ELSEVIER

Applied Mathematical Modelling 22 (1998) 569–581

APPLIED MATHEMATICAL MODELLING

Contact characteristics of viscoelastic bonded layers

G.R. Naghieh, Z.M. Jin, H. Rahnejat *

Department of Mechanical and Manufacturing Engineering, University of Bradford, Bradford, West Yorkshire BD7 1DP, UK

Received 27 June 1997; received in revised form 14 April 1998; accepted 2 June 1998

Abstract

A viscoelastic layered contact model has successfully been developed and solved analytically. The single layered linear viscoelastic material is assumed to be perfectly bonded to a rigid substrate in contact with a rigid indenter without friction under a step load. Two cases are considered: (a) a compressible layered material with a typical Poisson's ratio of 0.4 and (b) an incompressible layer with a Poisson's ratio of 0.5. Two viscoelastic models: Maxwell and three element standard linear solid are investigated. This paper highlights the methodology employed and the results obtained under various conditions.

Nomenclature

a	contact radius for viscoelastic material
a_0	contact radius for elastic material
d	layer thickness
E_0, E_∞	instantaneous and equilibrium elastic modulii
$G(t)$	relaxation function in shear
$H(t)$	unit step function
$k(t)$	relaxation function in dilatation
p	pressure distribution
p_0	maximum elastic pressure
R	radius of curvature
$R_p(t)$	kernel for pressure
t	time
u, w	displacements in x and z directions
ε_r, ε_θ, ε_z	normal strains in spherical coordinates
η	dynamic viscosity
ξ	substituted variable
λ, μ	Lame's constants, $(\lambda = Ev/(1+v)(1-2v), \mu = E/2(1+v))$
v	Poisson's ratio
τ_2	relaxation time

* Corresponding author. Tel.: +44 1274 384 547; fax: +44 1274 384 525; e-mail: h.rahnejat@bradford.ac.uk.

ISSN/98/$ – see front matter
PII: S 0 3 0 7 - 9 0 4 X (9 8) 1 0 0 5 2 - 5

1. Introduction

Tribological coatings and layered solids are increasingly employed in many engineering disciplines, such as in automotive, aerospace, medical and robotics fields. Various applications include overlay of soft lubricants to hard surfaces, sealing rubbers and viscoelastic load bearing members for instance in the motor vehicle industry. Rubber-lined bearings are used in automotive and other applications. Polymeric layers are also increasingly utilised in robotic and prosthetic applications as soft fingertip tactile devices or as friction enhancers in assortment of grippers. The dissimilar elastic properties of these layers to those of the substrate are of particular interest.

Many approaches are proposed in the literature to analyse the contact interaction between the soft layer and the indenting object surface. The indentation characteristics of soft layers are of particular interest in soft finger prehensile contacts with normal loads in the region 0.25–3 N (this being the typical range of contact loads) [1]. A number of analytic and finite element indentation studies have been carried out, specifically for soft fingertip contacts [1,2]. In Ref. [2], the layered skin sensor (distributed strain guages mounted on thin plates and protected by silicone rubber), emulating the human skin have been treated as an elastic layer. In Ref. [1], a viscoelastic layer is considered as a semi-infinite solid, described by a one-dimensional Maxwell model. These and other studies [3,4] have employed the semi-infinite solid assumption, based upon the Hertzian contact conditions. However, there are two main limitations with this approach; the nature of the layered structure and its viscoelastic behaviour. The soft layer should be considered as a viscoelastic material owing to their time dependent relaxation characteristics, therefore the limitation in Ref. [2]. Furthermore, the effect of a finite layer thickness renders void the half-space assumption, therefore the limitations in Refs. [1,3,4]. This latter problem has been remedied in Refs. [5–7] who has reported analytic solutions for bonded and unbonded layered elastic solids.

Kalker [8,9], Braat and Kalker [10] and Goryacheva et al. [11] have provided analysis of viscoelastic layered coatings in contact with a semi-infinite plane under rolling conditions. They have shown the dependence of the contact characteristics on the relative thickness of the multi-layered structure and the mechanical properties of the surface layer. In all these references the viscoelastic layer is bonded to a semi-infinite solid for soft viscoelastic coated layers in bearing applications. These studies have employed the layered viscoelastic analytic formulation under rolling conditions, as experienced by bearings coated with soft viscoelastic layers. Some of these coatings, for spacecraft use in fact behave as elastoplastic layers. This problem has been tackled recently by Naghieh et al. [12].

The semi-infinite assumption in the case of skin emulation does not hold, irrespective of good agreement apparently obtained with a Maxwell model with the experimental indentation results, since the ratio of contact radius to layer thickness is greater than 2 for typical finger prehension loads (this condition renders the semi-infinite assumption void as highlighted by Jafar [5]). Therefore, there is a need for a layered as opposed to semi-infinite solution of bonded viscoelastic layers, not only for soft fingertip emulation but for the generic case of thin layers particularly for compressible materials. Some initial solutions for the case of three element standard linear solid have been presented by Naghieh et al. [13]. More comprehensive solutions, including for Maxwell model are presented in this paper.

2. Mathematical formulation

The viscoelastic material properties are characterised by creep or relaxation functions, thus taking into account the time-dependent behaviour of the material. The approach is applicable

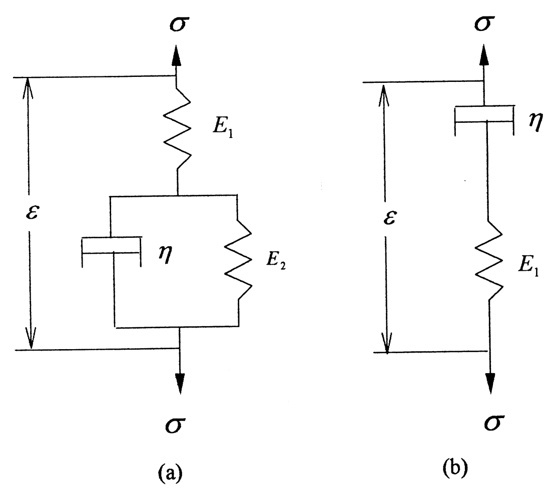


Fig. 1. (a) The standard linear solid, three elements model. (b) The Maxwell model.

to the situations, where the strain remains small. A standard linear solid and a Maxwell model are employed to describe the viscoelastic behaviour of the material in this study. Such models are an arrangement of springs and dashpots in parallel or in series, as shown in Fig. 1.

