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The role of Glauber exchange in soft collinear effective theory and the Balitsky–Fadin–Kuraev–Lipatov Equation

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ABSTRACT

In soft collinear effective theory (SCET) the interaction between high energy quarks moving in opposite directions involving momentum transfer much smaller than the center-of-mass energy is described by the Glauber interaction operator which has two-dimensional Coulomb-like behavior. Here, we determine this n–$\bar{n}$ collinear Glauber interaction operator and consider its renormalization properties at one loop. At this order a rapidity divergence appears which gives rise to an infrared divergent (IR) rapidity anomalous dimension commonly called the gluon Regge trajectory. We then go on to consider the forward quark scattering cross section in SCET. The emission of real soft gluons from the Glauber interaction gives rise to the Lipatov vertex. Squaring and adding the real and virtual amplitudes results in a cancelation of IR divergences, however the rapidity divergence remains. We introduce a rapidity counter-term to cancel the rapidity divergence, and derive a rapidity renormalization group equation which is the Balitsky–Fadin–Kuraev–Lipatov Equation. This connects Glauber interactions with the emergence of Regge behavior in SCET.

Factorization of high-energy interactions in QCD is the systematic separation of different momentum regions into universal factors to all orders in the strong coupling constant α_s. All-order proofs of factorization, which were first carried out by Collins, Soper and Sterman [1–4] rely on a set of powerful theoretical tools. Among these are: power counting, pinch analysis via the Landau equations [5], and the Coleman–Norton Theorem [6]. The Landau equations allow for the isolation of pinch singularities which, via the Coleman–Norton Theorem can be identified with long-distance (infrared) physics. Generically pinch singularities can be identified with one of three momentum regions: collinear, soft, or Glauber. In the collinear region internal propagators become collinear with external particles, and in the soft region they become soft relative to external particles. In either of these limits particles can approximately stay on their mass shell. The Glauber region, however, is special as it corresponds to off-shell modes (Glauber modes) with $k_\perp \gg k^+, k^-$, which leads to a two-dimensional Coulomb-like interaction between and amongst collinear and soft particles [1,7]. The presence of Glauber interactions is problematic because they can destroy factorization [7,8]. Fortunately, it has been shown that for sufficiently inclusive quantities the sum over final-state cuts cancels unwanted pinches, and thereby eliminates Glauber contributions [9–11].

An alternative approach in deriving factorization is to use effective field theory (EFT). The EFT that describes the soft and collinear degrees of freedom which arise in factorization is soft collinear effective theory (SCET) [12–15], and in Ref. [16] it was shown how the perturbative factorization theorems of QCD are reproduced in SCET. However, SCET as it was originally formulated did not include Glauber type interactions. An attempt to include Glauber interactions between collinear quarks moving in opposite directions in SCET was made in Ref. [17] where factorization of the Drell–Yan cross section was reconsidered. Unfortunately, this attempt did not account for the overlap between different moment regions and failed as a result. The analysis was taken up in Ref. [18] where it was concluded that "for the exclusive Drell–Yan amplitude the correct effective theory would require Glauber modes." Though the authors did not consider under which circumstances the contribution from Glauber interactions cancels. In addition, a number of groups have considered the role of Glauber interactions between collinear and soft degrees of freedom in dense QCD matter [19–21]. More recently, an attempt to include a Glauber interaction between two collinear particles moving in opposite directions has been presented [22,23].

A second, seemingly unrelated issue concerning the formulation of SCET was raised in Refs. [24,25], where it was pointed out that Regge behavior appears to fall outside of the usual organizing scheme of SCET. Specifically, Regge behavior refers to the emergence of power-law behavior for scattering amplitudes.

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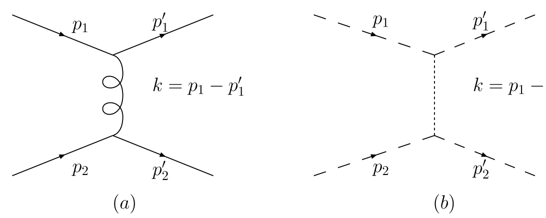

Fig. 1. Leading order contribution to forward quark–quark scattering at high energy: (a) QCD diagram, (b) SCET diagram (dashed lines indicate collinear quarks, and dotted lines Glauber gluons).

In perturbative QCD this arises out of a summation of ladder graphs which gives rise to the Balitsky–Fadin–Kuraev–Lipatov (BFKL) evolution equation [26,27] (see Ref. [28] for a very readable treatment). The solution of the leading logarithmic (LL) BFKL equation gives the total cross section for high energy scattering with just such a power law form. Clearly, as an EFT of QCD at high energy SCET needs to be able to reproduce the BFKL results.

In this work we will show that Glauber interactions between two collinear quarks moving in opposite directions and Regge behavior are intimately connected. One cannot have one without the other. We consider the simplest possible process: two high energy quarks undergoing forward scattering. This interaction is mediated by the exchange of a gluon with momentum that has Glauber scaling (in other words a Glauber gluon): $k_\perp^2 \gg k^+k^-$. We first determine the SCET operator responsible for the Glauber interaction, and then renormalize it. At one-loop a rapidity divergence appears which we treat in the formalism of Ref. [29,30]. The coefficient of the rapidity divergent term is called the gluon Regge trajectory which is infrared (IR) divergent. We then go on to consider the real emission of a soft gluon from the Glauber interaction and derive the Lipatov vertex. With these results in hand we calculate the total cross section for the forward scattering of high energy quarks. We find that at next-to-leading order in α_s this expression also has a rapidity divergence. Absorbing this rapidity divergence into a rapidity counter-term allows us to derive a rapidity RGE which is the famous BFKL equation. This then demonstrates the emergence of Regge behavior in SCET from Glauber interactions between collinear particles.

