Heavy baryons with strangeness in a soliton model

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A B S T R A C T

We present results from a chiral soliton model calculation for the spectrum of baryons with a single heavy quark (charm or bottom) and non-zero strangeness. We treat the strange components within a three flavor collective coordinate quantization of the soliton that fully accounts for light flavor symmetry breaking. Heavy baryons emerge by binding a heavy meson to the soliton. The dynamics of this heavy meson is described by the heavy quark effective theory with finite mass effects included.

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1. Motivation

Baryons containing heavy quarks such as charm or bottom form an excellent opportunity to study the binding of quarks to hadrons. Since there is no exact solution to quantum-chromo-dynamics (QCD), various models and approximations that focus on particular features of QCD are relevant. In the context of heavy baryons three are of particular importance. First, there is the heavy spin–flavor symmetry [1][1] that governs the dynamics of heavy quarks. Second, there is the chiral symmetry that dictates the interactions among the light quarks. In addition to dynamical chiral symmetry breaking, there is substantial flavor symmetry breaking when the strange quark is involved. It is thus particularly interesting to investigate baryons that, in the valence quark picture, are composed of a single heavy quark and two light, including strange, quarks. Thirdly, generalizing QCD from three to arbitrarily many color degrees of freedom suggests to consider baryons as soliton configurations in an effective meson theory [3].

Our point of departure is a chiral soliton of meson fields built from up and down quarks [4]. States with good baryon quantum numbers are generated by quantizing the fluctuations about the soliton. The modes associated with (flavor) rotations have large non-harmonic components and consequently are treated as collective excitations. The Hamiltonian for these collective coordinates contains flavor symmetry breaking terms that slightly suppress the non-harmonic contributions. The important feature is that this Hamiltonian can be diagonalized exactly, *i.e.* the spectrum can be determined beyond a perturbation expansion in the quark mass differences [5]. The resulting eigenvalues are associated with the strangeness contribution to the baryon masses. For the particular case of kaon–nucleon scattering this approach has been verified [6] to yield the correct resonance position. Subsequently fields representing mesons with a single heavy quark are included. While their heavy quark components are subject to the heavy spin–flavor symmetry, their light ones couple to the light meson fields according to chiral symmetry such that the soliton generates an attractive potential for the heavy meson fields [7]. Combined with the soliton, a bound state in this potential builds the heavy baryon. (This is a generalization of the so-called bound state approach [8] that, in the harmonic approximation, describes hyperons in the Skyrme model [9].[2]) The strangeness components of the heavy meson bound state are subject to the same collective coordinate treatment as the soliton.

Shortly after the bound state approach in the Skyrme model of pseudoscalar mesons was applied to hyperons it was extended to

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[2] While Ref. [10] comprehensively reviews soliton model studies, Ref. [6] thoroughly discusses the two above mentioned descriptions of strangeness in chiral soliton models, in particular with regard to the large N_C limit.

heavier baryons [11]. In those studies the relevance of the heavy spin–flavor symmetry was not yet recognized. Subsequently, also heavy vector meson fields were included [12]. More or less at the same time investigations were performed in the heavy limit scenario [13,14]. Those heavy limit studies included neither corrections to the heavy spin–flavor symmetry from finite masses nor strangeness degrees of freedom. In that case, baryons like Ξ_c cannot be addressed. Strangeness was indeed included in Ref. [15], however, light flavor symmetry breaking was treated in a perturbation expansion and finite mass effects were omitted. This does not distinguish between even and odd parity or charm and bottom baryons and typically overestimates the binding energy of the heavy meson [7]. Also the parameters of the final energy formula were fitted rather than calculated from a realistic soliton model. These widespread bound state studies derive a potential for the meson fields from the soliton that is fixed in position. We note that this picture is strictly valid only in the large number of colors limit when the soliton is more massive than the heavy meson. Though this approach is a systematic and consistent expansion in the number of colors, kinematical corrections should be expected in the real world with three colors.

Our soliton model calculation for the spectrum of heavy baryons will improve with regard to the following aspects: We take the parameters in the mass formula from an actual soliton model calculation (we allow for moderate adjustment of the light flavor symmetry breaking strength), go beyond the perturbation expansion in that symmetry breaking and construct the heavy meson bound state from a model that systematically incorporates finite mass corrections. Our model calculation will produce an extensive picture of baryons, from the nucleon up to the Ω_b. We will not consider doubly-heavy baryons, though.

The spectrum of heavy baryons has been investigated in other approaches as well. A comprehensive account of the (non-relativistic) quark model approach is given in Ref. [16] with some newer results reported in Ref. [17]. Relativistic effects are incorporated within quark–diquark models [18]. QCD sum rules were not only used to obtain the spectrum [19], but also to extract the heavy quark mass poles [20]. Lattice QCD calculations can be traced from Ref. [21] that also studies baryons with more than one heavy quark. Finally, Ref. [22] contains comprehensive reviews on baryon spectroscopy that discuss a variety of approaches and may be consulted for further references.