Boltzmann's superposition principle is used in order to sum up the stresses and the strains in a historical manner, thus simulating creep, relaxation, recovery, and other dynamic characteristics of the viscoelastic material. Linear viscoelastic behaviour may be specified in general by either a creep or a relaxation model according to Leaderman [14]. In the present analysis, a relaxation model is considered, where:

$$\sigma(t) = \int_0^t G(t-\tau)\frac{\mathrm{d}\varepsilon(\tau)}{\mathrm{d}\tau}\,\mathrm{d}\tau, \tag{1}$$

where $\sigma(t)$ is stress at time t and $G(t)$ the time dependent relaxation function, having the dimensions of the elastic modulus. This function represents the viscoelastic properties of the material and specifies the stress response to a unit change of strain $\mathrm{d}\varepsilon$ at time τ. Relaxation functions may be either obtained experimentally or deduced from appropriate spring-dashpot models as proposed by Lee et al. [15].

In this study a viscoelastic layered solid based on either a three element standard linear solid or a Maxwell model is employed. The viscoelastic layer is assumed to be firmly bonded to a rigid substrate and forms a contact with a rigid indenter under a normal step load. Friction between the viscoelastic layer and the indenter is assumed to be negligible and the layer material to be perfectly isotropic and homogeneous. Furthermore, the contact is assumed to be axisymmetric as shown in Fig. 2.

The logical approach to the solution of the viscoelastic contact problem is to replace the elastic constant in the corresponding elastic solution with the relaxation function in Eq. (1). Such an approach has been highlighted by Radok [16]. Lee et al. [15] have shown that this approach can be applied to the contact problems provided that the applied loading induces an increasing contact area.

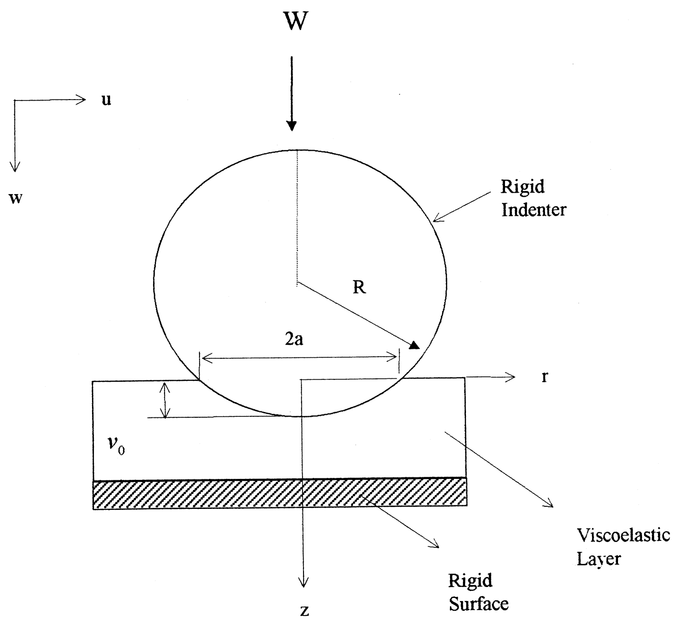


Fig. 2. Contact geometry for layered solids.

2.1. Elastic solutions for layered contacts

According to Johnson [17] for a thin bonded elastic layered contact where the contact radius is in general much larger than the layer thickness, the radial and the circumferential strains $(\varepsilon_r, \varepsilon_\theta)$ are negligible. Thus, the stress component in the z direction (see Fig. 2) reduces to:

$$\sigma_z = (\lambda + 2\mu)\varepsilon_z, \tag{2}$$

where the simplified strain-displacement relations in the directions of r, θ and z can be obtained from the general expressions given in Refs. [16,17], as follows:

$$\varepsilon_r = \frac{\partial u}{\partial r}, \qquad \varepsilon_\theta = \frac{u}{r}, \qquad \varepsilon_z = \frac{\partial w}{\partial z} \tag{3}$$

and the Lame's constants (μ and λ) are defined as:

$$\mu = \frac{E}{2(1+v)} \quad \text{and} \quad \lambda = \frac{Ev}{(1+v)(1-2v)}. \tag{4}$$

In contact problems, the following equations for contact pressure ($p(r)$) and vertical displacement ($w(r)$) hold true:

$$p(r) = 0, \quad r > a_0, \tag{5}$$

$$w(r) = v_0 - \frac{r^2}{2R}, \quad r \leqslant a_0, \tag{6}$$

where a_0 is the contact radius, R the radius of the indenter and v_0 the maximum displacement at $r = 0$.

Furthermore, the compressive strain in the element for the compressible layer, that is Poisson's ratio is not very close to 0.5, is given by the geometry of deformation:

$$\frac{\partial w}{\partial z} = -\frac{1}{d}\left(v_0 - \frac{r^2}{2R}\right), \tag{7}$$

where d is the layer thickness.

