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On the chiral expansion for parton distributions at small $x \sim O(m_\pi^2)$

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ABSTRACT

In the framework of the chiral perturbation theory (χPT) we computed two- and three-loop corrections to the pion parton distributions which posses δ-function singularities at $x_{Bj} \to 0$. From this calculation one can conclude that in the region of small $x_{Bj} \sim m_\pi^2/(4\pi F_\pi)^2$ standard χPT needs in the resummation of all orders to get correct description of the PDFs at small-x region. We demonstrate an example of such resummation.

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1. Introduction

Recently chiral perturbation theory (χPT) has been applied to parton distribution functions (PDFs) [1–4] in order to determined the dependence of these quantities on the mass of Goldstone bosons (m_π) and/or external small momenta (ξ and t in the case of generalized parton distributions (GPDs)). In contrast to standard χPT, in this case one deals with hadronic matrix elements of non-local light-cone operators ($n^2 = 0$):

$$O(\lambda) = \bar{q}\left(\frac{1}{2}\lambda n\right)\gamma_+\left\{1, \frac{1}{2}\tau^a\right\}q\left(-\frac{1}{2}\lambda n\right). \tag{1}$$

Therefore apart from chiral expansion parameters (m_π, small external momenta) one has the additional scale λ—the distance characterizing the non-locality of the operator. Interesting theoretical question is to study the interplay of this additional scale with the chiral expansion parameters. In the present Letter we investigate this problem for the quark parton distributions in the pion.

For the case of the quark operators there exist two independent distributions corresponding to the two possible values of the isospin I: $I = 0$ defines the singlet distribution $Q(x)$ and $I = 1$ non-singlet one $q(x)$, which are normalized as:

$$\int_{-1}^{1} dx\, q(x) = 1, \qquad \int_{-1}^{1} dx\, x Q(x) = M_2^Q. \tag{2}$$

Here M_2^Q is the momentum fraction carried by quarks. For convenience, the explicit definitions of the PDFs in terms of the matrix elements of the light-cone operators are presented in Appendix A.

The standard chiral expansion of these PDFs implies:

$$Q(x) = Q^{(0)}(x) + a_\chi Q^{(1)}(x) + a_\chi^2 Q^{(2)}(x) + \cdots, \tag{3}$$

$$q(x) = q^{(0)}(x) + a_\chi q^{(1)}(x) + a_\chi^2 q^{(2)}(x) + \cdots, \tag{4}$$

where $a_\chi = (m_\pi/4\pi F_\pi)^2$ ($F_\pi \approx 93$ MeV is the pion decay constant) is the chiral expansion parameter. Important observation is that in the above formulae one assumes that the momentum fraction x has no chiral power counting, i.e.

$$x \sim \mathcal{O}(a_\chi^0). \tag{5}$$

The one-loop non-analytic contributions have been computed in many papers [1–4]. They read:

$$Q^{(1)}(x) = 0 \times \ln[1/a_\chi] + \mathcal{O}(a_\chi), \tag{6}$$

$$q^{(1)}(x) = \left\{q^{(0)}(x) - \delta(x)\right\}\ln[1/a_\chi] + \mathcal{O}(a_\chi). \tag{7}$$

We observe that isovector PDF $q(x)$ has specific δ-singular behavior at small momentum fraction. This can be understood as following: for some small values of the momentum fractions $x \sim a_\chi$ the next-to-leading singular term formally is of the same order as the leading term $a_\chi\delta(x) \sim \mathcal{O}(a_\chi^0)$. Moreover, higher orders of χPT may posses even more singular structures, like derivatives of the δ-function: $\delta^{(n)}(x)$. This may indicate that in this situation the usual chiral expansion is not valid anymore. The question is: How to write correctly the chiral expansion for $x \sim a_\chi$ (equivalently for the large light-cone distance $\lambda \sim 1/a_\chi$)? Is the chiral expansion for this case controlled by the finite order of the standard χPT?

It is clear that in order to construct the improved chiral expansion for the region $x \sim a_\chi$ one has to perform resummation of all δ-singular terms. Then such reordering of the χPT expansion may lead to generation of the some finite size function instead of singular $\delta(x)$ and therefore provides more correct chiral description of the small-x behavior.

Let us formulate the above suggestion more accurately. We shall assume, that performing the chiral limit at fixed value of the momentum fraction x one obtains the standard chiral expansions (3),