2. The soliton model

In chiral Lagrangians the interaction terms are ordered by the number of derivatives acting on the pseudoscalar fields. The more derivatives there are, the more unknown parameters appear in the Lagrangian. Replacing these higher derivatives by resonance exchange terms is advantageous because more information is available to determine the parameters. We thus consider a chiral soliton that is stabilized by vector mesons ρ and ω [23] as a refinement of the Skyrme model [4,9]. Other shortcomings of the pseudoscalar soliton, like the neutron proton mass difference or the axial singlet matrix element of the nucleon are also solved when including light vector mesons [10].

The basic building block of the model is the chiral field $U = \exp\left(i\sum_{a=1}^{8}\phi_a(x)\lambda_a/f_a\right)$, i.e. the non-linear realization of the pseudoscalar octet field $\phi_a(x)$. Here f_a are the respective decay constants [$f_\pi = 93$ MeV (for $a = 1, 2, 3$), $f_k = 114$ MeV (for $a = 4, \ldots, 7$). The case $a = 8$ requires additional input [24] but is not relevant here.] and λ_a are the eight Gell-Mann matrices of $SU(3)$. The static field configuration of the soliton is the hedgehog ansatz

$$U_0(\boldsymbol{r}) = \exp\left[\boldsymbol{\tau} \cdot \hat{\boldsymbol{r}} F(r)\right], \quad \omega_\mu(\boldsymbol{r}) = \omega(r)\, g_{\mu 0} \quad \text{and}$$

$$\rho_{im}^{(0)}(\boldsymbol{r}) = \epsilon_{ikm}\hat{r}_k \frac{G(r)}{r}. \tag{1}$$

The isovector $\boldsymbol{\tau} = (\lambda_1, \lambda_2, \lambda_3)$ comprises the three Pauli matrices from the isospin subspace of flavor $SU(3)$. The spatial components of the ω_μ and the time components of the $\boldsymbol{\rho}_\mu$ fields are zero. For the latter, i is an isospin/flavor index and $m = 1, 2, 3$ labels its spatial components. The profile functions $F(r)$, $\omega(r)$ and $G(r)$ enter the classical energy functional, E_{cl}. The profiles are determined by the minimization of E_{cl}, subject to boundary conditions that ensure unit baryon number:

$$F(0) = 0, \quad \left.\frac{d\omega(r)}{dr}\right|_{r=0} = 0 \quad \text{and} \quad G(0) = -2. \tag{2}$$

All profile functions vanish asymptotically. Configurations that are suitable for quantization are obtained by introducing time dependent collective coordinates for the flavor orientation $A(t) \in SU(3)$

$$U(\boldsymbol{r}, t) = A(t)U_0(\boldsymbol{r})A^\dagger(t) \quad \text{and}$$

$$\boldsymbol{\tau} \cdot \boldsymbol{\rho}_\mu(\boldsymbol{r}, t) = A(t)\boldsymbol{\tau} \cdot \boldsymbol{\rho}_\mu^{(0)}(\boldsymbol{r})A^\dagger(t). \tag{3}$$

In addition profile functions are induced for the spatial components of ω_μ and the time components of ρ_μ [25,26]. Defining eight angular velocities Ω_a via the time derivative of the collective coordinates

$$\frac{i}{2}\sum_{a=1}^{8}\Omega_a\lambda_a = A^\dagger(t)\frac{dA(t)}{dt}, \tag{4}$$

allows a compact presentation of the Lagrange function for the collective coordinates from the light meson fields

$$L_l(\Omega_a) = -E_{cl} + \frac{1}{2}\alpha^2\sum_{i=1}^{3}\Omega_i^2 + \frac{1}{2}\beta^2\sum_{\alpha=4}^{7}\Omega_\alpha^2 - \frac{\sqrt{3}}{2}\Omega_8. \tag{5}$$

It is obtained from the spatial integral over the Lagrange density with the above described field configuration substituted. Note that the collective coordinates only appear via the angular velocities; A does not appear explicitly. The last term, which is only linear in the time derivative, originates from the Wess–Zumino–Witten action [27] that incorporates the QCD anomaly. The coefficients α^2 and β^2 are moments of inertia for rotations in isospace[3] and the strangeness subspace of flavor $SU(3)$, respectively. These moments of inertia are functionals of the profile functions and the variational principle determines the induced components of the vector meson fields. The structure of the collective coordinate Lagrangian, Eq. (5) is generic to all chiral models that support soliton solutions. The particular numerical values for the classical energy and the moments of inertia are, of course, subject to the particular model. Here we employ the calculation described in Appendix A of Ref. [26] for the entries of Eq. (5).

3. Heavy meson bound state

In effective meson theories, the heavy flavor enters via a heavy meson containing a single heavy quark (charm or bottom) of mass M. The dynamics of the heavy meson follows the heavy flavor effective theory [2] that treats the pseudoscalar (P) and vector meson (Q_μ) components equivalently. That is, in the limit $M \to \infty$ these components are part of a single multiplet (The constant four-velocity V^μ characterizes the heavy quark rest frame.)