Combining Eqs. (2)–(4) and (7) and noting that the contact pressure, $p(r) = -\sigma_z$, the solution can be obtained for a compressible material with a typical value of Poisson's ratio (v) not close to 0.5 as [3]:

$$k_0 p(r) = \frac{a_0^2(1-v)^2}{2Rd(1-2v)}\left[1 - \left(\frac{r}{a_0}\right)^2\right], \tag{8}$$

where $k_0 = (1 - v^2)/E_0$ and the contact radius can be obtained by considering the load (W) balance equation

$$a_0 = \left[\frac{(1-2v)4RdWk_0}{(1-v)^2\pi}\right]^{1/4}. \tag{9}$$

The corresponding maximum indentation can be obtained from the boundary condition imposed by Eq. (5):

$$v_0 = \left(\frac{(1-2v)dWk_0}{(1-v)^2\pi R}\right)^{1/2}. \tag{10}$$

For an incompressible layered material with a Poisson's ratio of 0.5, the pressure distribution has been obtained in Ref. [5] as:

$$\frac{1}{4E_0}p(r) = \frac{a_0^4}{128Rd^3}\left[1 - \left(\frac{r}{a_0}\right)^2\right]^2, \tag{11}$$

the contact radius

$$a_0 = \left(\frac{96d^3RW}{\pi E_0}\right)^{1/6} \tag{12}$$

and the maximum indentation as:

$$v_0 = \left(\frac{3d^3W}{2\pi R^2 E_0}\right)^{1/3}. \tag{13}$$

2.2. Viscoelastic solutions for layered contacts

2.2.1. Compressible three element, standard linear solid material

For the viscoelastic material, the contact radius ($a(t)$) and the pressure distribution ($p(r, t)$) are time dependent. Therefore, it is feasible to follow the approach highlighted in Ref. [16], incorporating time in Eqs. (8)–(10). The contact radius can be obtained

$$a(t) = \left[\frac{(1-2v)4RdWk(t)}{(1-v)^2\pi}\right]^{1/4} H(t), \tag{14}$$

where $H(t)$ is the step function and $k(t)$ is a function of time and can be shown to be [4]

$$k(t) = k_0\left[e^{-t/(2\tau_2)} + \frac{E_0}{E_\infty}\left(1 - e^{-t/(2\tau_2)}\right)\right], \tag{15}$$

where τ_2 is the relaxation time, (η/E_0), E_0 is the instantaneous elastic modulus and E_∞ is the asymptotic elastic modulus.

For simplicity in the case of three element standard linear solid in Fig. 1(a), let $E_1 = E_2 = E_0$, in this study and thus, $E_\infty = \frac{1}{2}E_0$, and

$$k(t) = k_0\left(2 - \mathrm{e}^{-t/(2\tau_2)}\right). \tag{16}$$

Eq. (14) can also be simplified as:

$$a(t) = a_0\left(2 - \mathrm{e}^{-t/(2\tau_2)}\right)^{1/4} H(t). \tag{17}$$

The corresponding maximum indentation is

$$v_0(t) = v_0\left(2 - \mathrm{e}^{-t/(2\tau_2)}\right)^{1/2} H(t) \tag{18}$$

and corresponding pressure distribution for the compressible viscoelastic material can be written as:

$$p(r,t) = \frac{(1-v)1}{(1-2v)2Rd}\int_0^t R_p(t-\tau)\frac{\partial}{\partial\tau}g(r,t)\,\mathrm{d}\tau, \tag{19}$$

where $g(r, t) = (a^2 - r^2)\,H(t)$ for the present compressible layer model and $R_p(t)$ can be found from Ref. [5] as:

$$R_p(t) = \frac{E_0}{2(1-v^2)}\left(1 + \mathrm{e}^{-t/\tau_2}\right) \tag{20}$$

and

$$p(r,t) = \frac{(1-v)E_0}{(1+v)(1-2v)4Rd}\int_0^t \left(1 + \mathrm{e}^{-(t-\tau)/\tau_2}\right)\frac{\partial}{\partial\tau}\left[(a^2 - r^2)H(\tau)\right]\mathrm{d}\tau. \tag{21}$$

Finally, viscoelastic contact pressure formula for three element layered compressible material of Poisson's ratio not close to 0.5 will be in the following form by further simplifying Eq. (21) as:

$$p(r,t) = \frac{p_0}{2}\left\{\mathrm{e}^{-t/\tau_2}\left[1 - \left(\frac{r}{a_0}\right)^2\right] + \left[\left(2 - \mathrm{e}^{-t/(2\tau_2)}\right)^{1/2} - \left(\frac{r}{a_0}\right)^2\right] + \mathrm{e}^{-t/\tau_2}\int_{f(0)}^{f(t)}\left[2 - \left(\xi + \left(\frac{r}{a_0}\right)^2\right)^2\right]^{-2}\mathrm{d}\xi\right\}, \tag{22}$$

where the corresponding maximum compressible elastic contact pressure,

$$p_0 = \frac{(1-v)E_0 a_0^2}{(1+v)(1-2v)2Rd} \quad \text{and} \quad f(t) = \left(2 - \mathrm{e}^{-t/(2\tau_2)}\right)^{1/2} - \left(\frac{r}{a_0}\right)^2.$$

2.2.2. Compressible Maxwell model

The pressure distribution and the contact radius equations are performed in a similar method as in the case of the compressible three-element model (standard linear solid material). The corresponding creep function for Maxwell material takes the form of:

$$k(t) = k_0\left(1 + \frac{t}{\tau_2}\right)H(t) \tag{23}$$

and the viscoelastic contact radius:

$$a(t) = a_0\left(1 + \frac{t}{\tau_2}\right)^{1/4} H(t). \tag{24}$$

The indentation of the Maxwell material caused by the weight of sphere of the rigid indenter:

$$v_0(t) = v_0\left(1 + \frac{t}{\tau_2}\right)^{1/2} H(t). \tag{25}$$

Following the same procedure as the three element model and noting,

$$R_p(t) = \frac{E_0}{(1 - v^2)} \mathrm{e}^{-t/\tau_2}. \tag{26}$$

The viscoelastic contact pressure formula for Maxwell layered compressible material of Poisson's ratio not close to 0.5:

$$p(r,t) = p_0 \mathrm{e}^{-t/\tau_2}\left[\left[1 - \left(\frac{r}{a_0}\right)^2\right] + \int_{f(0)}^{f(t)} \mathrm{e}^{\left(\xi + (r/a_0)^2\right)^2 - 1}\, \mathrm{d}\xi\right], \tag{27}$$

where $f(t) = (1 + t/\tau_2)^{1/2} - (r/a_0)^2$.