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(4) which can be rewritten as a sum of the regular $f^{\text{reg}}(x)$ and δ-singular parts

$$Q(x) = Q^{\text{reg}}(x) + \delta'(x)D_1 + \delta'''(x)D_3 + \cdots = Q^{\text{reg}}(x) + \sum_{i\geqslant 1,\text{odd}} D_i\delta^{(i)}(x), \tag{8}$$

$$q(x) = q^{\text{reg}}(x) + \delta(x)D_0 + \delta''(x)D_2 + \cdots = q^{\text{reg}}(x) + \sum_{i\geqslant 0,\text{even}} D_i\delta^{(i)}(x), \tag{9}$$

where the regular contributions can be represented in the framework of χPT by series similar to (3), (4) and where each term in the expansions is represented by some smooth function of the momentum fraction x. The coefficients D_i in front of derivatives of the δ-functions are some constants which also can be expanded with respect to small parameters a_χ:

$$D_i = D_i^{(0)} + a_\chi D_i^{(1)} + a_\chi^2 D_i^{(2)} + \cdots. \tag{10}$$

We shall assume that PDFs are smooth functions in the chiral limit, i.e., for all values $i = 0, 1, \ldots$

$$D_i^{(0)} = 0. \tag{11}$$

Then the singular terms can occur only from the corrections induced by the loop diagrams of the chiral perturbation theory. The power of the derivatives is dictated by the symmetry properties of the given PDF. Recall that singlet (non-singlet) pion PDFs can be understood as antisymmetrical (symmetrical) function with respect to exchange $x \leftrightarrow -x$ being extended to the whole region $-1 < x < 1$.

In order to check the assumption (8) or (9) it would be interesting to see that the singular contributions $\sim \delta^{(n)}(x)$ can occur at least in the first few orders of χPT expansion. In next section we compute such singular contributions to the PDFs and find the explicit contributions of the type $[a_\chi \ln(1/a_\chi)]^{n+1}\ \delta^{(n)}(x)$ for the cases $n = 1, 2$. We shall see that the mechanism of the generation of δ-functions is quite general and should work also at higher orders. In Section 4 we briefly discuss the main consequences which follow from representations (8) and (9) and possibilities to perform the resummation of the singular contributions. On the concrete example we demonstrate that resummed PDF is given by finite size function $f(x/a_\chi)$ which can be represented by the infinite sum of the δ-functions being expanded with respect to a_χ.

2. The singular δ-function contributions in ChPT up to three loops

Our goal is to investigate the higher orders of the chiral expansion in order to establish the existence δ-singular terms. We shall accept following strategy: we restrict our consideration only to leading non-analytic contributions. Such approach allows us to perform all calculations in the framework of the leading order chiral Lagrangian

$$\mathcal{L}_{(2)} = \frac{1}{4}F_\pi^2 \operatorname{tr}\left(\partial^\mu U \partial_\mu U^\dagger + \chi^\dagger U + \chi U^\dagger\right), \tag{12}$$

for the pion dynamics and leading order chiral representation for the light-cone operators (1). For convenience, all necessary technical details and definitions are presented in Appendix A. The accuracy of our approach, despite we are working at higher orders, allows us to neglect the renormalization of the pion mass and chiral couplings, hence below we always assume their physical values.

The calculation of the contributions of double chiral logarithms is well-known subject and have been already discussed in the literature, see for instance [5]. The main observation is that such terms occur due to the UV-divergencies of the χPT diagrams and can be effectively computed if one uses the basic property of locality of the UV-counterterms.

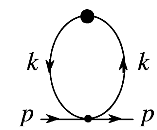

Fig. 1. One-loop diagram which generates δ-contribution to the pion PDF. The large blob denotes two-pion non-local operator vertex generated by corresponding chiral Lagrangian for the light-cone operator.

We shall use dimensional regularization with space–time dimension $d = 4 - 2\varepsilon$ in order to compute the bare matrix elements. For the one-loop diagram the coefficient in front of chiral logarithm is determined completely by the UV-pole, therefore it is enough to compute only the $1/\varepsilon$-pole part of the graph. This is clear from the general structure for the unrenormalized matrix element:

$$\int \frac{d\lambda}{2\pi} e^{-ip_+x\lambda} \langle p|O(\lambda)|p\rangle|_{1\text{loop}} \equiv \left\langle O^{(1)}\right\rangle = a_\chi \frac{1}{\varepsilon}\left(\frac{\mu_\chi^2}{m_\pi^2}\right)^\varepsilon D_0^{(1)}\,\delta(x) + \cdots \tag{13}$$

$$= a_\chi \left(\frac{1}{\varepsilon} + \ln\left[\frac{\mu_\chi^2}{m_\pi^2}\right]\right) D_0^{(1)}\delta(x) + \cdots, \tag{14}$$

where the chiral scale $\mu_\chi \sim 4\pi F_\pi$. The interesting for us δ-term is generated from the diagram in Fig. 1. Let us describe shortly corresponding calculations. The expression for the diagram is given

$$G_1 = i\varepsilon[abc]a_\chi q^{(0)}(\beta) * \int dk \frac{k_+\delta(xp_+ - \beta k_+)}{m_\pi^2[k^2 - m_\pi^2]^2}\left[-4(kp) + \cdots\right], \tag{15}$$

where the dots in the brackets denote irrelevant (regular) contributions from the four-pion vertex and by the asterisk we denote the convolution integral with respect to β. We also neglected various terms with ε which can provide only the finite terms and denote $dk \equiv d^dk/\pi^{d/2}$. Obviously, the integral in (15) is quadratically divergent. Moreover, from the structure of the denominator and numerator one observe that k_+ in the argument of the δ-function can be ignored due to the rotation invariance and one obtains

$$G_1 = -i\varepsilon[abc]a_\chi \delta(x) \int dk \frac{k^2}{m_\pi^2[k^2 - m_\pi^2]^2}, \tag{16}$$

where we used the normalization condition (2). The integral over the momentum k can be easily computed and the result defines the constant $D_0^{(1)}$ which reproduces the contribution with the δ-function from (7).

