–189 (1991).

[6] De Silva, C.W. "Research laboratory for fish processing automation", In *Proc. 3rd Int. Symposium on Robotics and Manufacturing*, M. Jamshidi, Ed., In *Robotics and Manufacturing*, vol. 3, pp. 935–940, ASME Press, Vancouver, B.C (1990).
[7] Karray, F. and de Silva, C.W., *Soft Computing and Intelligent Systems Design*, Addison-Wesley, New York, NY (2004).
[8] De Silva, C.W., *Mechatronics-A Foundation Course*, CRC Press/Taylor & Francis, Boca Raton, FL (2010).

Clarence W. de Silva is Professor of Mechanical Engineering and occupies the Tier 1 Canada Research Chair Professorship in Mechatronics & Industrial Automation at the University of British Columbia, Vancouver, Canada. A Professional Engineer (P. Eng.), he is also a Fellow of ASME, IEEE, the Canadian Academy of Engineering and the Royal Society of Canada. He is a Senior Fulbright Fellow, Lilly Fellow, ASI Fellow, Killam Fellow, and Erskine Fellow. He has authored 21 books and over 400 papers, half of which are in journals. He has received many awards including the Paynter Outstanding Investigator Award and the Takahashi Education Award of the ASME Dynamic Systems and Control Division, the Killam Research Prize and the Outstanding Engineering Educator Award of IEEE Canada. He has served as Editor/Associate Editor of 14 journals including ASME and IEEE Transactions, and as Editor-in-Chief of the International Journal of Control and Intelligent Systems. He has received Ph.D. degrees from Massachusetts Institute of Technology, USA (1978), and Cambridge University, UK (1998), and an Honorary DEng from Waterloo University, Canada (2008).