2.2.3. Incompressible three element, standard linear solid material

For a viscoelastic layer with a Poisson's ratio of 0.5, the pressure distribution and the contact radius can be derived in a similar manner as in the case of the compressible material. It is noted that the viscoelastic models will be the same and the main difference is in the elastic pressure distribution. Eqs. (17) and (18) can also be used for the incompressible viscoelastic layer provided that the corresponding incompressible elastic expressions are substituted.

The corresponding contact pressure can be derived from the following equation:

$$p(r,t) = \frac{E_0}{64 d^3 R} \int_0^t \left(1 + \mathrm{e}^{-(t-\tau)/\tau_2}\right) \frac{\partial}{\partial \tau}\left[(a^2 - r^2)^2 H(\tau)\right] \mathrm{d}\tau \tag{28}$$

or written as:

$$p(r,t) = \frac{p_0}{2}\left\{\mathrm{e}^{-t/\tau_2}\left(1 - \left(\frac{r}{a_0}\right)^2\right)^2 + \left[\left(2 - \mathrm{e}^{-t/(2\tau_2)}\right)^{1/3} - \left(\frac{r}{a_0}\right)^2\right]^2 + \mathrm{e}^{-t/\tau_2} \int_{f(0)}^{f(t)} \left[2 - \left(\xi^{1/2} + \left(\frac{r}{a_0}\right)^2\right)^3\right]^{-2} \mathrm{d}\xi\right\}, \tag{29}$$

where $f(t) = [\left(2 - \mathrm{e}^{-t/(2\tau_2)}\right)^{1/3} - (r/a_0)^2]^2$.

2.2.4. For incompressible Maxwell model

The contact pressure for the incompressible Maxwell model is

$$p(r,t) = p_0 e^{-t/\tau_2}\left[\left(1-\left(\frac{r}{a_0}\right)^2\right)^2 + \int_{f(0)}^{f(t)} e^{\left(\xi^{1/2}+(r/a_0)^2\right)^3-1}\,d\xi\right], \tag{30}$$

where $f(t) = [(1+t/\tau_2)^{1/3} - (r/a_0)^2]^2$.

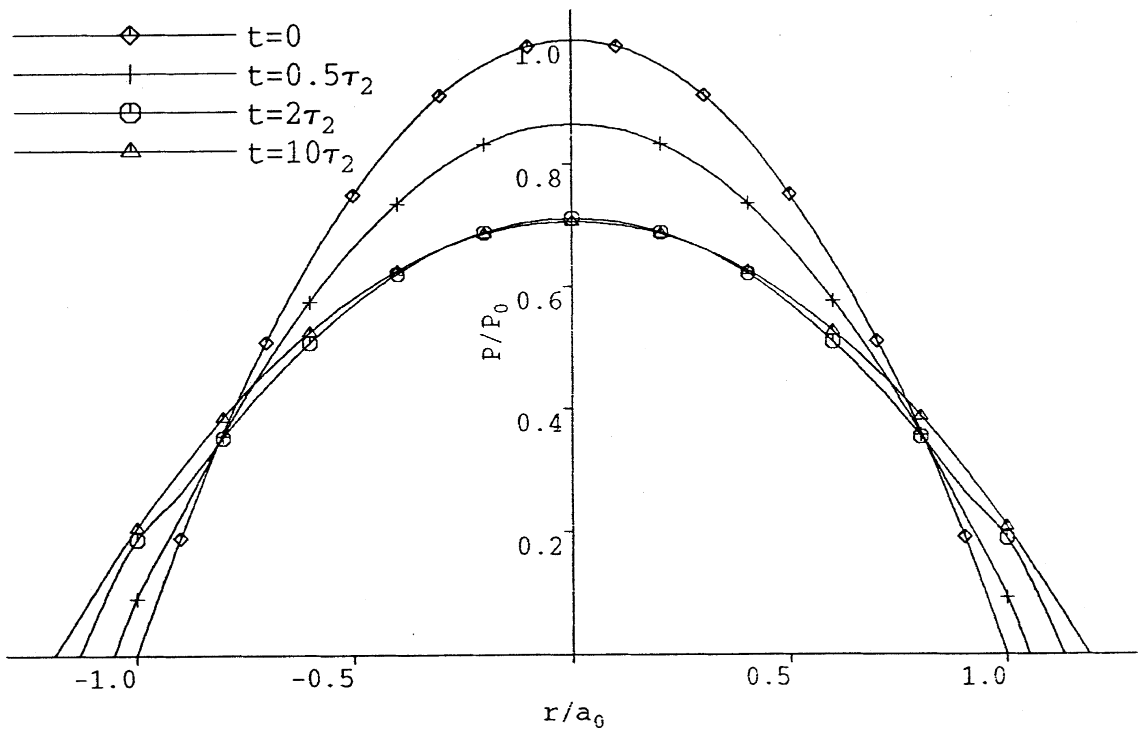
3. Results and discussion

The pressure distribution is normalised with respect to its maximum instantaneous initial value p_0 at $t = 0$ (elastic) in Fig. 3(a) and (b) for a compressible material with a typical Poisson's ratio of 0.4, and, Fig. 4(a) and (b) for an incompressible material with a Poisson's ratio of 0.5, for both standard linear solid and Maxwell models, respectively. It is interesting to note that the instantaneous pressure distributions conform to their corresponding elastic solutions given by Eqs. (8) and (11). This fact corroborates the numerical scheme adopted in this study. As time progresses, the maximum pressure falls due to viscoelastic relaxation of the layered material. There is a corresponding increase in the contact footprint radius. This effect is more pronounced for the case of compressible bonded layers as expected. For example, in the case of the compressible material, the maximum pressure is decreased by about 30% during a period equivalent to 10 times the duration of the material's relaxation whilst for an incompressible layer the corresponding decrease is about 23%. It is also interesting to note the difference in the pressure distribution profiles between the compressible and the incompressible materials. This can be explained from Eqs. (8) and (11), that in the case of the incompressible material, the pressure gradient diminishes towards the contact extremities.