The next important observation is very simple. One can expect that at the higher orders the non-trivial contributions to the coefficients D_i can be generated in similar way if the four-pion subgraphs generates the scalar products $(k\cdot p)^i$ with the highest possible power i in the numerator. Then the $i+1$-loop graph for the forward matrix element includes the following master integral

$$G_{i+1} \sim \frac{1}{\varepsilon^i} a_\chi^{i+1} q^{(0)}(\beta) * \int dk \frac{k_+\delta(xp_+ - \beta k_+)}{[k^2 - m_\pi^2]^2} \frac{(k\cdot p)^{(i+1)}}{m_\pi^{2(i+1)}} + \cdots. \tag{17}$$

This integral can be computed in the same way as before, but now one has to perform the expansion of the δ-function with respect to k_+ in order to obtain factor k_+^i which is necessary in order to contract momentum k in the scalar products. Hence the master

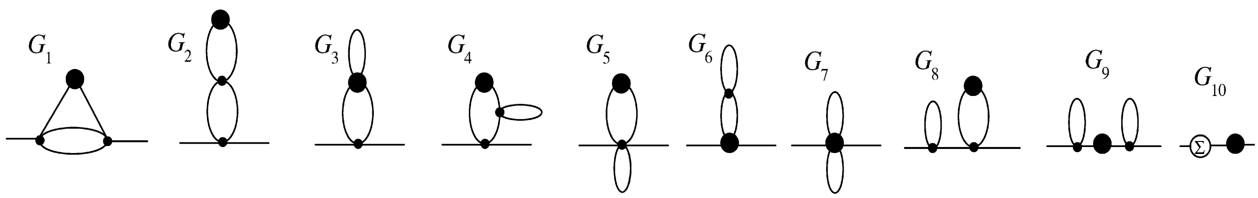

Fig. 2. Two-loop diagrams with the two-pion operator vertex contributing to the chiral expansion of the pion parton distributions.

integral can provide contribution with $\sim \delta^{(i)}(x) a_\chi^{i+1}$. At the same time, the integral is UV-divergent and therefore the result contains the contribution with the maximal power of the logarithm $\sim a_\chi^{i+1} \ln^{i+1}[1/a_\chi]$. This is exactly the contribution which we want to compute. All this is an illustration of the main idea how the higher derivatives of the δ-function may appear at the higher orders. In order to perform the explicit calculation of the model independent logarithmic contributions one has to take into account the presence of the subdivergencies in the multiloop graphs. The idea of the method is quite general and based on the locality of the UV-counterterms. For the two-loop case it was discussed in Ref. [5] but can be easily extended to arbitrary order of χPT. For convenience, we provide below the technical analysis for the two-loop calculation of chiral logarithms in our case.

The renormalized matrix element of an operator to two-loop accuracy can be written as:

$$\langle O_R \rangle = t^{2\varepsilon} \langle O^{(2)} \rangle + \frac{1}{\varepsilon} t^{\varepsilon} \langle \tilde{O}^{(1)} \rangle + \frac{1}{\varepsilon^2} \langle \tilde{O}^{(0)} \rangle. \tag{18}$$

Let us clarify the rhs of Eq. (18). The first term $\langle O^{(2)} \rangle$ is the bare matrix element (two-loop graph). The second term $\frac{1}{\varepsilon} \langle \tilde{O}^{(1)} \rangle$ denotes subtractions of the one-loop subdivergencies associated with the operator vertex or pion Green functions. The one-loop reduced matrix element $\langle \tilde{O}^{(1)} \rangle$ includes effective vertex from the corresponding one-loop counterterm. And the third term is proportional to the tree level matrix element $\langle \tilde{O}^{(0)} \rangle$ of some effective operator. This term corresponds to the local counterterm, which removes the total divergency. The dimensionless combination $t = \frac{\mu_\chi^2}{m_\pi^2}$ is introduced for convenience. To two-loop accuracy the structure of divergencies of each contribution in Eq. (18) has the following form:

$$\langle O^{(2)} \rangle = \frac{1}{\varepsilon^2} g_2 + \frac{1}{\varepsilon} g_1 + g_0, \qquad \langle \tilde{O}^{(1)} \rangle = \frac{1}{\varepsilon} h_1 + \cdots, \tag{19}$$

where functions g_i and h_i are polynomials of external momenta and the pion mass. Inserting these expansions into (18) one obtains

$$\langle O \rangle_R = \frac{1}{\varepsilon^2} g_2 \left[1 + 2\varepsilon \ln t + 2\varepsilon^2 \ln^2 t\right] + \frac{1}{\varepsilon^2} h_1 \left[1 + \varepsilon \ln t + \frac{1}{2} \varepsilon^2 \ln^2 t\right] + \cdots \tag{20}$$

$$= \frac{1}{\varepsilon^2} [g_2 + h_1] + \frac{1}{\varepsilon} [2g_2 + h_1] \ln t + \underbrace{\frac{1}{\varepsilon}[\cdots]}_{\text{local terms}} + \ln^2 t \left[2g_2 + \frac{1}{2} h_1\right] + \cdots. \tag{21}$$

