Dynamic Route Choice Based on Prospect Theory

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Abstract

Traveller route choice behaviour is influenced by many uncertain factors, which include both outer stochastic factors from road network and inner factors from traveller psychology. Traveller route choice behaviour is dynamic in practical travel process, namely traveller can change route on the way. Considered the characteristics of bounded rationality in human decision making process and change of route choice on the way, a dynamic route choice model is established based on prospect theory, and the method of dynamic route choice which is more suitable to human thinking habits and actual travel characteristics is proposed in this paper. The method allows the traveller to adjust the route according to road situation at any time. Decision making process reflects that the influence of traveller reference point on the decision result. Reference point changes with the situation of road sections passed. The examples are given to verify the validity and explain application of the method. The result shows that traveller can change route on the way when he meet congestion on the section passed, and if traveller’s psychological expected time can not be met, he will choose risky route, by which the probability of congestion is higher. The method describes traveller route choice behaviour process better and reflects the true situation of road network. It is beneficial to predict traveller behaviours and changes of traffic network situation, and have certain guiding significance for traffic network planning and traffic intelligent control.

Keywords: prospect theory; dynamic route choice; change on the way; reference point; bounded rational

1. Introduction

The problem of traveller’s route choice is always the focus in the field of traffic research. The reliability of the research result has direct influence on traffic network planning and traffic intelligent control. Traditionally the

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researches of traveller route choice problem are based on the expected utility theory (Gao et al., 2010). Traveller’s complete rationality is a prerequisite for the utility theory (De et al., 2011). However people often reflect less than fully rational behaviours in actual route choice decision due to interference of individual psychological factors. In recent years, some scholars introduce prospect theory into the research of route choice behaviour to overcome the prerequisite shortcoming of the utility theory, in which decision makers must be completely rational. De et al. (2011) presents a comparative analysis from the points of view of theory and application of the expected utility theory, prospect theory, and regret theory; Zhao and Zhang (2007) formulated a theoretical model for traveller’s route choice behaviour within a day; Zhang et al. (2013) established a stochastic dynamic user optimal (SDUO) traffic assignment model based on prospect theory. Liu et al. (2010) built model of traveller’s perceived utility under the condition of the route utility continuous random distribution based on prospect theory. Xia et al. (2012) studied reference point setting and influence of departure time on route choice under the condition of reference point unchanged. Wu and Yang (2013) proposed a traveller decision method in uncertain traffic environment. Jou and Chen (2013) applied cumulative prospect theory into studying of freeway drivers’ route choice behaviours. The above studies only consider travellers’ route decision before departure, and the problem of adjusting route on the way is not involved after departure.

It is possible that travellers change route on the way according to road condition. The route choice behaviours run through the whole travel process. So this paper builds a dynamic route choice model allowing traveller to change route on the way based on prospect theory.

2. Prospect Theory

Psychologist Kahneman proposed a new theory——prospect theory that apply psychology research into economics field, and made outstanding contribution for human judgment and decision under the uncertain condition. So he won the 2002 annual Nobel Prize in economics. Prospect theory opens up a new studying field of decision making under the uncertain condition. Prospect theory holds that people’s behaviours, revealing non-rational psychological factors, are predictable (Bromiley, 2010).

Prospect theory finds out the behaviour mode that is not realized in rational decision research, and obtains four basic conclusions: (1) most people show risk aversion when they are faced with gain (fixed effect); (2) most people show risk preferences (reflection effect); (3) most people usually judge gain and loss according to reference point (reference dependent); (4) most people are more sensitive to loss than gain (loss effect). These conclusions are reflected in people’s life, and people tend to follow the above rules in the face of decision making.

The utility function and the subjective probability function in classic utility theory are replaced with the value function and the decision weight function respectively. The value function related to reference point reflects decision-maker subjective feelings towards objective value. The decision weight function, representing people’s subjective evaluation towards objective probability, reflects the influence of probability on prospect value. Prospect theory can reasonably explain practical phenomenon (see, e.g. Hjorth and Fosgerau, 2012) and reflect real scene. Characteristics of prospect theory are also concerned (see, e.g. Zeisberger et al., 2012). Prospect theory currently is applied into the fields of economy, accounting and market decision behaviour researches (see, e.g. Li et al., 2012).