It should also be noted that the present analysis for the incompressible layered model can be applied for most elastomeric materials such as rubber with a Poisson's ratio close to 0.5, and the compressible layered model for polymers with a Poisson's ratio in the range between 0.4 and 0.45. The Maxwell model for the incompressible case conforms more closely with the behaviour of rubber. This has also been noted in Ref. [8]. For the case of carbon polymers experimental validation of the results is now being carried out and the preliminary findings indicate better agreement with the compressible three element standard linear solid model.

Another interesting observation relating to the Maxwell model of Figs. 3(b) and 4(b) is that as the material creeps the pressure distribution at the contact extremities alters significantly as the growth of the contact area pulls in new material into the deformation region. This new material responds elastically and follows the elastic pressure profile. Johnson [17] has also observed the same behaviour for semi-infinite solids. Furthermore, the Maxwell model exhibits a shorter relaxation time than that of the standard three element solid model, both starting with the same expected elastic pressure distribution.

It should be realised that the above results are presented in normalised coordinates in order to provide general solutions. In practice, the maximum pressure depends upon mainly material properties as shown in Figs. 5(a), 5(b), 6(a) and 6(b). For example, for a viscoelastic layer made of highly filled carbon polymer (HFCP) with an instantaneous modulus of 1 GPa, the maximum contact pressure after immediate contact is just above 25 MPa for both the three parameter model and the Maxwell model, and is decreased to about 18 MPa after 10 times relaxation time for the three parameter model and about 5 MPa after 2 times relaxation time for the Maxwell model. For a typical rubber with an instantaneous modulus of 2 MPa, the maximum contact pressure is predicted to be just below 0.8 MPa at time equal to zero and becomes 0.6 MPa, after 10 times relaxation time for the three parameter model and 0.18 MPa for the Maxwell model.



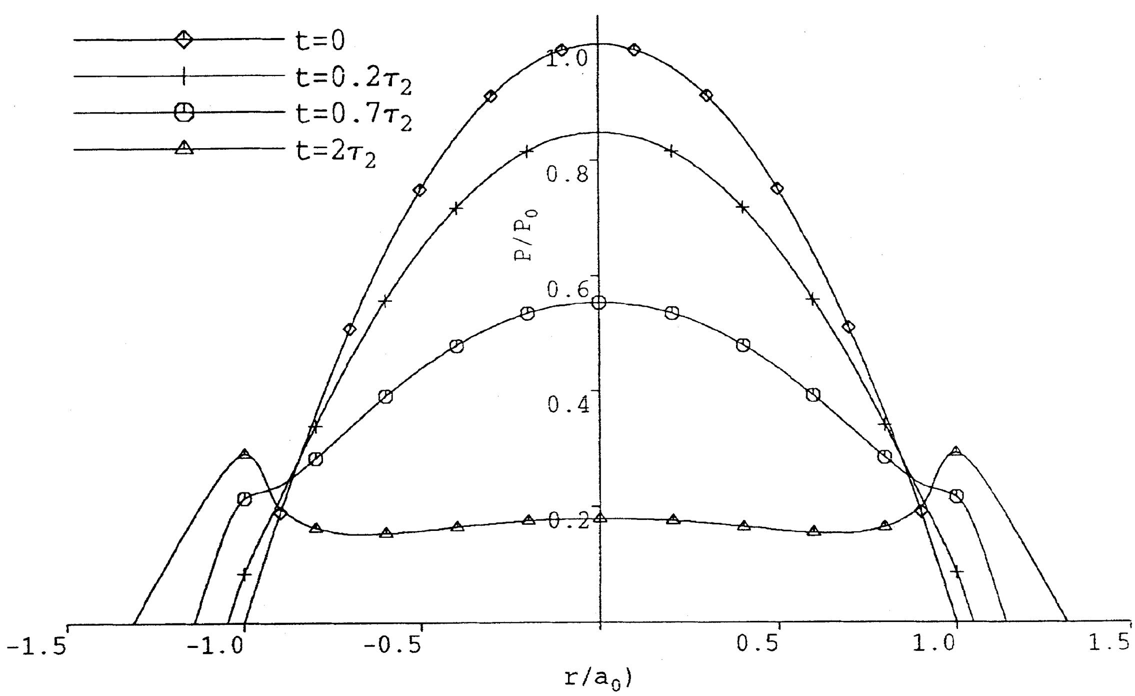
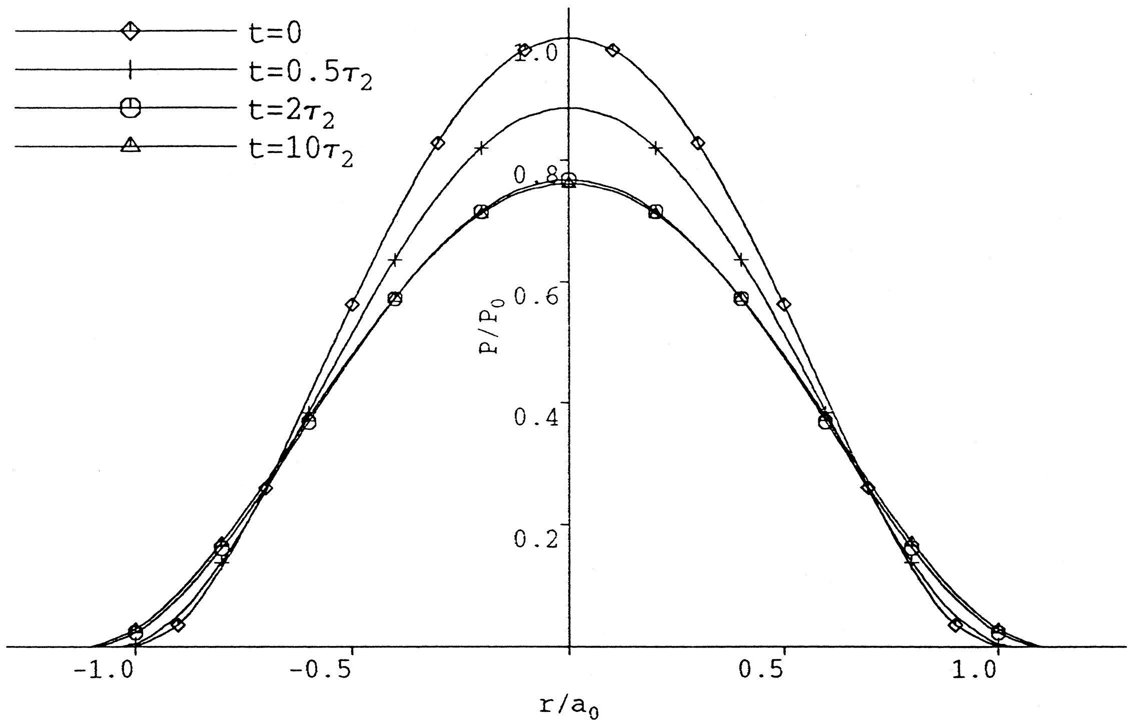