Now, the non-local $1/\varepsilon$ divergency proportional to $\ln t$ must cancel (locality of counterterms), that provides the relation:

$$2g_2 + h_1 = 0 \quad \Rightarrow \quad g_2 = -\frac{1}{2} h_1, \tag{22}$$

i.e., the residue of the $1/\varepsilon^2$ is defined in terms of the one-loop diagrams with insertions of one-loop counterterms. This defines completely the coefficient in front of double logarithmic contributions:

$$2g_2 + \frac{1}{2} h_1 = -\frac{1}{2} h_1. \tag{23}$$

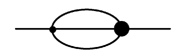

Fig. 3. Two-loop diagram with the true four-pion operator vertex (without tadpoles) contributing to the chiral expansion of the pion parton distributions. The singular δ-functions are absent in the contribution from this graph, see explanation in the text.

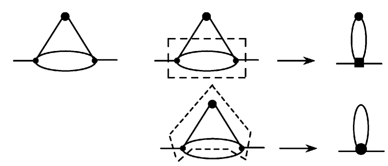

Fig. 4. One-loop sub-divergencies of the two-loop diagram G_1 and corresponding counterterm. The filled circle is an operator counter, filled square is the 4-pion Green function counterterm.

Therefore, in order to define the contribution of the type $\sim \ln^2(m_\pi^2)$ we can compute only $1/\varepsilon^2$ pole in the diagrams with insertion of the one-loop counterterms. In addition, we have to select such structures, which can produce δ-functions. From the observation considered above, one can expect that the two-loop diagrams can contain master integrals Eq. (17) with the power $i = 1$, hence such diagrams are relevant for the computation of the coefficient D_1 in front of $\delta'(x)$ from (8) ($I = 0$).

At the NNLO of χPT we have set of the two-loops diagrams presented in Figs. 2 and 3. Diagrams G_3–G_9 are insertions of the tadpole loops to the one-loop diagrams. From simple analysis one can conclude that these diagrams do not contain the master integral like (17) with the maximal power $i = 2$ for scalar product $(k \cdot p)$ because of dimensional reasons: the tadpole integral is proportional to the pion mass $\sim m_\pi^2$ and therefore reduces the possible power i by one unit. It means that these diagrams have the same singular structures as the one-loop graphs, i.e., they can have only $\delta(x)$ terms without derivatives. The more singular terms may appear only from the first two diagrams $G_{1,2}$.

Consider these graphs. There are two one-loop divergent subgraphs in these diagrams: operator vertex subgraph and pure pion subgraph. They are depicted in Fig. 4. There are two corresponding counterterms:

$$\frac{1}{\varepsilon} t^{\varepsilon} \langle \tilde{O}^{(1)} \rangle = \frac{1}{\varepsilon} t^{\varepsilon} \langle \tilde{O}_V^{(1)} \rangle + \frac{1}{\varepsilon} t^{\varepsilon} \langle \tilde{O}_\pi^{(1)} \rangle. \tag{24}$$

Computation of these contributions provides following result

$$D_1 = \left(-\frac{5}{3}\right) M_2^Q \ln^2\left[\frac{\mu_\chi^2}{m_\pi^2}\right]. \tag{25}$$

No new singular structures of the isovector PDF $q(x)$ appear, as the isovector PDF is symmetric.

Let us briefly discuss the diagram on Fig. 3. This diagram includes the four-pion operator vertex.[1] Such vertex can be generated either from the operators (A.6) and (A.7) or from the multiparticle terms which we do not write explicitly in Eqs. (A.6), (A.7).

[1] We do not consider here the operator vertices with the tadpole loops as the true multiparticle vertices.

These additional multiparticle terms of the chiral operator include new chiral constants and do not generate two-pion vertices. Their expansion begins from the four-pion contributions as a minimum. We checked that the diagram Fig. 3 does not produce the contributions with the δ-functions in the case when the vertex is generated from the operators (A.6) and (A.7). This can be seen from the fact that one pion from the operator vertex describes the external state and therefore provides the non-trivial argument for the distribution of the fraction x that excludes occurrence $\delta(x)$. The contributions of the multiparticle terms with the new chiral constants have been ignored for the same reason.