We use SCET to study the scattering of two high energy quarks moving in opposite directions $q(p_1) + q(p_2) \to q(p_1') + q(p_2')$ with large invariant mass $s = (p_1 + p_2)^2$ and small momentum transfer $t = (p_1 - p_1')^2 \ll s$. We also restrict ourselves to perturbative values of t, where $t \gg \Lambda \sim 1$ GeV. At leading order in the SCET power counting such an interaction can be described by the exchange of an off-shell gluon between the quarks, resulting in a two-dimensional Coulomb like potential in transverse momentum. To see how such an operator arises in SCET we start with QCD and match onto SCET degrees of freedom. The QCD diagram is given in Fig. 1(a). For the sake of matching we can take all the quarks to be massless and on-shell. In addition, the momentum $\vec{p}_1$ defines the z-axis. Then, the incoming momentum can be expressed in terms of two light-like vectors $n^\mu = (1, 0, 0, 1)$ and $\bar{n}^\mu = (1, 0, 0, -1)$:

$$p_1^\mu = \frac{\sqrt{s}}{2} n^\mu \qquad p_2^\mu = \frac{\sqrt{s}}{2} \bar{n}^\mu. \tag{1}$$

The outgoing momentum can be expressed in a Sudakov decomposed form as well:

$$p_1'^\mu = \frac{1}{2}(\sqrt{s} - \bar{n}\cdot k) n^\mu - \frac{1}{2} n\cdot k \bar{n}^\mu - k_\perp^\mu$$
$$p_2'^\mu = \frac{1}{2}\bar{n}\cdot k n^\mu + \frac{1}{2}(\sqrt{s} + n\cdot k)\bar{n}^\mu + k_\perp^\mu. \tag{2}$$

The outgoing quarks are taken to be on-shell so they must have

$$n\cdot k = \frac{\vec{k}_\perp^2}{\sqrt{s} - \bar{n}\cdot k} \qquad \bar{n}\cdot k = \frac{\vec{k}_\perp^2}{\sqrt{s} + n\cdot k}. \tag{3}$$

In the forward region we have $k^2 = n\cdot k \bar{n}\cdot k + \vec{k}_\perp^2 = t$ so that $k_\perp^\mu \sim \sqrt{t}$ and the above equation implies $n\cdot k \sim \bar{n}\cdot k \sim t/\sqrt{s} \ll k_\perp^\mu$. In this region the out-going momenta reduce to

$$p_1'^\mu \approx \frac{\sqrt{s}}{2} n^\mu + \frac{\vec{k}_\perp^2}{2}\bar{n}^\mu - k_\perp^\mu$$
$$p_2'^\mu \approx + \frac{\vec{k}_\perp^2}{2} n^\mu + \frac{\sqrt{s}}{2}\bar{n}^\mu + k_\perp^\mu, \tag{4}$$

where $k^2 \approx -\vec{k}_\perp^2$. We carry out the matching depicted in Fig. 1 by expanding the QCD amplitude in the forward region

$$\mathcal{A}_{QCD} = -\frac{g^2}{\vec{k}_\perp^2} \bar{u}(p_1') T^a \gamma^\mu u(p_1) \bar{u}(p_2') T^a \gamma_\mu u(p_2)$$
$$\approx -\frac{n\cdot\bar{n} g^2}{\vec{k}_\perp^2} \bar{\xi}_n T^a \frac{\bar{n}\!\!\!/}{2} \xi_n \bar{\xi}_{\bar{n}} T^a \frac{n\!\!\!/}{2} \xi_{\bar{n}}, \tag{5}$$

where ξ_n and $\xi_{\bar{n}}$ are the high-energy limit of the QCD spinors for quarks moving in the n^μ and $\bar{n}^\mu$ direction respectively. This amplitude is reproduced by the SCET operator first derived in Ref. [22]

$$\mathcal{O}_G^{n\bar{n}} = -\frac{2g^2}{\vec{k}_\perp^2} \bar{\xi}_{p_1',n} T^a \frac{\bar{n}\!\!\!/}{2} \xi_{p_1,n} \bar{\xi}_{p_2',\bar{n}} T^a \frac{n\!\!\!/}{2} \xi_{p_2,\bar{n}}, \tag{6}$$

where $\xi_{p_1,n}$ and $\xi_{p_2,\bar{n}}$ are SCET quark fields. This operator is not gauge invariant under separate gauge transformations in the n and $\bar{n}$ sectors, but can be made so by adding the appropriate SCET collinear Wilson lines [14]

$$W_n = \sum_{\text{perms}} \exp\left(-\frac{g}{\bar{n}\cdot\mathcal{P}} \bar{n}\cdot A_{q,n}\right) \quad \text{and}$$
$$W_{\bar{n}} = \sum_{\text{perms}} \exp\left(-\frac{g}{n\cdot\mathcal{P}} n\cdot A_{q,\bar{n}}\right). \tag{7}$$