Fig. 3. (a) The transient pressure distribution by a rigid sphere indenting a three parameter compressible layer ($\upsilon = 0.4$). (b) The transient pressure distribution by a rigid sphere indenting a Maxwell compressible layer ($\upsilon = 0.4$).



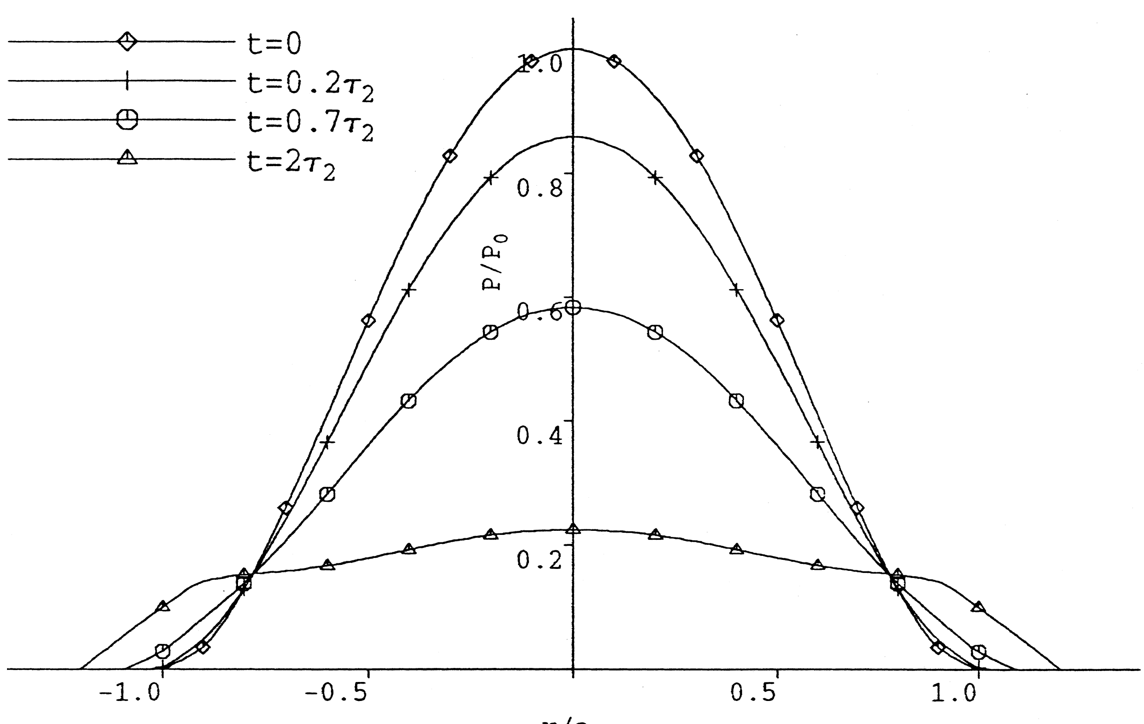


Fig. 4. (a) The transient pressure distribution by a rigid sphere indenting a three parameter compressible layer ($v = 0.5$). (b) The transient pressure distribution by a rigid sphere indenting a Maxwell compressible layer ($v = 0.5$).

