Solitary wave interaction with porous structures

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Abstract

This paper presents the interaction of solitary waves with porous structures. To get the energy-dissipation, reflection, transmission, and diffraction of solitary waves to porous structures, we use Lee et al.’s (2014) model. They developed extended Boussinesq equations for waves in porous media considering inertial and drag resistances. When these resistances are neglected, the model is reduced to the extended Boussinesq equations of Madsen and Sørensen (1992). For the initial condition we use an analytical solution of solitary waves for Madsen and Sørensen’s (1992) Boussinesq equations. For verification, numerical results of Lee et al.’s (2014) model are compared to the numerical and experimental results of Lara et al. (2012) in a case of solitary waves normally incident to a semi-infinite porous breakwater. Numerical simulations are conducted for various cases such as obliquely incident solitary waves to a semi-infinite porous breakwater, solitary waves to a detached porous breakwater or through a porous breakwater gap.

1. Introduction

The interaction of nonlinear waves to porous breakwater plays an important role to coastal engineering. Rubble-mound breakwaters have been built to absorb and reduce wave energy transmitting into harbour. Rubble-
mound breakwaters are popular since they are easy to build, safe against severe wave climate, and efficient to protect facilities behind the breakwater. Solitary wave has been known having constant form when it propagates in a constant water depth over a long distance. In one-dimensional domain, when solitary waves interact with a porous breakwater, the solitary waves partly transmit through and reflect from the breakwater depending on the porosity and the width of the breakwater. In a wave basin, there are not only reflected and transmitted waves but also diffraction waves behind the porous breakwater. There are three types of diffraction waves, i.e., one from the incident wave to the transmitting wave and another from the incident wave into the calm water behind the transmitted wave in the lee of the breakwater and the other from the reflected wave to the calm water in front of the breakwater. Lynett et al. (2000) described the first two diffraction waves. The transmitted, reflected, and diffraction waves have the same form with solitary waves.

Liu and Wen (1997) simulated wave propagation through a porous breakwater using two types of equations, i.e., Peregrine’s (1967) equations outside the breakwater and their equations inside the breakwater. They used matching conditions at the interface. Lee et al. (2014) developed an extended Boussinesq model for waves propagating inside porous media. Their model can simulate waves inside and outside porous media simultaneously and can be applied from shallow to deep water (i.e., \( kh = 1.2\pi \)). In this study, we apply Lee et al.’s (2014) extended Boussinesq equations to simulate solitary waves interacting with a porous breakwater without using any matching conditions at the interface boundaries. The numerical solutions are verified by comparison with exact solutions and physical experiment data.

2. Governing equations

The extended Boussinesq equations of Lee et al. (2014) for waves propagating inside porous media are given by

\[
\frac{\partial \eta}{\partial t} + \nabla \cdot \left[ (h + \eta) \bar{u} \right] = 0
\]

\[
\left( \beta \frac{\partial}{\partial t} + \alpha \right) \bar{u} + g \eta + \beta \bar{u} \cdot \nabla \bar{u} + \frac{1}{6} \left( \beta \frac{\partial}{\partial t} + \alpha \right) h^2 \nabla (\nabla \cdot \bar{u}) - \left( \frac{1}{2} + \gamma \right) \left( \beta \frac{\partial}{\partial t} + \alpha \right) h \nabla \left[ \nabla \cdot (h \bar{u}) \right] - \gamma gh \nabla \left[ \nabla \cdot (h \nabla \eta) \right] = 0
\]

where \( \bar{u} \) is the depth averaged seepage velocity, \( \eta \) is the water surface elevation, \( g \) is the gravitational acceleration, \( h \) is the mean water depth, \( \gamma ( = 1/18 ) \) is a correction factor which is close to Madsen and Sørensen’s (1992) suggested value of 1/15, and \( \alpha \) and \( \beta \) are the drag and inertial coefficients, respectively, given by

\[
\alpha = \alpha_t \left( \frac{1 - \lambda}{\lambda} \right)^2 \frac{\nu}{d^2} + \alpha_r \frac{1 - \lambda}{\lambda} \frac{1}{d} |\bar{u}|
\]

\[
\beta = \lambda + (1 - \lambda) (1 + \kappa)
\]

\[
= 1 + (1 - \lambda) \kappa
\]

In the above equations, \( \alpha_r \) and \( \alpha_t \) are coefficients which represent the laminar and turbulent flow resistances, respectively, \( \lambda \) is the porosity, \( \kappa \) is the added mass coefficient, \( \nu \) is the kinematic viscosity of water, and \( d \) is the size of the porous material.

In unsteady flow, the inertial resistance term is necessary to consider the divergence and convergence of streamlines in the presence of the solid material. And, the resistance term should be considered in both the local and convective accelerations. In Eq. (4), the value one which is added to the added mass coefficient is to consider the inertial resistance of the water with the volume of solid material, and the added mass coefficient is to consider the inertial resistance in view of geometrical smoothness of the solid material.
For clean water (i.e., $\lambda = 1$), $\alpha = 0$ and $\beta = 1$ and the above governing equations are reduced to the conventional extended Boussinesq equations of Madsen and Sørensen’s (1992).