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[3] Because of the hedgehog structure it is equivalent to coordinate space.

$$H = \frac{1}{2}\left(1 + \gamma_\mu V^\mu\right)\left(i\gamma_5 P' + \gamma^\mu Q'_\mu\right) \quad \text{where}$$

$$P' = e^{-iMV \cdot x} P \quad \text{and} \quad Q'_\mu = e^{-iMV \cdot x} Q_\mu . \tag{6}$$

The Lagrangian that describes the coupling of this multiplet to the light mesons including the vector mesons ρ and ω and respects the heavy spin–flavor symmetry is [28]

$$\frac{1}{M}\widetilde{\mathcal{L}}_H = iV_\mu \text{Tr}\left\{HD^\mu\bar{H}\right\} - d\,\text{Tr}\left\{H\gamma_\mu\gamma_5 p^\mu\bar{H}\right\}$$

$$- i\frac{\sqrt{2}c}{m_\rho}\text{Tr}\left\{H\gamma_\mu\gamma_\nu F^{\mu\nu}\bar{H}\right\} + \dots , \tag{7}$$

where $\bar{H} = \gamma_0 H^\dagger \gamma_0$. We take the covariant derivative to be[4] $D_\mu = \partial_\mu + iv_\mu$. The chiral currents of the light pseudoscalar mesons are $v_\mu, p_\mu = \frac{i}{2}\left(\sqrt{U}\partial_\mu\sqrt{U^\dagger} \pm \sqrt{U^\dagger}\partial_\mu\sqrt{U}\right)$ and $F^{\mu\nu}$ is the field strength tensor of the light vector mesons. The heavy–light coupling constants $d \approx 0.53$ and $c \approx 1.60$ were determined from heavy meson decays. A field theory model that minimally extends to finite M and M^* for the pseudoscalar and vector components, respectively, has also been constructed in Ref. [28]

$$\mathcal{L}_H = \left(D_\mu P\right)^\dagger D^\mu P - \frac{1}{2}\left(Q_{\mu\nu}\right)^\dagger Q^{\mu\nu} - M^2 P^\dagger P + M^{*2} Q^\dagger_\mu Q^\mu$$

$$+ 2iMd\left(P^\dagger p_\mu Q^\mu - Q^\dagger_\mu p^\mu P\right)$$

$$- \frac{d}{2}\epsilon^{\alpha\beta\mu\nu}\left[(Q_{\nu\alpha})^\dagger p_\mu Q_\beta + Q^\dagger_\beta p_\mu Q_{\nu\alpha}\right]$$

$$- \frac{2\sqrt{2}icM}{m_V}\left\{2Q^\dagger_\mu F^{\mu\nu} Q_\nu\right.$$

$$\left. - \frac{i}{M}\epsilon^{\alpha\beta\mu\nu}\left[\left(D_\beta P\right)^\dagger F_{\mu\nu} Q_\alpha + Q^\dagger_\alpha F_{\mu\nu} D_\beta P\right]\right\}, \tag{8}$$

so that $\mathcal{L}_H \to \widetilde{\mathcal{L}}_H$ in the heavy limit. Here $Q_{\mu\nu}$ is the field strength tensor of the heavy vector mesons. The central feature is that, through the coupling to the light meson soliton, solutions for the heavy meson fields emerge with energy $0 < \omega < M$, i.e. bound states. (Negative energy bound states are also possible. Eventually they build pentaquark baryons that will not be considered here.) The most strongly bound solution has P-wave structure in the pseudoscalar component:

$$P = \frac{e^{i\omega t}}{\sqrt{4\pi}}\Phi(r)\hat{\boldsymbol{r}}\cdot\hat{\boldsymbol{\tau}}\chi, \qquad Q_0 = \frac{e^{i\omega t}}{\sqrt{4\pi}}\Psi_0(r)\chi \quad \text{and}$$

$$Q_i = \frac{e^{i\omega t}}{\sqrt{4\pi}}\left[i\Psi_1(r)\hat{r}_i + \frac{1}{2}\Psi_2(r)\epsilon_{ijk}\hat{r}_j\tau_k\right]\chi . \tag{9}$$

Here P and Q_μ are three component spinors whose flavor content is parameterized by the (constant) spinor χ. Since the coupling to the light mesons occurs via a soliton in the isospin subspace, only the first two components of χ are non-zero. The four radial functions in Eq. (9) couple to the profiles of the static soliton, Eq. (1) in linear differential equations. Normalizable solutions exist only for certain values of ω. These solutions are the bound wave-functions. Their construction, in particular with regard to finite M corrections, and their normalization to carry unit heavy charge is explained in Refs. [7] and [14], respectively. A heavy baryon is then a compound system of the soliton for the light flavors and the bound state of the heavy meson [8]. There are also bound states

in the S-wave channel in which the heavy meson field is parameterized as (see Ref. [12] for parameterizations of higher angular momenta)

$$P = \frac{e^{i\omega t}}{\sqrt{4\pi}}\Phi(r)\chi, \qquad Q_0 = \frac{e^{i\omega t}}{\sqrt{4\pi}}\Psi_0(r)\hat{\boldsymbol{r}}\cdot\hat{\boldsymbol{\tau}}\chi \quad \text{and}$$

$$Q_i = \frac{e^{i\omega t}}{\sqrt{4\pi}}\left[\Psi_1(r)\hat{r}_i\hat{\boldsymbol{r}}\cdot\hat{\boldsymbol{\tau}} + \Psi_2(r)r\boldsymbol{\tau}\cdot\partial_i\hat{\boldsymbol{r}}\right]\chi . \tag{10}$$

They combine with the soliton to form negative parity heavy baryons [7,14]. For convenience we have used equal symbols for the S and P-wave profile functions but, of course, they are different. The computation of the bound state energies ω from identifying localized solutions to the equations of motions that arise by substituting the parameterizations, Eqs. (9) and (10), into the Euler–Lagrange equations of Eq. (8) is detailed in Appendix A of Ref. [7]. That reference also provides figures of the resulting profile functions.