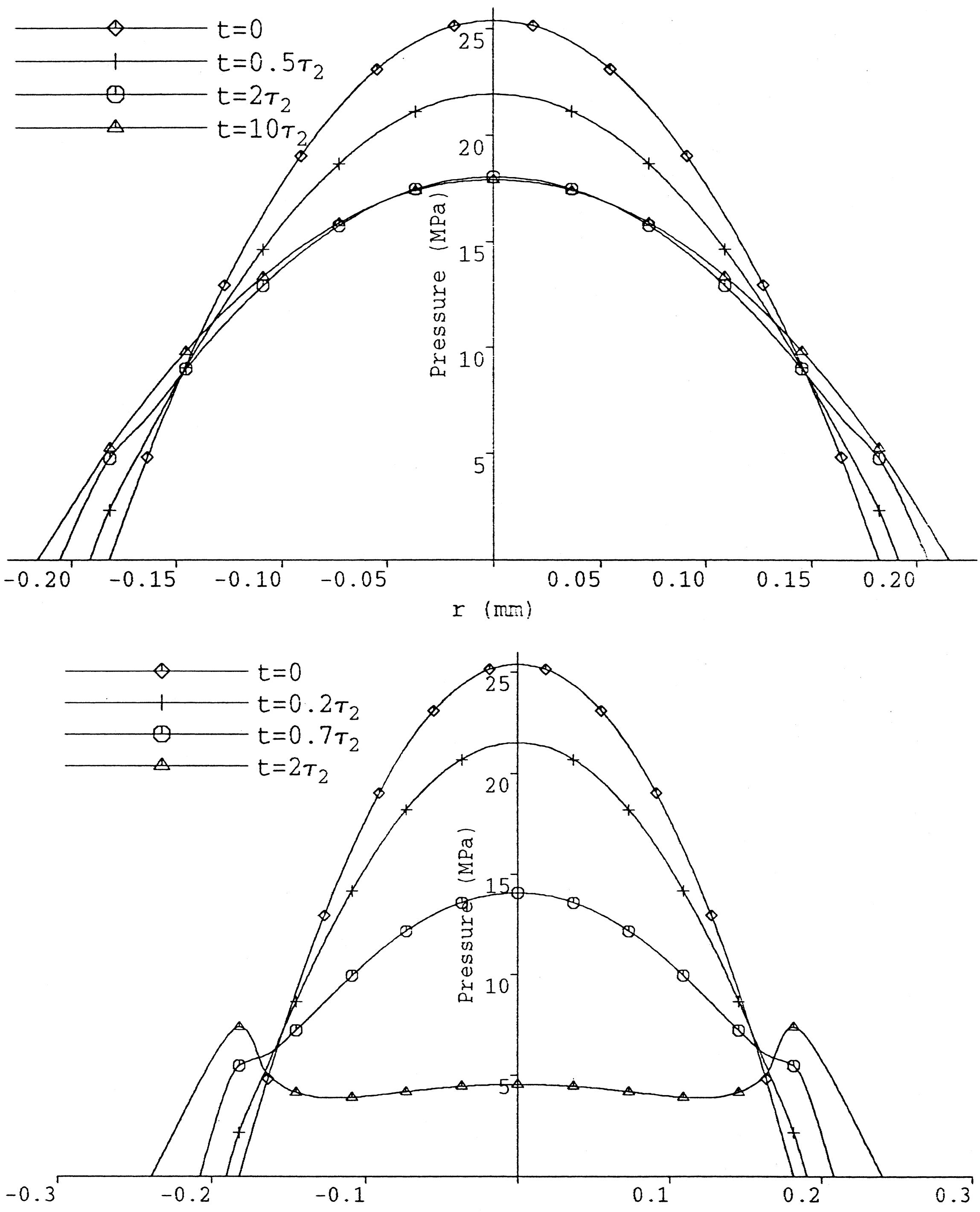


Fig. 5. (a) The transient pressure distribution by a rigid sphere indenting a three parameter compressible layer (HFCP) ($v = 0.4$, $d = 0.11$ mm, $W_s = 1.3$ N, $E_0 = 1$ GPa, $R = 12.7$ mm, $\tau_2 = 0.826$ s). (b) The transient pressure distribution by a rigid sphere indenting a Maxwell compressible layer (HFCP) ($v = 0.4$, $d = 0.11$ mm, $W_s = 1.3$ N, $E_0 = 1$ Gpa, $R = 12.7$ mm, $\tau_2 = 0.826$ s).