In addition, soft gluons with momentum that scales as $k_s^\mu \sim \sqrt{t}$ can be radiated from the collinear quarks. While such an interaction puts the collinear quark off-shell, it is order one in the power counting and must be summed into a soft Wilson line [15]

$$S_n = \sum_{\text{perms}} \exp\left(\frac{-g}{n\cdot\mathcal{P}} n\cdot A_{s,q}\right)$$
$$S_{\bar{n}} = \sum_{\text{perms}} \exp\left(\frac{-g}{\bar{n}\cdot\mathcal{P}} \bar{n}\cdot A_{s,q}\right). \tag{8}$$

Including both collinear and soft Wilson lines we arrive at the n–$\bar{n}$ collinear Glauber operator

$$\mathcal{O}_G^{n\bar{n}} = -8\pi\alpha_s(\mu) \bar{\xi}_{p_2',\bar{n}} W_{\bar{n}} Y_{\bar{n}}^\dagger T^a \frac{n\!\!\!/}{2} Y_{\bar{n}} W_{\bar{n}}^\dagger \xi_{p_2,\bar{n}}$$
$$\times \frac{1}{\vec{\mathcal{P}}_\perp^2} \bar{\xi}_{p_1',n} W_n Y_n^\dagger T^a \frac{\bar{n}\!\!\!/}{2} Y_n W_n^\dagger \xi_{p_1,n}. \tag{9}$$

There are also collinear-soft Glauber operators which were considered in detail in Refs. [19–21]. These operators have leading order Feynman diagrams depicted in Fig. 2 (not shown is the coupling to a soft ghost), however, they are not needed here.

Next, we renormalize the operator in Eq. (9). The diagrams that contribute are shown in Fig. 3. The double lines in the diagrams in (a) indicate that a soft gluon is emitted from one of the soft Wilson lines. The diagrams in (a) are ultraviolet (UV) finite, but contain a

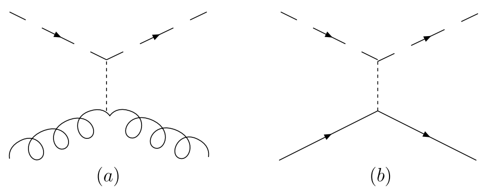

Fig. 2. Leading order Feynman diagrams corresponding to operators that couple collinear and soft degrees of freedom via Glauber exchange: (a) collinear quark coupling to a soft gluon, (b) collinear quark coupling to a soft quark (solid line). Not shown is the collinear quark coupling to a soft ghost.

rapidity divergence. The first two diagrams in (b) (plus a ghost-loop that is not shown) are time-ordered products of two of the operators that give the tree-level diagrams in Fig. 2. They are UV divergent but do not have a rapidity divergence, and are needed to give the correct RG for the Glauber gluon coupling constant. An explicit calculation of these diagrams has not been carried out so far, and clearly would be an important check on the formalism. The third diagram in (b) comes from a time ordered product of the collinear Glauber operator with terms from the SCET Lagrangian that couple collinear gluons to collinear quarks. This diagram also has a UV divergence, which is canceled by the collinear Lagrangian vertex counter-term. As this diagram involves only collinear degrees of freedom moving in the same direction it is the same as the renormalization of the QCD quark–gluon vertex [31].

The physics of interest is associated with the rapidity divergence, so we will focus on the diagrams in Fig. 3(a). The sum of these four diagrams gives

$$\mathcal{A} = -8\pi\alpha_s(\mu)\bar{\xi}_n T^a \frac{\bar{\not{n}}}{2}\xi_n \bar{\xi}_{\bar{n}} T^a \frac{\not{n}}{2}\xi_{\bar{n}}\big[iN_c\alpha_s(\mu)\mathcal{I}(\vec{k}_\perp)\big], \qquad (10)$$

where

$$\mathcal{I}(\vec{k}_\perp) = \int \frac{dq^-}{q^-}\int \frac{d^2q_\perp}{(2\pi)^2}\frac{1}{\vec{q}_\perp^{\,2}}\frac{1}{(\vec{q}+\vec{k})_\perp^2}. \qquad (11)$$

In obtaining the expression in Eq. (10) a symmetry factor of one-half needs to be included as the first two diagrams in Fig. 3(a) are identical to the second two diagrams. The integral over q^- results in a rapidity divergence, while the integral over $q_\perp$, which in the literature is called the gluon Regge trajectory, contains IR divergences. To evaluate this integral we will need to introduce regulators for both types of divergences. Here we will regulate the rapidity divergence using the methods developed in Ref. [29,30], and use a gluon mass (or dimensional regularization) to regulate IR divergences. With these modifications the integral above becomes