The heavy meson fields must also account for the collective flavor rotation introduced in Eq. (3). This enforces the substitution

$$P \longrightarrow A(t)P \qquad \text{and} \qquad Q_\mu \longrightarrow A(t)Q_\mu , \tag{11}$$

where the right hand sides contain the fields introduced in Eqs. (9) and (10) for P and S wave channels, respectively. This gives non-zero strange components of the heavy mesons and couples the heavy meson strange quark to that of the soliton. Substituting this flavor rotating configuration into the Lagrange density and integrating over space provides the collective coordinate Lagrange function from the heavy fields

$$L_h(\Omega_a) = -\omega\chi^\dagger\chi + \frac{1}{2\sqrt{3}}\chi^\dagger\Omega_8\chi + \rho\chi^\dagger\left(\boldsymbol{\Omega}\cdot\frac{\boldsymbol{\tau}}{2}\right)\chi . \tag{12}$$

Again, the flavor rotation matrix A does not appear explicitly. With the time dependence of the collective coordinates, terms that involve $\sum_{a=1}^{8}\lambda_a\Omega_a$ enter. In the heavy meson sector the quadratic terms provide the bound state contributions to the moments of inertia α^2 and β^2. Since the bound state wave-functions are strongly localized around the center of the soliton[5] the latter dominates the moments of inertia. It is thus safe to only retain the linear terms in Eq. (12). At that order only $a = 1, 2, 3$ and $a = 8$ survive because the bound states do not have any strangeness components. The normalization of the bound state wave-function dictates the coefficients in the first and second terms. The hyperfine splitting parameter ρ is a functional of all profile functions, including some of the induced light vector fields. Its explicit expression is given in Eqs. (B.1)–(B.4) of Ref. [14], where it is called χ_P and χ_S for P- and S-wave channels, respectively.

4. Quantization in $SU(3)$, symmetry breaking and hyperfine splitting

Before we construct a Hamilton operator for the collective coordinates via Legendre transformation of the Lagrangian $L_l + L_h$ we recall that the rotations introduced in Eq. (3) are not exact zero modes in any sensitive model. The reason is that $SU(3)$ flavor symmetry is explicitly broken by different (current) quark masses. This breaking is measured by the ratio

$$x = \frac{2m_s}{m_u + m_d} , \tag{13}$$

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[4] Symmetry allows to also include the light vector meson in this derivative at the expense of an unknown coupling constant. The bound state energies only show moderate sensitivity on that constant [7] so we omit it here.

[5] Their asymptotic behavior is $e^{-|\omega|r} \sim e^{-Mr}$ compared to $e^{-m_\pi r}$ of the chiral field.

where the m_q are the current quark masses of the respective quarks. It can be estimated from meson data [24,29,30]. In early soliton model studies this ratio was considered to be quite large, $x \approx 30$ [24], or even bigger [31]. This was accompanied by sizable symmetry breaking among the hyperons [26]. Later this ratio was re-evaluated and found to be somewhat smaller: $20 \leq x \leq 25$ [30]. Thus it is appropriate to consider this ratio for the (light) flavor symmetry breaking as a tunable parameter. Then symmetry breaking adds to the collective coordinate Lagrangian

$$L_{\text{sb}}(A) = -\frac{x}{2}\widetilde{\gamma}\left[1 - D_{88}(A)\right], \tag{14}$$

where $D_{ab} = \frac{1}{2}\text{tr}\left[\lambda_a A \lambda_b A^\dagger\right]$ parameterizes the adjoint representation of the collective rotations. The coefficient $\widetilde{\gamma}$ is again a functional of the profile functions and acquires its main contribution from the classical fields, Eq. (1). It can be computed in any soliton model. (In the literature $\gamma = x\widetilde{\gamma}$ is typically used.) The heavy mesons also contribute to the symmetry breaking parameter by appropriately substituting mass matrices in Eq. (8). For example, for the charm heavy meson in the P-wave channel we have

$$\gamma = \gamma_{\text{soliton}} + \int\limits_0^\infty drr^2\left[\left(m_D^2 - m_{D_s}^2\right)\Phi^2\right.$$
$$\left. + \left(m_{D^*}^2 - m_{D_s^*}^2\right)\left(-\Psi_0^2 + \Psi_1^2 + \frac{1}{2}\Psi_2^2\right)\right]. \tag{15}$$

Numerically this contribution is small and can easily be compensated by a slight change of x.