Now let us briefly discuss the general mechanism of generation δ-singular terms. It turned out, that such terms appear only from the diagrams with four-pion counterterms and only in the diagram G_1. We checked that the operator vertex counterterm, provides only simple $\delta(x)$ without derivatives. The term with the pion counterterm exactly reproduces the master integral (17) with $i = 2$. It also naturally explains the absence of $\delta'(x)$ at one loop: the four-pion vertex is quadratically divergent and in the corresponding master integral can occur only simple δ-function structure considered in Eq. (15). Note also that the δ-producing master integrals have always maximal value of the UV-divergency index, i.e., power divergent terms play the key role in the generation of the δ-singular contributions.

Previous analysis can be easily extended to the three-loop diagrams. Following the method discussed above one can again reduce the calculation of the $a_\chi^3 \ln^3[1/a_\chi]$ contribution to the reduced one-loop diagram with effective vertex corresponding to the insertion of the two-loop counterterms.

We again have the subset of the graphs with the four-pion operator vertices shown in Fig. 5. In the case of isospin $I = 1$ such vertices appear only from the multiparticle part of the effective twist-2 operator with the new chiral constants. Therefore such contribution without loss of generality can be considered separately. In this Letter we shall ignore them what, however, cannot change our main conclusions.

The set of the "two-pion" relevant graphs is given in Fig. 6. In Fig. 7 we show the two-loop subgraphs for the case of diagram G_1 and resulting one-loop graphs with effective vertices. We do not show all diagrams with tadpole loops and wave function renormalization. It is clear that such contributions can produce only δ-functions which occur in the one- and two-loop graphs. But now we are interested in the new, more singular terms with the second derivative $D_2\delta^{(2)}(x)$ in Eq. (9). The main steps of the calculation are the same as for the two-loop case. Again the singular contributions occur only from the reduced diagrams with effective vertex generated by the four-pion subgraphs (the upper line in Fig. 7). The substitution of the effective counterterm vertex reduce the calculation to the master integral (17). Only the diagrams G_1, G_4 and G_5 provides non-trivial contribution to the coefficient D_2. The three-loop result is

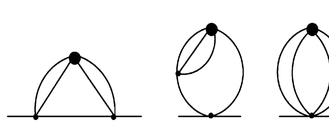

Fig. 5. Three-loop diagrams with the four-pion operator vertices.

$$D_2 = a_\chi^3 \ln^3[1/a_\chi]\langle x^2\rangle\left(-\frac{25}{108}\right), \tag{26}$$

where $\langle x^2\rangle$ denotes the moments of the PDFs in the chiral limit

$$\langle x^{n-1}\rangle = \int_0^1 dx\, x^{n-1} q^{(0)}(x). \tag{27}$$

It is obvious, that given analysis can be continued further order by order in χPT. We do not find any special reasons that could suppress contributions with the higher derivatives $\delta^{(n)}(x)$ in (8) and (9). Hence, we may conclude that structure of the chiral expansions for pion PDFs proposed in formulas (8) and (9) is established.

3. Calculation of the singular terms in the pion GPDs

In this section we discuss shortly the results for the singular contributions to the pion GPDs, counterparts of the δ-contributions to the pion PDFs, see Eq. (37). We shall consider, for simplicity, the limit

$$t \to 0, \qquad \xi \neq 0. \tag{28}$$

Logarithmic one-loop calculation gives [2,4]:

$$\begin{aligned} &H^{I=1}(x,\xi)\big|_{\xi\neq 0, t=0} \\ &\quad = \mathring{H}^{I=1}(x,\xi)\big(1 + a_\chi \ln[1/a_\chi]\big) \\ &\qquad - a_\chi \ln[1/a_\chi]\frac{\theta(|x| \leqslant \xi)}{\xi}\mathring{\varphi}_\pi\left(\frac{x}{\xi}\right). \end{aligned} \tag{29}$$

Here the second term in the forward limit ($\xi \to 0$) tends to the $\delta(x)$ contribution for the pion PDF

$$\frac{\theta(|x| \leqslant \xi)}{\xi}\mathring{\varphi}_\pi\left(\frac{x}{\xi}\right) \xrightarrow{\xi\to 0} \delta(x). \tag{30}$$

In order to obtain above result we used that pion distribution amplitude $\mathring{\varphi}_\pi(z)$ in the chiral limit, due to the soft-pion theorem [6], can be represented in terms of double distribution as

$$\mathring{\varphi}_\pi(z) = F^{I=1}(\beta,\alpha) * \delta(\alpha + \beta - z). \tag{31}$$

The isoscalar pion GPD does not have logarithmic contribution at the one-loop order. Calculations of the singular higher loop contribution goes along the line described in previous section, the final result is as follows:

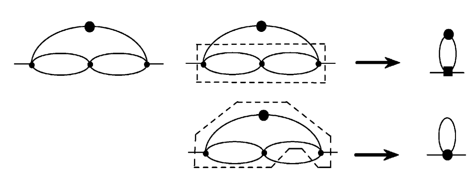

Fig. 7. Diagram G_1 (left) and two reduced graphs with effective vertices (right) which have to be computed in order to define δ-function logarithmic contribution. By the dashed line we show the two-loop subgraphs which has to be substituted by its UV-counterterm.