$$\begin{aligned}
\mathcal{I}(\vec{k}_\perp) &= \nu^{2\eta}w(\nu)^2\int \frac{d^4q}{(2\pi)^4}\frac{1}{q^-}\frac{1}{q^+}\frac{(q^3)^{-2\eta}}{q^2-m_g^2}\frac{1}{(\vec{q}+\vec{k})_\perp^2+m_g^2}\\
&= -i\frac{\nu^{2\eta}w(\nu)^2}{\eta}\frac{\Gamma(\frac{1}{2}-\eta)\Gamma(1+\eta)}{(4\pi)^2\sqrt{\pi}}\frac{1}{(k_\perp^2)^{1+\eta}}\\
&\quad\times\int_0^1 dx\frac{x^\eta}{[x(1-x)+m_g^2/k_\perp^2]^{1+\eta}}\\
&\approx \frac{-2i}{(4\pi)^2}\frac{w(\nu)^2}{\vec{k}_\perp^2}\bigg[\frac{1}{\eta}\ln\bigg(\frac{\vec{k}_\perp^2}{m_g^2}\bigg)+\ln\bigg(\frac{\vec{k}_\perp^2}{4\nu}\bigg)\ln\bigg(\frac{\vec{k}_\perp^2}{m_g^2}\bigg)\\
&\quad-\frac{1}{4}\ln^2\bigg(\frac{\vec{k}_\perp^2}{m_g^2}\bigg)+i\pi\ln\bigg(\frac{\vec{k}_\perp^2}{m_g^2}\bigg)\bigg],
\end{aligned} \qquad (12)$$

where $w(\nu)$ is a bookkeeping parameter that has been introduced for convenience in deriving the rapidity RGE, and will eventually be set to one [29,30]. For completeness we also give an expression

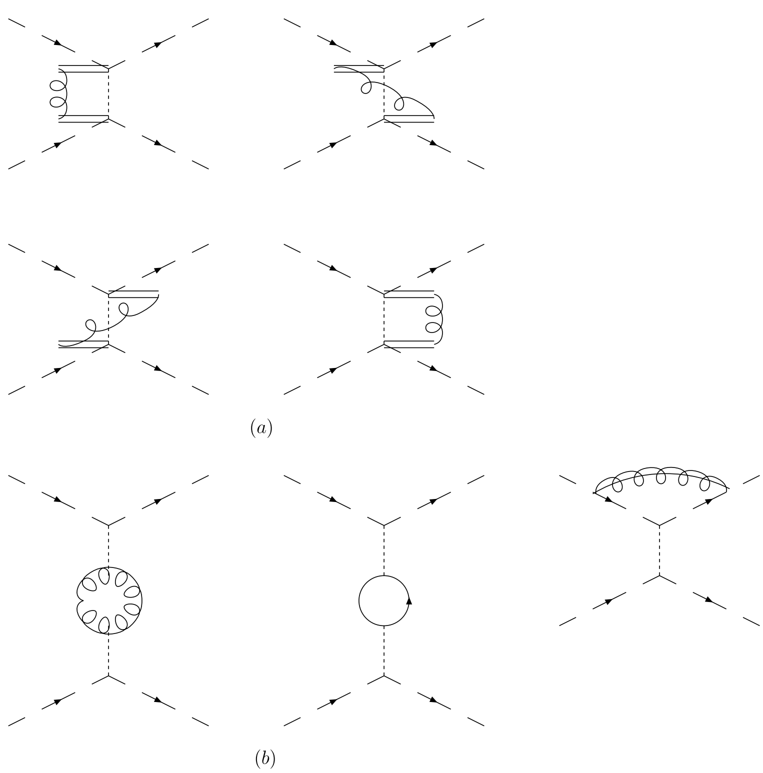

Fig. 3. One loop Feynman diagrams contributing to the renormalization of $\mathcal{O}_G^{n\bar{n}}$. The double line in the diagrams in (a) indicates soft gluon emission from a Wilson line. These diagrams have a rapidity divergence which gives the gluon Regge trajectory. The diagrams in (b) have no rapidity divergence, but have UV divergences. The first two diagrams involve soft gluons and soft quarks (the soft-ghost loop diagram is not shown), and the UV divergence in these diagrams is canceled by a soft Lagrangian counter-term. The last diagram involves the exchange of a collinear gluon (spring with a line) and the UV divergence is canceled by a collinear Lagrangian counter-term.

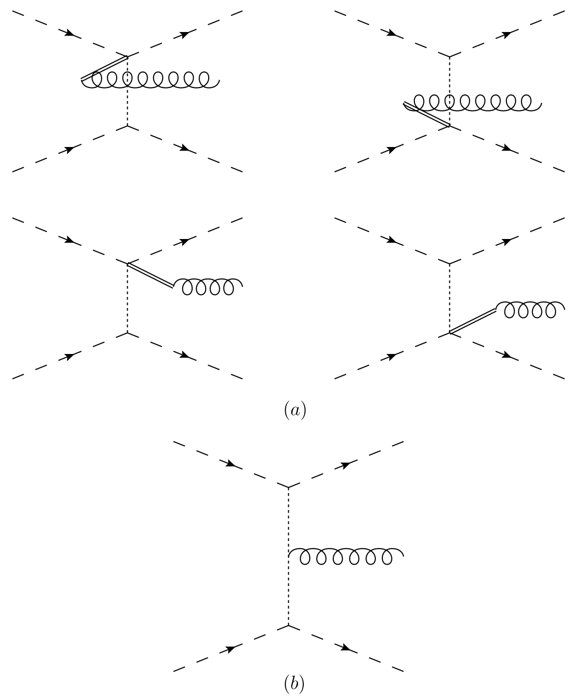

Fig. 4. Real emission of soft gluons from the n–$\bar{n}$ Glauber interaction: (a) emission from the soft Wilson lines, (b) emission from the Glauber gluon.