We have now collected all terms for the collective coordinate Lagrangian $L(A, \Omega) = L_l(\Omega) + L_h(\Omega) + L_{\text{sb}}(A)$ and can construct the Hamilton operator by Legendre transformation,

$$H(A, R_a, \chi) = E_{\text{cl}} + \frac{1}{2}\left(\frac{1}{\alpha^2} - \frac{1}{\beta^2}\right)\sum_{i=1}^3 R_i^2$$
$$+ \frac{1}{2\beta^2}\sum_{a=1}^8 R_a^2 + \frac{x}{2}\widetilde{\gamma}\left[1 - D_{88}(A)\right]$$
$$- \frac{3}{8\beta^2}\left(1 - \frac{1}{3}\chi^\dagger\chi\right)^2 + |\omega|\chi^\dagger\chi + H_{\text{hf}}, \tag{16}$$

where $R_a = \frac{\partial L}{\partial \Omega_a}$ defines the said Legendre transformation. The R_a are the right generators of $SU(3)$ since $[A, R_a] = A(\lambda_a/2)$ upon canonical quantization. The spinors χ contain annihilation and creation operators for the heavy meson bound state. They are quantized as ordinary harmonic oscillators. In particular $\chi^\dagger\chi$ is the number operator for the heavy meson bound state. Since we are considering hadrons with a single heavy quark, contributions that are quartic in χ have been omitted for consistency. (In the square a term that is explicitly of quartic order is maintained because it cancels a similar term in $\sum_a R_a^2$, cf. subsection below.) The hyperfine splitting part, H_{hf}, that emerges from the last term in Eq. (12), will be discussed later.

4.1. SU(3) diagonalization

The Hamiltonian, Eq. (16) is not complete without the constraint

$$Y_R = \frac{2}{\sqrt{3}}R_8 = 1 - \frac{1}{3}\chi^\dagger\chi, \tag{17}$$

that arises from the terms linear in Ω_8 in Eqs. (5) and (12). Thus the heavy baryons have right hypercharge $2/3$. Since the zero strangeness components of any $SU(3)$ representation has equal hypercharge and right hypercharge, the $SU(3)$ coordinates must be

quantized as diquarks for heavy baryons [15]. The most relevant diquark representations are the antisymmetric anti-triplet and the symmetric sextet.

When symmetry breaking is included, elements of higher dimensional representations with the same flavor and $R_{1,2,3}$ quantum numbers are admixed. We first determine the quantum number r in the intrinsic spin $\sum_{i=1}^3 R_i^2 = r(r+1)$: In addition to its dimensionality, an $SU(3)$ representation is characterized by two sets of quantum numbers (I, I_3, Y) for the flavor and (r, r_3, Y_R) for the R_a degrees of freedom, respectively. The flavor generators are $L_a = \sum_{b=1}^8 D_{ab} R_b$ with $L_{1,2,3} = I_{1,2,3}$ and $Y = \frac{2}{\sqrt{3}}L_8$ being the observables. Low-dimensional representations (such as the anti-triplet and the sextet) are non-degenerate and their elements with $Y = Y_R$ have $|\mathbf{I}| = |\mathbf{R}|$. Thus r equals the isospin (I) of the zero strangeness element within an $SU(3)$ representation: the anti-triplet has $r = 0$ and the sextet has $r = 1$. Symmetric and antisymmetric $SU(3)$ representations do not mix under symmetry breaking. Hence $r = 0$ and $r = 1$ for a heavy baryon whose diquark component builds up from the anti-triplet and sextet, respectively. The admixture of higher dimensional representations has been estimated in a perturbation expansion for hyperons [32] and heavy baryons [15]. It can also be done exactly within the so-called Yabu–Ando approach [5]. The starting point is an Euler angle representation of the collective coordinates A in which the conjugate momenta R_a are differential operators. Then the eigenvalue equation

$$\left\{\sum_{a=1}^8 R_a^2 + (x\widetilde{\gamma}\beta^2)\left[1 - D_{88}(A)\right]\right\}\Psi(A) = \epsilon\Psi(A) \tag{18}$$

is cast into a set of coupled ordinary second order differential equations. The single variable is the strangeness changing angle in A. The particular setting of the differential equations depends on the considered flavor quantum numbers. For ordinary baryons $(Y_R = 1)$ this treatment is reviewed in Ref. [10] and the results for diquark wave-functions that enter the heavy baryon wave-functions $(Y_R = 2/3)$ are reported in Ref. [33]. Having obtained the $SU(3)$-flavor eigenvalue ϵ from the differential equations we simplify the $SU(3)$ part and write

$$H(A, R_a, \chi) \longrightarrow H(\chi) = E_{\text{cl}} + \left(\frac{1}{\alpha^2} - \frac{1}{\beta^2}\right)\frac{r(r+1)}{2}$$
$$+ \frac{\epsilon}{2\beta^2} - \frac{3}{8\beta^2}\left(1 - \frac{1}{3}\chi^\dagger\chi\right)^2$$
$$+ |\omega|\chi^\dagger\chi + H_{\text{hf}}. \tag{19}$$

The dependence of the eigenvalues ϵ on x varies with spin and isospin. Hence there is implicit hyperfine splitting, however, it also appears explicitly as we discuss next.