3. Initial and boundary conditions

The solitary waves are initially set outside the porous breakwater. And, the water surface elevation and particle velocity are initially set outside the porous breakwater. Wei and Kirby (1995) derived an approximation analytical solution for solitary waves using the extended Boussinesq equations of Nwogu (1993). Following their procedure, we derive the analytical solution of solitary waves for Madsen and Sørensen’s (1992) Boussinesq equations and use as an initial condition. The dimensional forms of the depth-averaged velocity and water surface elevation of the solitary waves are given by

$$u = A \text{sech}^2 \left[ K \left( x - Ct \right) \right]$$

$$\eta = A_{1} \text{sech}^2 \left[ K \left( x - Ct \right) \right] + A_{2} \text{sech}^2 \left[ K \left( x - Ct \right) \right]$$

where

$$A = \frac{C^2 - gh}{C}$$

$$A_{1} = \frac{(C^2 - gh) \left[ gh - 3\gamma \left( C^2 - gh \right) \right]}{gh \left[ C^2 - 3\gamma \left( C^2 - gh \right) \right]}$$

$$A_{2} = \frac{(C^2 - gh)^2 \left[ 2C^2 + 3\gamma \left( C^2 - gh \right) \right]}{2gh \left[ C^2 - 12\gamma \left( C^2 - gh \right) \right] \left[ C^2 - 3\gamma \left( C^2 - gh \right) \right]}$$

$$K = \frac{1}{h} \sqrt{\frac{3 C^2 - gh}{4 C^2}}$$

Liu and Wen (1997) and other authors used two types of governing equations to generate waves propagating through a porous breakwater. One is to generate waves outside the porous breakwater and the other for waves inside the porous breakwater. That’s why along the interface between the open-water and the porous breakwater they need to apply matching conditions for both free surface displacement and velocity. The free surface and
velocity as well as their spatial derivatives are continuous across the interfaces. However, in our study, in order to simulate waves propagating through a permeable breakwater we use only one type of the derived governing equations for waves propagating inside and outside porous media by adjusting the media porosity. That is, outside the breakwater (clean water area) we equate the porosity as unity and inside the breakwater (porous media) we equate the porosity less than unity, as can be seen in Fig. 1. So we do not need any matching conditions at the interface between the open-water and the porous breakwater.

4. Numerical verification

Lee et al (2014) developed and verified their model in 1-dimension domain. In this study, we apply their model to generate solitary waves in horizontal 2-dimensional domain.

Numerical results are compared to the experimental data of Lara et al. (2012) who conducted laboratory experiments in the University of Cantabria’s wave basin. The positions of the porous breakwater and wave gauges are presented in Fig.2 and Table 1. The characteristics of the breakwater are $\lambda = 0.51$, $\alpha_i = 1000$, $\alpha_e = 3$, $d = 15 \text{ cm}$, the width of the breakwater $w_b = 50 \text{ cm}$, the wave height $H = 9 \text{ cm}$, and the water depth $h = 40 \text{ cm}$ ($H/h = 0.225$).

A vertical wall is placed at the end of the wave basin. The full reflective boundary conditions at the wall are specified as

$$u_{m,x} = 0$$

(11)

where $m_x$ is the last point in $x$-direction and $j = 1, 2, ..., ny$ are grid points along $y$-direction.

We show the comparison of surface elevation between numerical and experimental results at 15 wave gauges as in Fig.3. Gauges 1, 14, and 15 show very good agreement of incident solitary wave with experimental data. The reflection from the breakwater is well captured at gauges 2, 3 and 4. The transmission waves through the breakwater are well modelled as shown in gauges 7 to 13. However, the reflected waves from the side walls and end wall are not very well captured as shown at the end of each time series due to reflections from the side and end walls of the wave basin. It should be noted that $t = 0$ sec is not the time that solitary wave is generated from the wave maker. It is the time that the measurements start.

<table>
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Fig.2. Computation layout for solitary waves interacting with porous breakwater (units are in meter).
Fig. 3. Comparison of water surface elevation of current model (solid line) with experimental data (dashed line) and numerical data (dotted line) of Lara et al. (2012) at 15 gauges.

To date, there have been no physical experiments for obliquely solitary wave interaction with neither porous infinite breakwater nor porous detached breakwater. This research shows some images of obliquely incident solitary wave interaction with porous infinite breakwater (Fig. 4), and normal incident solitary wave interaction with porous detached breakwater (Fig. 5). Three types of diffraction can be observed from these two figures. The conventional diffraction that can also be found when wave propagating to a solid breakwater appears clearly in Fig. 4 (t=7.5 sec) and Fig. 5 (at t=9 sec). The incident wave energy diffracts into the calm water behind the porous breakwater. From these figures the second type of diffraction is shown, that is the diffraction of the reflected waves in front of the breakwater. These two types of diffraction create circular crest lines. The last one that only appears when solitary wave propagates to porous breakwater is the diffraction of the incident wave into the transmitted wave, i.e., Fig. 4 (at t=7.5 sec) and Fig. 5 (at t=7.5 sec, and t=9. sec).

Fig. 4 (at t=2.5 sec, and 3.6 sec) show the development of stem wave when oblique incident solitary wave meets the right wall. The transmitted and reflected stem waves are recorded when they propagate to the porous breakwater.
Fig. 4. Oblique ($\theta = 20^\circ$) incident solitary wave interaction with porous semi-infinite breakwater at different time steps.
5. Conclusions

In this study the model Lee at al. (2014) for waves in porous media has been verified for two-dimensional problems and a new analytical solution for solitary waves using Boussinesq equations of Madsen and Sørensen (1992) has been derived. Using the new derived solitary waves as initial conditions, solitary waves were generated and interacted with porous structures. The numerical results of the normal incident solitary waves are very good agreement with the physical and numerical data of Lara et al. (2012). When obliquely incident solitary waves interact with porous structures, stem waves, refracted waves, transmitted waves, and diffraction waves are well observed.

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References