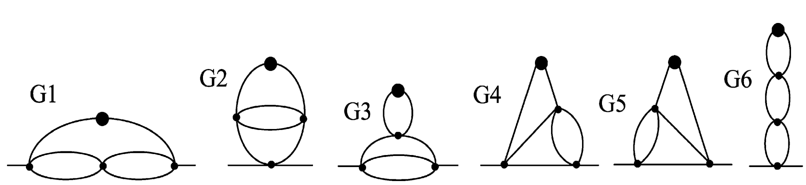

Fig. 6. Three-loop diagrams which may produce δ-function contributions.

$$H^{I=0}(x,\xi) = H^{I=0}_{\rm reg}(x,\xi) + \frac{5}{3}a_\chi^2\ln^2[1/a_\chi]\frac{\theta(|x|\leqslant\xi)}{\xi^2}\,D\left(\frac{x}{\xi}\right), \tag{32}$$

$$H^{I=1}(x,\xi) = H^{I=1}_{\rm reg}(x,\xi) - \theta\big(|x|\leqslant\xi\big)\left\{a_\chi\ln[1/a_\chi]\frac{1}{\xi}\mathring{\varphi}_\pi\left(\frac{x}{\xi}\right) - \frac{25}{108}a_\chi^3\ln^3[1/a_\chi]\frac{1}{3\xi^3}\psi\left(\frac{x}{\xi}\right)\right\}. \tag{33}$$

Here $D(z)$ is the so-called D-term [7], which in the chiral limit, due to the soft-pion theorem [6], can be expressed as:

$$D(z) = 2F^{I=0}(\beta,\alpha) * \delta(\alpha+\beta-z), \tag{34}$$

and function $\psi(z)$ is expressed in terms of double distribution as follows:

$$\psi(z) = \lim_{\eta\to1}\frac{\partial^2}{\partial\eta^2}\eta F^{I=1}(\beta,\alpha) * \beta\,\delta(\alpha+\beta\eta-z). \tag{35}$$

One can easily check that in the forward limit ($\xi\to0$) Eq. (32) is reduced to the result for the pion PDFs (37).

The singular terms of GPDs (32) result in the contributions to the amplitudes for hard exclusive processes of the following type:

$$\sim\left(\frac{a_\chi\ln(1/a_\chi)}{\xi}\right)^n, \quad n=1,2,3,\ldots. \tag{36}$$

This clearly demonstrates that any finite order of chiral corrections to physical observables at $\xi\sim a_\chi\ln(1/a_\chi)$ gives contribution which is not suppressed by small chiral parameter. Therefore in this region of kinematical variable ξ one needs to perform infinite order resummation of the chiral perturbation theory.

4. Discussion

In present Letter we have demonstrated that chiral expansion of the pion PDFs includes the contributions of the δ-function and its derivatives. We have also computed the leading non-analytic contributions to the coefficients $D_{1,2,3}$ in (8) and (9). The results read:

$$Q(x) = Q^{\rm reg}(x) + \left(-\frac{5}{3}\right)M_2^Q a_\chi^2\ln^2[1/a_\chi]\delta(x) + \mathcal{O}\big(a_\chi^3\big), \tag{37}$$

$$q(x) = q^{\rm reg}(x) - a_\chi\ln[1/a_\chi]\delta(x) + \langle x^2\rangle\left(-\frac{25}{108}\right)a_\chi^3\ln^3[1/a_\chi]\delta''(x) + \mathcal{O}\big(a_\chi^4\big). \tag{38}$$

It is also clear that higher order terms of the chiral Lagrangian for pion dynamics $\mathcal{L}_{6,8,\ldots}$ and for the light-cone operators can also produce δ-singular terms but they are suppressed by powers of the small chiral parameter with respect to *leading chiral logarithms* ($\sim[a_\chi\ln(1/a_\chi)]$) by additional powers of a_χ.

From the obtained results we can conclude that in the region of small momentum fraction x application of the standard chiral expansion faces with the difficulties. In this region one cannot truncate the perturbation expansion and hence need to perform some sort of resummation. This reorganization of the chiral expansion may combine the singular contributions to some generating function $f(z)$:

$$\sum D_n\varepsilon^n\delta^{(n)}(x) = f(x/\varepsilon), \tag{39}$$

where small parameter $\varepsilon = a_\chi\ln[1/a_\chi]$. The function $f(z)$ is some stable in the chiral limit function, which, probably, can be defined in the whole real axes $-\infty<z<\infty$ and generates contributions to the moments in the form

$$\int_{-1}^{1}dx\,x^{n-1}f(x/\varepsilon) \approx \varepsilon^n\int_{-\infty}^{\infty}dz\,z^{n-1}f(z) \simeq \varepsilon^n f_n = \varepsilon^n(-1)^n(n-1)!D_{n-1}. \tag{40}$$

The obvious conclusion which follows from this discussion is that any finite order calculation of the PDF moments cannot provide enough information in order to reconstruct PDF in the whole region of variable x. One has to perform resummation of singular δ-function terms but this is equivalent to calculation of all orders χPT.