4.2. Hyperfine splitting

The eigenstates of the Hamiltonian, Eq. (16) are combinations in which each term is a product of two factors, one is a function of A and the other of χ. The combinations are such that eigenstates of flavor and total spin are generated. The flavor information is completely contained in A because flavor transformations correspond to multiplying A by unitary matrices from the left. To construct total spin eigenstates we consider the effect of spatial rotations. The soliton is the hedgehog configuration and spatial rotations are equivalent to multiplying A by unitary $SU(2)$ matrices from the right. For the heavy meson bound state this multiplication must be compensated by an additional flavor transformation of the spinor χ. Thus the total spin is

Table 1

Model results for the mass differences of the charm and bottom baryons: $\Delta_N = M - M_N$, $\Delta_c = M - M_{\Lambda_c}$ and $\Delta_b = M - M_{\Lambda_b}$ with the M's computed from Eq. (22) in comparison with available experimental data. The spin and isospin of a considered baryon are I and j. The $SU(3)$ quantum number r is defined in the text. All data are in MeV. See text for explanation of question mark on Ξ_c.

(I, j, r)		$x = 25$				$x = 30$				Expt. [34]			
		Pos. par.		Neg. par.		Pos. par.		Neg. par.		Pos. par.		Neg. par.	
		Δ_N	Δ_c	Δ_N	Δ_c	Δ_N	Δ_c	Δ_N	Δ_c	Δ_N	Δ_c	Δ_N	Δ_c
$(0, 1/2, 0)$	Λ_c	1230	0	1479	249	1233	0	1482	249	1347	0	1653	306
$(1, 1/2, 1)$	Σ_c	1423	193	1664	434	1425	192	1666	433	1515	168	–	–
$(1/2, 1/2, 0)$	Ξ_c	1446	216	1695	465	1486	253	1735	502	1529	186	1851	504
$(0, 1/2, 1)$	Ω_c	1693	463	1934	704	1756	523	1997	764	1756	409	–	–
$(1/2, 1/2, 1)$	Ξ_c	1557	328	1798	569	1588	355	1829	596	1637	290	–	–
$(1, 3/2, 1)$	Σ_c	1464	234	1717	487	1466	233	1719	486	1579	232	–	–
$(1/2, 3/2, 1)$	Ξ_c	1598	369	1851	622	1629	396	1882	649	1706	359	1876	529(?)
$(0, 3/2, 1)$	Ω_c	1734	504	1987	757	1797	564	2050	817	1831	484	–	–
		Δ_N	Δ_b	Δ_N	Δ_b	Δ_N	Δ_b	Δ_N	Δ_b	Δ_N	Δ_b	Δ_N	Δ_b
$(0, 1/2, 0)$	Λ_b	4391	0	4560	168	4394	0	4563	168	4681	0	4973	292
$(1, 1/2, 1)$	Σ_b	4601	210	4771	380	4603	209	4773	379	4872	191	–	–
$(1/2, 1/2, 0)$	Ξ_b	4608	216	4776	385	4647	253	4816	421	4855	174	–	–
$(0, 1/2, 1)$	Ω_b	4871	480	5041	650	4935	540	5105	710	5110	429	–	–
$(1/2, 1/2, 1)$	Ξ_b	4736	345	4906	514	4766	372	4936	542	–	–	–	–
$(1, 3/2, 1)$	Σ_b	4617	226	4785	393	4619	225	4787	392	4983	212	–	–
$(1/2, 3/2, 1)$	Ξ_b	4751	360	4919	528	4782	387	4950	555	5006	325	–	–
$(0, 3/2, 1)$	Ω_b	4887	496	5055	664	4950	556	5118	724	–	–	–	–

$$\boldsymbol{J} = -\boldsymbol{R} - \chi^{\dagger} \frac{\boldsymbol{\tau}}{2} \chi \,. \tag{20}$$

Calling j the spin of the considered baryon this implies $\boldsymbol{R} \cdot \langle \boldsymbol{\tau} \rangle = j(j+1) - r(r+1) - \frac{3}{4} \sim j(j+1) - r(r+1)$, where the expectation value refers to the heavy meson bound state. In the approximation we have again omitted terms that formally are quartic in χ. This scalar product appears in the Legendre transformation with respect to $\boldsymbol{\Omega}$,

$$\frac{\partial L}{\partial \boldsymbol{\Omega}} \cdot \boldsymbol{\Omega} - \frac{1}{2\alpha^2}\boldsymbol{\Omega}^2 - \rho \chi^{\dagger}\left(\boldsymbol{\Omega} \cdot \frac{\boldsymbol{\tau}}{2}\right)\chi = \frac{1}{2\alpha^2}\boldsymbol{R}^2 + \frac{\rho}{\alpha^2}\boldsymbol{R} \cdot \chi^{\dagger}\frac{\boldsymbol{\tau}}{2}\chi \,. \tag{21}$$

Collecting pieces we get the mass formula

$$M = \left(\frac{1}{\alpha^2} - \frac{1}{\beta^2}\right)\frac{r(r+1)}{2} + \frac{\epsilon}{2\beta^2} - \frac{3}{8\beta^2}\left(1 - \frac{N}{3}\right)^2$$
$$+ |\omega|N + \frac{\rho}{2\alpha^2}[j(j+1) - r(r+1)]N \,, \tag{22}$$

where $N = 0, 1$ counts the number of heavy valence quarks contained in the considered baryon. It has been included in the hyperfine splitting term since ordinary baryons have $r = j$. We have collected the leading contributions to the baryon energy in the large number of colors (N_C) expansion. However, a contribution $\mathcal{O}(N_C^0)$ is missing, the vacuum polarization energy E_{vac}. It is the quantum correction to the classical energy E_{cl} that cannot be rigorously computed because the theory is not renormalizable. Estimates in the Skyrme model suggest that E_{vac} considerably reduces E_{cl} [35]. We circumvent this limitation by only considering mass differences for which E_{cl} and E_{vac} cancel and consequently omit these terms from Eq. (22).