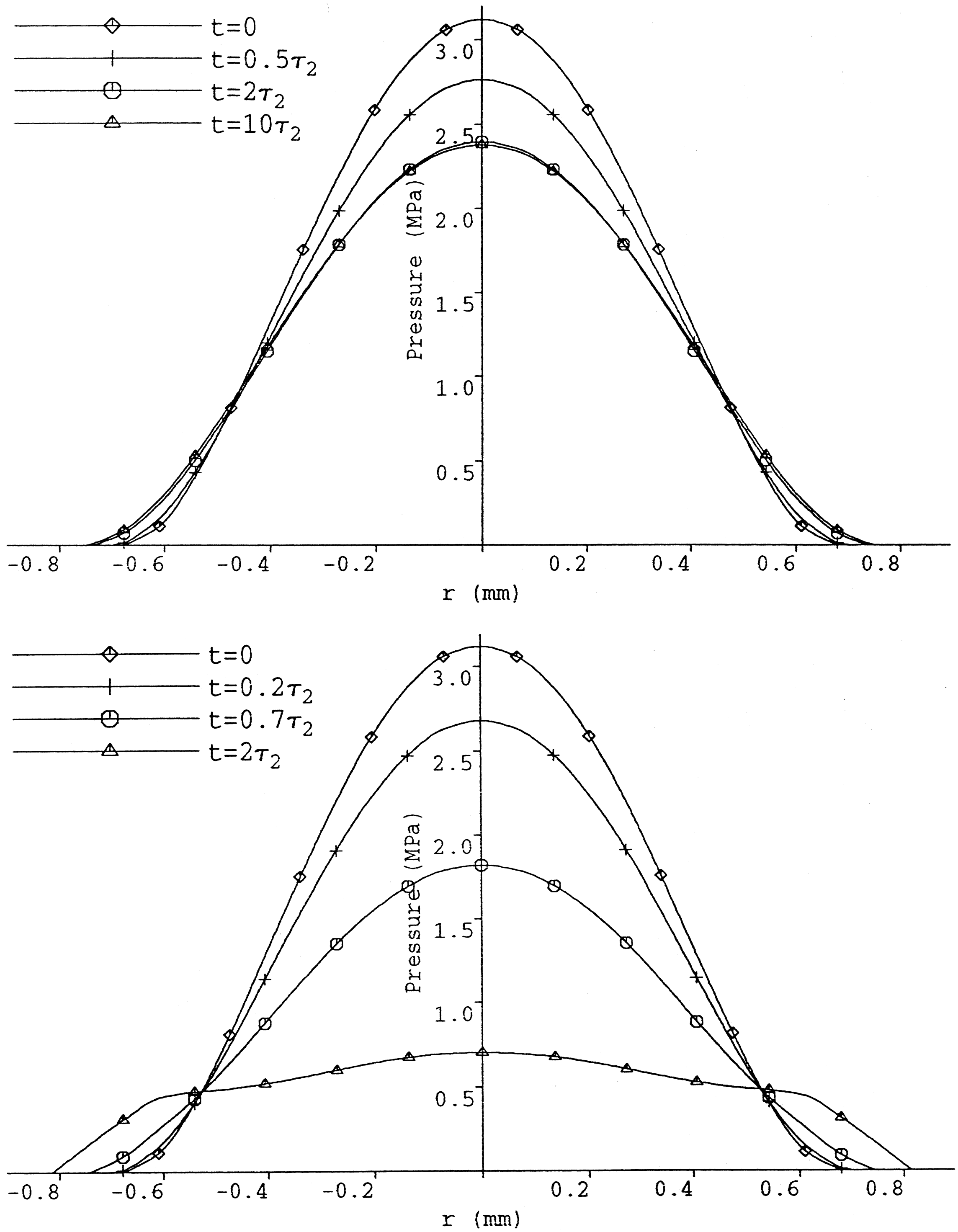


Fig. 6. (a) The transient pressure distribution by a rigid sphere indenting a three parameter compressible layer (Rubber) ($\upsilon = 0.5$, $d = 0.11$ mm, $W_s = 1.3$ N, $R = 12.7$ mm, $E_0 = 2$ MPa, $\tau_2 = 0.826$ s). (b) The transient pressure distribution by a rigid sphere indenting a Maxwell compressible layer (Rubber) ($\upsilon = 0.5$, $d = 0.11$ mm, $W_s = 1.3$ N, $E_0 = 2$ MPa, $R = 12.7$ mm, $\tau_2 = 0.826$ s).

4. Conclusion

The axi-symmetric contact of a perfectly isotropic, homogeneous, viscoelastic layer bonded to a rigid substrate of finite thickness, contacting a frictionless rigid indenter under a constant step load has been examined in this study. Non-dimensional general solutions of the contact radius and the contact pressure have been obtained for both the compressible and incompressible viscoelastic layered models based on either three parameter or Maxwell model. The main applications of the present results can be found in elastomeric and polymeric coatings used in various engineering disciplines. These analytic solutions can be used as reference solutions for validation of finite element models which can be extended to more complex geometries such as inclusion of soft coatings on bearing elements or rubber lined bearings, some solutions for these are provided in Ref. [12]. The solutions presented in this paper, however, are of practical use in many applications where quite simple contact geometries exist. These include precision prehensile grasp of regular objects such as cubes, balls and small cylinders (such as a pen). Furthermore, the finite thickness viscoelastic films in contact with rolling elements or "indenters" are of interest as in tape drives (roller against magnetic tape) or in floppy drives.

References

[1] D.T.V. Pawluk, A viscoelastic model of the human fingerpad and a holistic model of human touch, Ph.D. Thesis, Harvard University, USA, May 1997.
[2] T. Maeno, K. Kobayashi, N. Yamazaki, Sensing mechanisms of the partial incipient slip at the surface of the cylindrical fingers during the precision grip, Bioengineering Conf., ASME, BED-Vol. 35, 1997, pp. 117–118.
[3] E.H. Lee, J.R.M. Radok, The contact problems for viscoelastic bodies, J. Appl. Math. 27 (1960) 438–444.
[4] W.H. Yang, The contact problem for viscoelastic bodies, Trans. ASME, Ser. E, J. Appl. Math. 33 (1966) 395–401.
[5] M.J. Jaffar, Asymptotic behaviour of thin elastic layers bonded and unbounded to a rigid foundation, Int. J. Mech. Sci. 31 (1989) 229–235.
[6] M. Hannah, Contact stress and deformation in a thin elastic layer, Quart. Appl. Math. 4 (1951) 94–105.
[7] V.M. Aleksandrov, Asymptotic methods in contact problems of elasticity theory, P.M.M. 32 (1968) 691–703.
[8] J.J. Kalker, Elastic and viscoelastic analysis of two multiply-layered cylinder rolling over each other with Coulomb friction, in: Proceedings of the Leeds–Lyon Symposium on Tribology, 1989, pp. 27–34.
[9] J.J. Kalker, Viscoelastic multiply-layered cylinder rolling with dry friction, ASME J. Appl. Mech. 58 (1991) 666–679.
[10] G.F.M. Braat, J.J. Kalker, Theoretical and experimental analysis of the rolling contact between two cylinders coated with multi-layered viscoelastic rubber, in: M.H. Aliabadi, C.A. Brebbia (Eds.), Contact Mechanics'93, Computational Techniques, Computational Mechanics Publication, 1993, pp. 115–126.
[11] I. Goryacheava, F. Sadeghi, D.A. Nickel, Internal stresses in contact of a rough body and a viscoelastic layered semi-infinite plane, Trans. ASME, J. Tribol. 118 (1996) 131–136.
[12] G.R. Naghieh, Z.M. Jin, H. Rahnejat, Contact mechanics analysis for layered elastic, elasto-plastic and viscoelastic coatings, in: NordTrib'98, Ebeltoft, Denmark, June 1998.
[13] G.R. Naghieh, H. Rahnejat, Z.M. Jin, Contact mechanics of viscoelastic layered surface, in: M.H. Aliabadi, A. Samartin (Eds.), Contact Mechanics'97, Computational Mechanics Publication, 1997, pp. 59–68.
[14] H. Leaderman, Proposed nomenclature for linear viscoelastic behaviour, Trans. Soc. Rheol. I (1957) 213–222.
[15] E.H. Lee, T.G. Rogers, Solution of viscoelastic stress analysis problems using measured creep or relaxation functions, Trans. ASME, Ser. E, J. Appl. Mech. 30 (1963) 127–133.
[16] J.R.M. Radok, Viscoelastic stress analysis, Quart. Appl. Math. 15 (1957) 198–202.
[17] K.L. Johnson, Contact Mechanics, Cambridge University Press, Cambridge, 1985.