Of course, such resummation is very difficult task. Even if we restrict our consideration only by the leading non-analytic terms we must compute straightforwardly order by order all terms D_i which is not possible. Hence one has to introduce some simplifications or models in order to achieve the goal. As an example, let us provide here the result of such simplified resummation in the leading order of the $1/N$ expansion, where N is number of pions ($N=3$ for the real two flavor QCD). This expansion is obtained by the generalization of the $SU(2)\times SU(2)=O(4)$ chiral model to the $O(N+1)$ model. The latter model can be solved in the large N limit. The result of the calculations for the isovector PDF is[2]:

$$q(x) \simeq q_{\rm reg}(x) - \frac{2}{N}\theta\big(N\varepsilon-|x|\big)\int_{|x|/(N\varepsilon)}^{1}\frac{dz}{z}q^{(0)}(z). \tag{41}$$

[Note, that parametrically $\varepsilon = 1/2a_\chi\ln[1/a_\chi]\sim1/N$.] This example of chiral resummation shows clearly that calculation of any finite order of χPT contributions to the Mellin moments of PDF does not allow to restore correctly the distribution function for $x<3/2a_\chi\ln[1/a_\chi]$ (numerically for the physical pion mass $x<0.09$ that is not very small). For the chiral extrapolation of the lattice data for PDFs this implies that usage of the moments without resummation (see e.g. [9]) can provide us the incomplete information about parton distributions in the region of $x\sim a_\chi\ln[1/a_\chi]$. In other words, one can use χPT in order to compute systematically the local moments of the PDFs but one cannot use these moments in order to describe the PDFs in the region of the small momentum fraction where all orders of the χPT contribute to the same accuracy.

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Appendix A. Twist-2 operators and the matrix elements

In this section we briefly describe the definitions and some technical details used in the Letter. We introduce two light-like vectors n, $\bar{n}$:

$$n^2=\bar{n}^2=0, \qquad n\cdot\bar{n}=1, \qquad a_+=a\cdot n. \tag{A.1}$$

There exist two QCD quark light-cone operators of twist-2:

$$P_R=\frac{1}{2}(1-\gamma_5), \qquad P_L=\frac{1}{2}(1+\gamma_5), \tag{A.2}$$

$$[O_R]_{fg} = \bar{q}_g\left(\frac{1}{2}\lambda n\right)\gamma_+P_Rq_f\left(-\frac{1}{2}\lambda n\right), \tag{A.3}$$

$$[O_L]_{fg} = \bar{q}_g\left(\frac{1}{2}\lambda n\right)\gamma_+P_Lq_f\left(-\frac{1}{2}\lambda n\right), \tag{A.4}$$

where indexes f, g stand for flavor. These operators transform under global chiral rotations as

$$O_L\to V_LO_LV_L^\dagger, \qquad O_R\to V_RO_RV_R^\dagger. \tag{A.5}$$

[2] Details of the calculations and discussion will be given elsewhere [8].

In χPT these QCD operators are described by effective chiral operator with unknown chiral constants. In the pure pion sector (U as usually denotes pion field) one finds [2–4]

$$O^L_{fg}(\lambda) = -\frac{iF_\pi^2}{4}\mathcal{F}(\beta,\alpha) * \left[U\left(\frac{\alpha+\beta}{2}\lambda n\right) n\cdot \overleftrightarrow{\partial}\, U^\dagger\left(\frac{\alpha-\beta}{2}\lambda n\right)\right]_{fg}, \tag{A.6}$$

$$O^R_{fg}(\lambda) = -\frac{iF_\pi^2}{4}\,\mathcal{F}(\beta,\alpha) * \left[U^\dagger\left(\frac{\alpha+\beta}{2}\lambda n\right) n\cdot \overleftrightarrow{\partial}\, U\left(\frac{\alpha-\beta}{2}\lambda n\right)\right]_{fg}, \tag{A.7}$$

where by asterisk we denote the integral convolution with respect to β and α:

$$\mathcal{F}(\beta,\alpha) * O(\beta,\alpha) \equiv \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha\, \mathcal{F}(\beta,\alpha) O(\beta,\alpha). \tag{A.8}$$