This quantization scheme predicts two heavy Ξ baryons with spin $j = 1/2$: one has $r = 0$ and the other $r = 1$. In an $SU(3)$ symmetric world the former would be an anti-triplet state and the latter a sextet state. There is no mixing between these baryons because $[H, \boldsymbol{R}^2] = 0$. For $j = 3/2$ only one heavy Ξ baryon emerges in this scheme since then $r = 1$ is required. For the Ξ hyperon there is also only a single option with $j = 1/2$ that is build from the octet state. This counting suggests to relate r to the intermediate spin J_m defined in Ref. [11].

5. Numerical results

As mentioned above, we consider mass differences, because the model predictions for the absolute masses are subject to uncontrollable quantum contributions.

We find the energy eigenvalues ϵ in Eq. (18) for all baryons and then compute their energies according to Eq. (22). We adopt the $SU(3)$ parameters from Ref. [26]: $\alpha^2 = 5.144/\text{GeV}$, $\beta^2 = 4.302/\text{GeV}$ and[6] $\widetilde{\gamma} = 47$ MeV. For the heavy sector, the same soliton model was used in Ref. [14] to compute the bound state energies ω and hyperfine parameters ρ for both the P- and S-wave channels. From the model calculation described in Section 3 the following bound state parameters were obtained

$$\omega_P = 1326 \,\text{MeV}, \quad \rho_P = 0.140 \,,$$
$$\omega_S = 1572 \,\text{MeV}, \quad \rho_S = 0.181 \tag{23}$$

and

$$\omega_P = 4494 \,\text{MeV}, \quad \rho_P = 0.053 \,,$$
$$\omega_S = 4663 \,\text{MeV}, \quad \rho_S = 0.046 \tag{24}$$

in the charm and bottom sector, respectively (Ref. [14] lists the binding energies $\omega_{P,S} - M_D$ and $\omega_{P,S} - M_B$.). Then we are left with a single parameter, the effective symmetry breaking x defined in Eq. (13), that is not fully determined. We present our results for the charm and bottom baryon spectra in Table 1, that also contains the data for experimentally observed candidates [34]. We note that most of the quantum numbers listed in Ref. [34] are adapted from the quark model and stress that r is not a physical observable. Hence assigning the experimental results for Ξ type baryons to a particular r value is a prediction. Ref. [34] furthermore lists $\Lambda_c(2625)$ and $\Lambda_b(5920)$ with spin $j = 3/2$ that are not contained in our approach: We require $|j - r| = 1/2$ but these Λ's have neither strangeness nor isospin so they must have $r = 0$ and $j = 1/2$. We complete the picture by including the corresponding results for the low-lying non-heavy baryons in Table 2.

When comparing our model results to data in Table 1 and Fig. 1 we see that the mass differences within a given heavy quark sector

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[6] In the notation of Ref. [26] $\widetilde{\gamma} = (\delta'/\delta'')\gamma$.
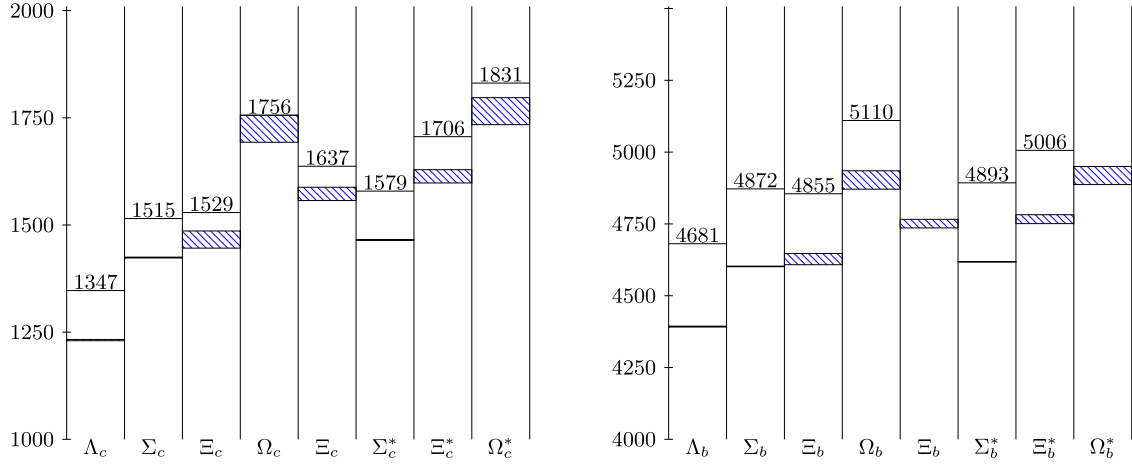
Fig. 1. (Color online.) Model results and experimental data for the mass differences of positive parity heavy baryons and the nucleon. Left panel: charm baryons, right panel: bottom baryons. The shaded areas are the model results for $x \in [25, 30]$ and data are indicated by lines and the number (in MeV) is written explicitly. As for ordinary hyperons, the asterisks denote total spin $j = \frac{3}{2}$. Note the different scales and off-sets. No experimental datum for Ω_b^* is available.