for $\mathcal{I}(\vec{k}_\perp)$ regulating the IR divergences with dimensional regularization:

$$\begin{aligned}\mathcal{I}(\vec{k}_\perp) &= -i\big(4\pi\mu^2\big)^\epsilon \frac{\nu^{2\eta} w(\nu)^2}{\eta} \frac{\Gamma(\frac{1}{2}-\eta)\Gamma(1+\eta+\epsilon)}{(4\pi)^2\sqrt{\pi}} \\ &\quad\times \frac{1}{(k_\perp^2)^{1+\eta+\epsilon}} \frac{\Gamma(-\epsilon)\Gamma(-\eta-\epsilon)}{\Gamma(-\eta-2\epsilon)} \\ &\approx \frac{-2i}{(4\pi)^2} \frac{w(\nu)^2}{\vec{k}_\perp^2} \left\{ \frac{\Gamma(-\epsilon)}{\eta} \left(\frac{\bar{\mu}^2 e^{\gamma_E}}{\vec{k}_\perp^2} \right)^\epsilon \frac{\Gamma(1+\epsilon)\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \right. \\ &\quad + \frac{1}{2\epsilon^2} + \frac{1}{2\epsilon}\left[\ln\left(\frac{\bar{\mu}^2}{4\nu^2}\right) + \ln\left(\frac{\vec{k}_\perp^2}{4\nu^2}\right) \right] \\ &\quad \left. + \frac{1}{4}\ln^2\left(\frac{\vec{k}_\perp^2}{\bar{\mu}^2}\right) - \ln\left(\frac{\vec{k}_\perp^2}{4\nu^2}\right)\ln\left(\frac{\vec{k}_\perp^2}{\bar{\mu}^2}\right) - \frac{\pi^2}{24} \right\}, \end{aligned} \tag{13}$$

where $\bar{\mu}^2 = 4\pi\mu^2 e^{-\gamma}$. The rapidity divergence corresponds to the term that diverges as $\eta \to 0$. This rapidity pole must be subtracted by a rapidity counter-term. However, as the rapidity divergent term contains IR divergences a sensible rapidity RGE cannot be derived. This issue is fixed if we consider forward scattering and include real emission diagrams.

The emission of a real soft gluon can occur from any of the soft Wilson lines as shown in Fig. 4(a) or from the exchanged Glauber gluon as shown in Fig. 4(b). The amplitude for the sum of the four diagrams in Fig. 4(a) is

$$\begin{aligned}\sum_{i=1}^{4} \mathcal{A}^i_{\rm real} &= -2g^2 \frac{1}{\vec{k}_\perp^2} \frac{1}{\vec{k}_\perp^{\prime 2}} \bar{\xi}_n T^a \frac{\bar{n}\!\!\!/}{2} \xi_n \bar{\xi}_{\bar{n}} T^b \frac{n\!\!\!/}{2} \xi_{\bar{n}} \big(-igf^{abc}\big) \\ &\quad\times \left(\frac{n^\alpha}{n\cdot k'} \vec{k}_\perp^2 + \frac{\bar{n}^\alpha}{\bar{n}\cdot k} \vec{k}_\perp^{\prime 2} \right) \end{aligned} \tag{14}$$

and the amplitude for the diagram in Fig. 4(b) is

$$\begin{aligned}\mathcal{A}^5_{\rm real} &= -2g^2 \frac{1}{\vec{k}_\perp^2} \frac{1}{\vec{k}_\perp^{\prime 2}} \bar{\xi}_n T^a \frac{\bar{n}\!\!\!/}{2} \xi_n \bar{\xi}_{\bar{n}} T^b \frac{n\!\!\!/}{2} \xi_{\bar{n}} \big(igf^{abc}\big) \\ &\quad\times \left(k_\perp^\alpha + k_\perp^{\prime\alpha} - \frac{1}{2}\bar{n}^\alpha n\cdot k' - \frac{1}{2} n^\alpha \bar{n}\cdot k \right), \end{aligned} \tag{15}$$

where the soft gluon momentum is $q^\mu = k^\mu - k'^\mu \approx \frac{1}{2}\bar{n}\cdot k n^\mu - \frac{1}{2} n\cdot k' \bar{n}^\mu + (k_\perp - k'_\perp)^\mu$. Adding these up we arrive at the Lipatov vertex

$$\begin{aligned}\mathcal{A}_{\rm L} &= -2g^2 \frac{1}{\vec{k}_\perp^2} \frac{1}{\vec{k}_\perp^{\prime 2}} \bar{\xi}_n T^a \frac{\bar{n}\!\!\!/}{2} \xi_n \bar{\xi}_{\bar{n}} T^b \frac{n\!\!\!/}{2} \xi_{\bar{n}} \big(igf^{abc}\big) \\ &\quad\times \left(k_\perp^\alpha + k_\perp^{\prime\alpha} - \frac{1}{2}\bar{n}^\alpha n\cdot k' - \frac{1}{2} n^\alpha \bar{n}\cdot k \right. \\ &\quad \left. - \frac{n^\alpha}{n\cdot k'} \vec{k}_\perp^2 - \frac{\bar{n}^\alpha}{\bar{n}\cdot k} \vec{k}_\perp^{\prime 2} \right). \end{aligned} \tag{16}$$

This vertex is gauge invariant, as can be explicitly verified by contracting with the external gluon momentum.

