Direct Parameter Extraction for Piezoresistively-sensed MEMS Resonators Embedded in Parasitic Capacitive Feedthrough

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Abstract

In this paper, we report a method to extract important parameters such as quality factor, motional transconductance and resonant frequency directly from the measured electrical transmission characteristic of a piezoresistively-sensed MEMS resonator embedded in feedthrough. The method has been applied to measurements for a square-extensional mode resonator where the resonant peak is only 0.22dB due to substantial feedthrough. The extracted parameters, derived using this method compare well with the corresponding values obtained using a Lorentzian fit to within 5%.

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Keywords: piezoresistive; resonator; feedthrough

1. Introduction

MEMS resonators, which function as timing elements in sensory and frequency control applications, typically suffer from the direct coupling of the AC drive input into output port due to parasitic elements. Such interference is known as feedthrough, commonly due to a parasitic capacitive path lying between the input and output. Its effect is manifested in a distortion of the transmission bandpass, where in severe cases the resonant peak is reduced to the extent that direct inference of the quality factor is not possible. As such, we have previously demonstrated a method to directly infer all the parameters of a resonator for the case where the motional current is sensed capacitively [1]. More recently, piezoresistive-sensing has been shown to yield better electromechanical transduction that scales better with frequency compared to

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capacitive-sensing [2]. To follow up on the recent interest in piezoresistive-sensing in MEMS resonators, this paper presents a variant of the previous direct extraction method, herein modified for application to the case of piezoresistively-sensed MEMS resonators.

2. Analysis of Method

The frequency response function (FRF) characterizing a piezoresistively-sensed MEMS resonator, that also defines the device’s transconductance ($g_m$), is given by:

$$g_m(\omega) = \frac{1}{\sqrt{1 - (\omega/\Omega)^2}} + j(\omega/\Omega)(1/Q) - j\omega C_F$$

$\Omega$ denotes the angular resonant frequency; $C_F$ is the parasitic capacitance; $\Pi$ is the electromechanical transduction factor which accounts for the coupling conversion at both actuation and sensing; and $Q$ is the mechanical quality factor. The first term in the expression corresponds to the electromechanical current while the second term represents the parasitic current through the feedthrough capacitor. Compared to the case of capacitive-sensing, there is a phase shift of $\pi/2$ relative to the electromechanical current associated with piezoresistive-sensing. In the former, the current is associated with the velocity. In the latter, the current is associated with stress which in turn is in phase with the displacement.

The FRF can be represented alternatively in the form of a Nyquist plot (Fig. 1a) and a susceptance plot (Fig. 1b). The plots in Fig. 1 have been generated for an example where $Q = 8000$. Although not shown in this paper, in the limit where $Q \gg 1$, the locus on the Nyquist plot approximates to a circle of diameter defined by the product $\Pi Q$, which defines the motional transductance $G_m$. As can be seen from Fig. 1a, the coordinates of the circle’s center are $(0, \Pi Q/2)$.

Fig. 1. Modeled frequency of a piezoresistive resonator represented as in a (a) Nyquist plot; (b) susceptance plot
The effect of $C_F$ is seen through a uniform shift of $\Omega C_F$ along the imaginary axis for both plots. In the limit of substantial feedthrough, since the FRF comprises primarily of the susceptance, the resonant peak observed in the feedthrough-embedded FRF corresponds to the mechanical resonance. The minima and maxima in the FRF phase are defined by the tangent to the circle extending from the origin in the Nyquist plot of Fig. 1a. It can thus be seen that in the limit defined by $\omega_0 C_F > IIQ$ ($\omega_0 C_F / IIQ = 2$ for the example in Fig. 1), the phase maxima and minima approximate to the -3dB frequencies where no feedthrough is present. Hence, the frequency separation between them approximates to the -3dB bandwidth. With both the resonant frequency and -3dB bandwidth known, $Q$ can be obtained. Finally, the difference between the maximum and minimum distance from the origin equals the circle diameter. These correspond to the resonant peak and floor in the FRF. Taking the difference between the two points gives the product $IIQ$, from which $Q$ can be obtained. From the Nyquist plot, it is evident that the floor of the FRF corresponds to the feedthrough contribution where the resonator is operating frequencies away from resonance. Hence in summary: (1) $\Omega$: equals to frequency at the resonant peak; (2) $Q$: separation between the phase maxima and minimal approximates to -3dB bandwidth; (3) $II$: difference between peak and floor equals $IIQ$.

3. Experimental Verification

This method was applied to the measured electrical transmission of a square-extensional (SE) mode resonator, illustrated in Fig. 2a. The resonator was electrostatically actuated and piezoresistively-sensed using the configuration described in the circuit schematic of Fig. 2b, adapted from [3]. The resonator was fabricated in a foundry SOI MEMS process, which a minimum linewidth of 2μm thus leading to large motional resistances relative to feedthrough. This provides for a useful case study to apply the extraction method. The device was packaged in 28-pin dual-in-line package and measured in vacuum at 0.2mbar.

Fig. 2. (a) SE mode shape simulated in COMSOL (b) Circuit schematic for characterization of the piezoresistive resonator

Fig. 3a shows the measured electrical transmission magnitude of the SE mode resonator. As may be seen, the measured characteristic is heavily embedded in feedthrough, such that the resonant peak height is only 0.22dB in the magnitude plot. As such, it is not possible to directly obtain the -3dB bandwidth in order to obtain $Q$. The feedthrough component was subtracted from the measured transmission to yield the extracted characteristic shown in Fig. 4. For greater accuracy, the data was fitted to a Lorentzian to obtain the resonator parameters. Table 1 compares the resonator parameters obtained using the direct extraction method against those values derived using the Lorentzian fit. The two sets of values are shown to agree well to within a 5% disparity margin.
Table 1. Comparison of resonator parameter values obtained by the direct extraction method to that from a Lorentzian fit

<table>
<thead>
<tr>
<th>Resonator Parameter</th>
<th>Direct extraction</th>
<th>Lorentzian fit</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resonant frequency (MHz)</td>
<td>12.010488</td>
<td>12.010488</td>
<td>0.0%</td>
</tr>
<tr>
<td>Quality factor, $Q$</td>
<td>133450</td>
<td>137260</td>
<td>2.8%</td>
</tr>
<tr>
<td>Overall Transduction factor, $I$ (pS)</td>
<td>24.3</td>
<td>25.4</td>
<td>4.3%</td>
</tr>
</tbody>
</table>

4. Conclusions

A method for directly and efficiently extracting parameters for a piezoresistive resonator embedded in feedthrough (where the resonant height is less than 3dB) has been presented. Good agreement within 5% to results using a Lorentzian fit was observed. Given such levels of accuracy and efficiency of extraction, this method provides a useful approach for analyzing large batches of electrical characteristic data for MEMS resonators. This compliments the earlier method applicable for capacitive resonators to include piezoresistive resonators that offer better transduction and are opening a way to achieve RF resonators.

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References