Table 2
Mass differences for non-heavy baryons with respect to the nucleon in MeV.

	Λ	Σ	Ξ	Δ	Σ^*	Ξ^*	Ω	$\Xi - \Sigma$
$x = 25$	134	218	320	324	438	551	661	101
$x = 30$	162	253	404	323	461	601	740	151
expt.	177	254	379	293	446	591	733	125

is overestimated. For example $M_{\Omega_c} - M_{\Lambda_c} = 463$ MeV for $x = 25$, while the empirical value is 409 MeV. Further increase of x worsens the picture. On the other hand, a sizable value ($x \sim 30$) for the symmetry breaking is required for a good agreement for non-heavy baryons. Simultaneously the splitting between different sectors is predicted on the low side. The Λ_c and Λ_b are about 100 MeV and 300 MeV too low, respectively. This is inherited from the heavy flavor calculation which overestimates the binding energies in the sense that it is too close to the estimate from exact heavy flavor symmetry. This can also be seen from the parity splitting which is underestimated by about 50 MeV (it vanishes in the heavy limit). Together with the effect of $SU(3)$ symmetry breaking the overestimated binding combines to acceptable agreement for the mass differences between the double strange baryons Ω_c and Ω_b and the nucleon, at least for $x = 30$. It has been argued [14] that kinematical corrections due to the soliton not being infinitely heavy change the predictions for $\omega_{P,S}$ appropriately. And indeed, replacing the heavy meson masses by the reduced mass built in conjunction with the classical soliton energy increases ω_P by roughly 100 MeV and ω_S by almost 200 MeV.

For $j = 1/2$ and positive parity there is an interesting effect in the Σ–Ξ system. The observed mass difference decreases and even changes sign when the heaviest flavor turns from strange via charm to bottom: $M_\Xi - M_\Sigma = 125, 14, -17$ MeV. Partially the model calculation reproduces this effect. For example, for $x = 25$ the mass differences 101, 23 and 6 MeV are predicted. Since the hyperfine splitting only has a moderate effect, the model exhibits a similar scenario for the negative parity channel. Unfortunately, there are no data to compare with.

Finally we discuss our results for the masses of those strange heavy baryons that have previously not been considered in a heavy meson soliton model with realistic heavy meson masses: the Ξ's and Ω's. For the positive parity heavy strange baryons we again observe that the mass splittings within a heavy multiplet are overestimated. A moderate reduction of the symmetry breaking ratio x would be sufficient to match the experimental data. For the neg-

ative parity Ξ_c with $j = 1/2$ the too large binding of the S-wave reverses this picture. This is not the case for its spin $3/2$ counterpart. Interestingly enough, Ref. [34] assigns the quantum number of this resonance by assuming it to join an $SU(4)$ multiplet with the negative parity $\Lambda_c (j = 3/2)$. We have argued above that this Λ_c is not contained in our approach but should be associated with a D-wave heavy meson. Thus, as indicated in Table 1, it is questionable to identify $(I, j, r, p) = (1/2, 3/2, 1, -)$ with $\Xi_c(2815)$. Rather it is a prediction for an even heavier resonance like the observed $\Xi_c(2930)$ or $\Xi_c(2980)$ whose quantum numbers still need to be determined [34].

6. Conclusion

We have presented a model calculation for the baryon spectrum that comprises light and heavy flavors. In particular we have focused on the role of light flavor symmetry breaking which is manifested by the strange quark being neither light nor heavy. When quantizing the flavor degrees of freedom, the corresponding deviations from the up-down sector are handled (numerically) exactly. In the heavy flavor sector the model is inspired by the heavy flavor symmetry, with subleading effects arising from finite masses included. The approach also includes the hyperfine splitting for the heavy baryons; a moderate effect that vanishes in the heavy limit. The model calculation is all-embracing as it contains spin $1/2$ and $3/2$ baryons starting from the lightest baryon (nucleon), including hyperons and extending to heavy baryons of either parity that have the valence quark content strange–strange–bottom. The spectrum is computed from a single mass formula where essentially all parameters are determined using data from the baryon number zero sector. We have also calculated masses for heavy baryons that are yet to be observed. Though we can only provide an estimate for their masses, we find a realistic indication for their positions relative to observed baryons.

The overall agreement with data is as expected for chiral soliton model estimates. As known from earlier studies, the mass predictions for the heavy baryons are on the low side when compared to the nucleon. Within a heavy baryon multiplet the computed mass differences are larger than the experimental data. This appears to be caused by too strong a remnant of the heavy spin–flavor symmetry in the approach. An understanding that goes beyond adopting reduced masses in the bound state approach is required. Furthermore the fine-tuning of the symmetry breaking ratio x as well as other model parameters that influence the soliton properties ap-

pears as an obvious endeavor. Actually, a complete analysis within a vector meson soliton model (but also a chiral quark model as the Nambu–Jona-Lasinio model [36]) shows that additional symmetry breaking operators such as $\sum_{i=1}^{3} D_{8i} R_i$ [26] arise in the Hamiltonian, Eq. (16). Their effects on the heavy baryon spectrum will be reported in a forthcoming paper.

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