Now we have all the pieces needed to calculate the quark scattering cross section in the forward region. Squaring the amplitude in Eq. (5) we obtain the tree level cross section

$$\sigma^{LO} = \frac{2\alpha_s^2 C_F}{N_c} \int \frac{d^2\vec{k}_\perp^2}{\vec{k}_\perp^2} \int \frac{d^2\vec{k}_\perp^{\prime 2}}{\vec{k}_\perp^{\prime 2}} \delta^{(2)}\big(\vec{k}_\perp - \vec{k}'_\perp\big). \tag{17}$$

The NLO virtual corrections give

$$\begin{aligned}\sigma_V^{NLO} &= \frac{2\alpha_s^2 C_F}{N_c} \int \frac{d^2\vec{k}_\perp}{\vec{k}_\perp^2} \int \frac{d^2\vec{k}'_\perp}{\vec{k}_\perp^{\prime 2}} \delta^{(2)}\big(\vec{k}_\perp - \vec{k}'_\perp\big) \\ &\quad\times \left(-\frac{\alpha_s N_c}{2\pi^2} \right) \nu^{2\eta} w(\nu)^2 \frac{\Gamma(\eta)\Gamma(\frac{1}{2}-\eta)}{\sqrt{\pi}} \\ &\quad\times \int d^2 q_\perp \frac{\vec{k}_\perp^2}{\vec{q}_\perp^2} \frac{1}{[(\vec{q}_\perp - \vec{k}_\perp)^2]^{1+\eta}}. \end{aligned} \tag{18}$$

The NLO real corrections can be obtained by the standard method of squaring the amplitude and summing over final states, or by taking the cut of the forward scattering graph in the Glauber regime. In order to incorporate the rapidity regulator we use the latter method to obtain

$$\begin{aligned}\sigma_R^{NLO} &= \frac{2\alpha_s^2 C_F}{N_c} \int \frac{d^2\vec{k}_\perp}{\vec{k}_\perp^2} \int \frac{d^2\vec{k}'_\perp}{\vec{k}_\perp^{\prime 2}} \\ &\quad\times \left(\frac{\alpha_s N_c}{\pi^2} \right) \nu^{2\eta} w(\nu)^2 \frac{\Gamma(\eta)\Gamma(\frac{1}{2}-\eta)}{\sqrt{\pi}} \\ &\quad\times \int d^2 q_\perp \frac{\delta^{(2)}(\vec{q}_\perp - \vec{k}'_\perp)}{[(\vec{q}_\perp - \vec{k}_\perp)^2]^{1+\eta}}. \end{aligned} \tag{19}$$

In order to ensure that there is no double counting in SCET the soft-Glauber overlap region needs to be subtracted from the above results, however in this case the overlap region vanishes. Adding these up we arrive at an expression for the forward scattering cross section accurate to NLO

$$\begin{aligned}\sigma &= \frac{2\alpha_s^2 C_F}{N_c} \int \frac{d^2\vec{k}_\perp}{\vec{k}_\perp^2} \int \frac{d^2\vec{k}_\perp}{\vec{k}_\perp^{\prime\prime 2}} \left\{ \delta^{(2)}\big(\vec{k}_\perp - \vec{k}'_\perp\big) \right. \\ &\quad + \left(\frac{\alpha_s N_c}{\pi^2} \right) \frac{\Gamma(\eta)\Gamma(\frac{1}{2}-\eta)}{\sqrt{\pi}} \nu^{2\eta} w(\nu)^2 \\ &\quad\times \int \frac{d^2 q_\perp}{[(\vec{q}_\perp - \vec{k}_\perp)^2]^{1+\eta}} \left[\delta^{(2)}\big(\vec{q}_\perp - \vec{k}'_\perp\big) \right. \\ &\quad \left.\left. - \frac{\vec{k}_\perp^2}{2\vec{q}_\perp^2} \delta^{(2)}\big(\vec{k}_\perp - \vec{k}'_\perp\big) \right] \right\}. \end{aligned} \tag{20}$$

Expanding around $\eta = 0$ we can isolate the rapidity divergent term

$$\sigma = \frac{2\alpha_s^2 C_F}{N_c}\int \frac{d^2\vec{k}_\perp}{\vec{k}_\perp^2}\int \frac{d^2\vec{k}_\perp}{\vec{k}_\perp''^2}\Big\{\delta^{(2)}\big(\vec{k}_\perp - \vec{k}'_\perp\big) + \left(\frac{\alpha_s N_c}{\pi^2}\right)\frac{w(\nu)^2}{\eta}\int \frac{d^2 q_\perp}{(\vec{q}_\perp - \vec{k}_\perp)^2}\Big[\delta^{(2)}\big(\vec{q}_\perp - \vec{k}'_\perp\big) - \frac{\vec{k}_\perp^2}{2\vec{q}_\perp^2}\delta^{(2)}\big(\vec{k}_\perp - \vec{k}'_\perp\big)\Big] + \dots\Big\}, \tag{21}$$