Here $\mathcal{F}(\beta,\alpha)$ represents the real generating function of the tower of low-energy constants and $\overleftrightarrow{\partial}_\mu$ denotes combination of derivatives $\overrightarrow{\partial_\mu} - \overleftarrow{\partial_\mu}$. The non-local structure of the operators (A.6) and (A.7) implies that the light-cone distance λ has chiral counting $\lambda \sim \mathcal{O}(p^{-1})$ as suggested in [2]. Then the combination $\lambda\partial_+$ is dimensionless that provides the non-locality with respect to light-cone direction. Important to note that given Lagrangian is not complete: the expressions (A.6) and (A.7) describe correctly only operators vertices with two attached pions (including the tadpoles loops). The low-energy constants $\mathcal{F}(\beta,\alpha)$ are characteristics of the structure of the pion, they are not determined in the effective field theory. According to the isospin $I = 0, 1$ one can construct two independent functions:

$$F^{I=0}[\beta,\alpha] = \frac{1}{2}\big(\mathcal{F}[-\beta,\alpha] - \mathcal{F}[\beta,\alpha]\big), \tag{A.9}$$

$$F^{I=1}[\beta,\alpha] = \frac{1}{2}\big(\mathcal{F}[-\beta,\alpha] + \mathcal{F}[\beta,\alpha]\big), \tag{A.10}$$

which are convenient to describe the pion matrix elements. Pion PDFs are defined as

$$\int \frac{d\lambda}{2\pi} e^{-ip_+ x\lambda} \big\langle \pi^b(p) \big| \mathrm{tr}\big[\tau^c O_{L+R}(\lambda)\big] \big| \pi^a(p) \big\rangle = 4i\varepsilon[abc] q(x), \tag{A.11}$$

$$\int \frac{d\lambda}{2\pi} e^{-ip_+ x\lambda} \big\langle \pi^b(p) \big| \mathrm{tr}\big[O_{L+R}(\lambda)\big] \big| \pi^a(p) \big\rangle = 2\delta^{ab} Q(x) \tag{A.12}$$

that in the chiral limit can be written as

$$2\int_{-1+|\beta|}^{1-|\beta|} d\alpha\, F^{I=0}(\beta,\alpha) = \big[\theta(\beta) q(\beta) - \theta(-\beta)\bar{q}(-\beta)\big] = Q(\beta), \tag{A.13}$$

$$\int_{-1+|\beta|}^{1-|\beta|} d\alpha\, F^{I=1}(\beta,\alpha) = \theta(\beta) q(\beta) + \theta(-\beta)\bar{q}(-\beta) = q(\beta). \tag{A.14}$$

The more general functions GPDs are defined as

$$\int \frac{d\lambda}{2\pi} e^{-iP_+ x\lambda} \big\langle \pi^b(p') \big| \mathrm{tr}\big[\tau^c O_{L+R}(\lambda)\big] \big| \pi^a(p) \big\rangle = 4i\varepsilon[abc] H^{I=1}(x,\xi,t), \tag{A.15}$$

$$\int \frac{d\lambda}{2\pi} e^{-iP_+ x\lambda} \big\langle \pi^b(p) \big| \mathrm{tr}\big[O_{L+R}(\lambda)\big] \big| \pi^a(p) \big\rangle = 2\delta^{ab} H^{I=0}(x,\xi,t) \tag{A.16}$$

with $P = \frac{1}{2}(p + p')$, $\xi = -\frac{(p'-p)_+}{(p'+p)_+}$, $t = (p' - p)^2$. In the forward limit $\xi \to 0$, $t \to 0$:

$$H^{I=1}(x,0,0) = q(x), \qquad H^{I=0}(x,0,0) = Q(x). \tag{A.17}$$

And the pion distribution amplitude is given

$$\delta^{ab} F_\pi \phi_\pi(u) = \frac{i}{4} \int \frac{d\lambda}{2\pi} e^{-iu(p.n)\lambda/2} \langle \pi^a(p) |\, \mathrm{tr}\big[\tau^b O_{R-L}(\lambda)\big] |0\rangle. \tag{A.18}$$

Defined distribution amplitude satisfies normalization

$$\int_{-1}^{1} du\, \phi_\pi(u) = 1. \tag{A.19}$$

References

[1] D. Arndt, M.J. Savage, Nucl. Phys. A 697 (2002) 429, nucl-th/0105045.
[2] N. Kivel, M.V. Polyakov, hep-ph/0203264.
[3] J.W. Chen, X.d. Ji, Phys. Rev. Lett. 87 (2001) 152002, hep-ph/0107158; J.W. Chen, X.d. Ji, Phys. Rev. Lett. 88 (2002) 249901, Erratum; J.W. Chen, X.d. Ji, Phys. Lett. B 523 (2001) 107, hep-ph/0105197.
[4] M. Diehl, A. Manashov, A. Schafer, Phys. Lett. B 622 (2005) 69, hep-ph/0505269.
[5] G. Colangelo, Phys. Lett. B 350 (1995) 85, hep-ph/9502285; G. Colangelo, Phys. Lett. B 361 (1995) 234, Erratum.
[6] M.V. Polyakov, Nucl. Phys. B 555 (1999) 231, hep-ph/9809483.
[7] M.V. Polyakov, C. Weiss, Phys. Rev. D 60 (1999) 114017, hep-ph/9902451.
[8] N. Kivel, M.V. Polyakov, in preparation.
[9] W. Detmold, W. Melnitchouk, A.W. Thomas, Phys. Rev. D 68 (2003) 034025, hep-lat/0303015.