where the dots represent NLO terms that are finite in the $\eta \to 0$ limit. This result raises the important question of how the rapidity divergence is subtracted. In SCET without Glauber gluons collinear and soft degrees of freedom factor and observables can often be expressed as convolutions of matrix elements of operators involving only collinear or soft degrees of freedom. If the factorization of soft and collinear holds in the presence of Glauber gluons then it may be that the above cross section can also be expressed as a convolution of the matrix element of a soft operator with the matrix element of an n-collinear operator and the matrix element of an $\bar{n}$-collinear operator. In this case the counter-term for the soft operator would cancel the rapidity divergence. Such a factorization is suggested by the standard treatment in the literature [28], where the two-dimensional Dirac delta function in transverse-momentum space is interpreted as the BFKL Green function. The rapidity divergence is then canceled by a counter-term for this Green function. However, factorization of the Glauber interaction in SCET requires an all orders summation of soft gluons, which has not yet been accomplished. A first step in this direction has recently been made in Ref. [32] where it is shown that in a scalar theory with n-collinear modes, $\bar{n}$-collinear modes, and Glauber modes an all orders summation of ladder graphs gives the leading Regge behavior. We will leave the summation of soft gluons for a future work, and motivated by the BFKL approach will for the time being conjecture that the cross section factors. We renormalize the rapidity divergence by identifying the two-dimensional Dirac delta function in transverse-momentum space as the leading order vacuum matrix element of a (currently unknown) operator, O_G^{soft}, involving soft fields: $G(\vec{k}_\perp - \vec{k}'_\perp) \equiv \langle O_{G,\text{soft}}\rangle$. Then

$$\begin{aligned} G\big(\vec{k}_\perp - \vec{k}'_\perp, \nu\big) &= \int d^2\ell_\perp \mathcal{Z}^{-1}(\vec{k}_\perp - \vec{\ell}_\perp; \eta, \nu) G\big(\vec{\ell}_\perp - \vec{k}'_\perp; \nu\big)^{(0)} \\ &= \int d^2\ell_\perp \mathcal{Z}^{-1}(\vec{k}_\perp - \vec{\ell}_\perp; \eta, \nu)\delta^{(2)}\big(\vec{\ell}_\perp - \vec{k}'_\perp\big) \\ &= \delta^{(2)}\big(\vec{k}_\perp - \vec{k}'_\perp\big) + \text{counter-terms}, \end{aligned} \tag{22}$$

where the superscript (0) indicates the matrix element of the bare operator. Inverting the above equation leads to

$$\delta^{(2)}\big(\vec{k}_\perp - \vec{k}'_\perp\big) = \int d^2\ell_\perp \mathcal{Z}(\vec{k}_\perp - \vec{\ell}_\perp; \eta, \nu) G\big(\vec{\ell}_\perp - \vec{k}'_\perp; \nu\big). \tag{23}$$

The rapidity divergence term in Eq. (21) is canceled by setting

$$\mathcal{Z}(\vec{k}_\perp - \vec{\ell}_\perp; \eta, \nu) = \delta^{(2)}(\vec{k}_\perp - \vec{\ell}_\perp) - \left(\frac{\alpha_s N_c}{\pi^2}\right)\frac{w(\nu)^2}{\eta}\left[\frac{1}{(\vec{k}_\perp - \vec{\ell}_\perp)^2} - \frac{1}{2}\delta^{(2)}(\vec{k}_\perp - \vec{\ell}_\perp)\int \frac{d^2 q_\perp}{(\vec{q}_\perp - \vec{k}_\perp)^2}\frac{\vec{k}_\perp^2}{\vec{q}_\perp^2}\right]. \tag{24}$$

Inserting this expression into Eq. (23) we find

$$\delta^{(2)}\big(\vec{k}_\perp - \vec{k}'_\perp\big) = G\big(\vec{k}_\perp - \vec{k}'_\perp; \nu\big) - \left(\frac{\alpha_s N_c}{\pi^2}\right)\frac{w(\nu)^2}{\eta}\left[\int d^2 q_\perp \frac{G(\vec{q}_\perp - \vec{k}'_\perp; \nu)}{(\vec{q}_\perp - \vec{k}_\perp)^2} - \frac{1}{2}G\big(\vec{k}_\perp - \vec{k}'_\perp; \nu\big)\int \frac{d^2 q_\perp}{(\vec{q}_\perp - \vec{k}_\perp)^2}\frac{\vec{k}_\perp^2}{\vec{q}_\perp^2}\right], \tag{25}$$

which when used in Eq. (21) gives

$$\sigma = \frac{2\alpha_s^2 C_F}{N_c}\int \frac{d^2\vec{k}_\perp}{\vec{k}_\perp^2}\int \frac{d^2\vec{k}'_\perp}{\vec{k}'^2_\perp} G\big(\vec{k}_\perp - \vec{k}'_\perp; \nu\big) + \dots \tag{26}$$

where the singular terms in η cancel and the dots indicate NLO terms that do not vanish in the $\eta \to 0$ limit. The dependence of $G(\vec{k}_\perp - \vec{k}'_\perp; \nu)$ on ν is given by the rapidity RGE

$$\frac{d}{d\ln\nu}G\big(\vec{k}_\perp - \vec{k}'_\perp; \nu\big) = \int d^2\ell_\perp \gamma_\nu\big(\vec{k}'_\perp - \vec{\ell}_\perp\big) G\big(\vec{\ell}_\perp - \vec{k}'_\perp; \nu\big), \tag{27}$$

where the rapidity anomalous dimension is determined from

$$\gamma_\nu\big(\vec{k}_\perp - \vec{k}'_\perp\big) = \int d^2\ell_\perp \mathcal{Z}\big(\vec{\ell}_\perp, -\vec{k}'_\perp; \eta, \nu\big)^{-1}\frac{d}{d\ln\nu}\mathcal{Z}(\vec{k}_\perp - \vec{\ell}_\perp; \eta, \nu). \tag{28}$$

Using

$$\frac{d}{d\ln\nu} = \frac{\partial}{\partial\ln\nu} - w(\nu)^2\eta\frac{\partial}{\partial w^2} \tag{29}$$

we find the leading-log (LL) rapidity anomalous dimension

$$\gamma_\nu\big(\vec{k}_\perp - \vec{k}'_\perp\big) = \left(\frac{\alpha_s N_c}{\pi^2}\right)\left[\frac{1}{(\vec{k}_\perp - \vec{k}'_\perp)^2} - \frac{1}{2}\delta^{(2)}\big(\vec{k}_\perp - \vec{k}'_\perp\big)\int \frac{d^2 q_\perp}{(\vec{q}_\perp - \vec{k}_\perp)^2}\frac{\vec{k}_\perp^2}{\vec{q}_\perp^2}\right], \tag{30}$$

where we set $w(\nu) = 1$. Using this LL expression in Eq. (27) gives

$$\frac{d}{d\ln\nu}G\big(\vec{k}_\perp - \vec{k}'_\perp; \nu\big) = \left(\frac{\alpha_s N_c}{\pi^2}\right)\int \frac{d^2 q_\perp}{(\vec{q}_\perp - \vec{k}_\perp)^2}\left[G\big(\vec{q}_\perp - \vec{k}'_\perp; \nu\big) - \frac{\vec{k}_\perp^2}{2\vec{q}_\perp^2}G\big(\vec{k}_\perp - \vec{k}'_\perp; \nu\big)\right]. \tag{31}$$

This is the BFKL equation [compare to Eq. (3.58) in Ref. [28]]. It can be solved by expanding $G(\vec{k}_\perp - \vec{k}'_\perp; \nu)$ in eigenfunctions

$$G\big(\vec{k}_\perp - \vec{k}'_\perp; \nu\big) = \sum_{n=-\infty}^{\infty}\int_{a-i\infty}^{a+i\infty}\frac{d\gamma}{2\pi i}C_{n,\gamma}(\nu)|\vec{k}_\perp|^{2(\gamma-1)}\big|\vec{k}'_\perp\big|^{2(\gamma^*-1)}e^{in(\phi-\phi')}, \tag{32}$$

running in rapidity from $\ln\nu_i \sim 0$ to $\ln\nu_f \sim \ln s$, and then taking the inverse transform [28]. The last step can only be done approximately. For large $\ln\nu_f$ one finds

$$G\big(\vec{k}_\perp - \vec{k}'_\perp; s\big) = \frac{1}{2\pi^2|\vec{k}_\perp||\vec{k}'_\perp|}\sqrt{\frac{\pi^2}{14\zeta(3)\alpha_s(\mu)N_c s}} \times \text{Exp}\left[\frac{4\alpha_s(\mu)N_c}{\pi}\ln 2\ln s - \frac{\pi\ln^2(|\vec{k}_\perp|/|\vec{k}'_\perp|)}{14\zeta(3)\alpha_s(\mu)N_c s}\right], \tag{33}$$

with the leading piece being the first term in the exponent. Using this result in Eq. (26) gives a cross section that has Regge behavior since it grows as a power of s

$$\sigma \sim s^{\alpha_p - 1} \qquad \alpha_p = 1 + \frac{4\alpha_s(\mu)N_c}{\pi}\ln 2, \tag{34}$$

where α_p is the pomeron intercept.

Two important issues confront SCET: the role of Glauber interactions in factorization and the emergence of Regge behavior. Here, we have shown that at the perturbative level these two issues are actually the same. Glauber interactions between n and $\bar{n}$ collinear quarks exhibit large rapidity logarithms at NLO in perturbation theory, and the rapidity RGE that resumes these large rapidity logarithms is the LL BFKL equation which gives rise to Regge behavior. While this is not the first attempt at incorporating Glauber interactions into SCET, it is the first time Glauber interactions in SCET have been connected to the emergence of Regge behavior in the theory. Clearly, the analysis presented here is based on a perturbative approach, and can only be considered a first (small) step towards a comprehensive treatment of Glauber interactions and hence Regge behavior in SCET. The ultimate goal is to include Glauber interactions to all orders so that conclusions about the role of Glauber interactions in factorization can be made to all orders, and that Regge behavior can be understood at the non-perturbative level. Both of these goals remain open questions in strong interactions, though a considerable effort has been devoted to them (especially the question of non-perturbative Regge behavior [28,33]). The hope is that reformulating these questions in an effective field theory language will allow us to bring new tools to bear and arrive at a solution.

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