3. Description of Dynamic Route Choice Problem Based on Prospect Theory

Symbols related to the problem of dynamic route choice based on prospect are given below:

- $L_i = \{l_{i1}, l_{i2}, \ldots, l_{in}\}$: feasible route set when traveller at intersection $i$, in which $l_{ij}$ is route $j$ in the set $L_i$, $j = 1, 2, 3 \ldots n$
- $l_{ij} = \{d_{iq1}, d_{iq2}, \ldots, d_{iqz}\}$: road sections set of route $l_{ij}$, that is, route $l_{ij}$ is composed of road sections sequence $d_{iq1}, d_{iq2}, \ldots, d_{iqz}$, in which $d_{iq}$ is road section $q$ of route $l_{ij}$, $q = 1, 2, 3 \ldots z$
• \( T_y = \{ t_y^1, t_y^2, \ldots, t_y^n \} \): scene state set of route \( l_y \), that is, possible travel time set of route \( l_y \), in which \( t_y^x \) is scene state \( x \) of route \( l_y \), \( x = 1, 2, 3 \ldots w_y \);

• \( T_{yq} = \{ t_{yq}^1, t_{yq}^2, \ldots, t_{yq}^n \} \): scene state set of section \( d_{yq} \), that is, possible travel time set of section \( d_{yq} \), in which \( t_{yq}^x \) is scene state \( x \) of section \( d_{yq} \), \( x = 1, 2, 3 \ldots w_q \);

• \( K = \{ k_0, k_1, k_2, \ldots, k_n \} \): traveller’s reference point set, in which \( k_i \) is traveller’s reference point at intersection \( i \), reflecting traveller’s psychological expected time for remaining journey, \( i = 0, 1, 2, 3 \ldots m \), \( i = 0 \) means traveller is at starting location;

• \( P = | p_{wy} |_{wy} \): scene probability matrix, in which \( p_{wy} \) is the probability of scene \( t_y^x \) emerging due to running on route \( l_y \), and \( \sum_{x=1}^{w_y} p_{wy} = 1 \).

### 4. Dynamic Route Choice Based on Prospect Theory

#### 4.1. Traveller decision-making process

Traveller chooses an initial route which is the most satisfied one at the starting location before departure and drive according to initial route. The choice of initial route depends on traveller’s psychological expected time and parameters of feasible routes. Traveller can choose again a route to run according to the situation of passed section when he reaches an intersection. Meanwhile traveller’s psychological expected time and the current feasible routes changed. Traveller will select the most satisfied route which is closest to his psychological expected time at this time, and then drive on the selected road until reaching the destination. The detailed process is illustrated in Fig. 1.

![Fig.1. Traveller dynamic decision-making process](image)

#### 4.2. Initial route choosing

Reference point of traveller at starting location is \( k_0 \), feasible route set is \( L_0 = \{ l_0^1, l_0^2, \ldots, l_0^m \} \) and possible scene state set of route \( l_{0j} \) is \( T_{0j} = \{ t_{0j}^1, t_{0j}^2, \ldots, t_{0j}^n \} \). Gain of scene \( t_{0j}^x \) for reference point \( k_0 \) is

\[
y_{0j}^x = k_0 - t_{0j}^x
\]
Loss of scene $t_{ij}'$ is less than that of reference point, and decision maker’s psychological perception is gain when $y_{ij}' \geq 0$; Loss of scene $t_{ij}'$ is more than that of reference point, and decision maker’s psychological perception is loss when $y_{ij}' < 0$.

According to the value function form Tversky and Kahneman proposed, value of scene $t_{ij}'$ is defined as

$$v_{ij}' = \begin{cases} (y_{ij}')^\alpha & y_{ij}' \geq 0 \\ -\lambda (-y_{ij}')^\beta & y_{ij}' < 0 \end{cases}$$

(2)

Where $\alpha$ and $\beta$ ($0 \leq \alpha, \beta \leq 1$) are risk attitude coefficients, which reflect concave convex degree of value function, describing travel time gain and loss. The bigger $\alpha$ and $\beta$, the greater concave convex degree of value function, the more adventurous decision maker tends to. Value function shows the psychological behaviour of decreasing sensitivity to travel time by degrees. $\lambda$ is loss aversion coefficient, and reflects that decision maker is more sensitive to loss when $\lambda > 1$. The bigger $\lambda$ the greater loss aversion degree. Some researchers prove that the value of $\alpha=0.89$, $\beta=0.92$, $\lambda=2.25$ are more consistent with decision maker psychological characteristic by a large number of experiments (Zhang and Fan, 2012).

Let $\pi(p_{ij}')$ is scene weight of scene $t_{ij}'$ emerging due to running on route $l_{ij}$ as

$$\pi(p_{ij}') = \begin{cases} \frac{P_{ij}^\chi}{(p_{ij}^\chi + (1-p_{ij})^\chi)^{\chi/\delta}} & y_{ij}' \geq 0 \\ \frac{p_{ij}^\delta}{(p_{ij}^\delta + (1-p_{ij})^\delta)^{\delta/\delta}} & y_{ij}' < 0 \end{cases}$$

(3)

Where $p_{ij}'$ is probability of scene $t_{ij}'$ due to route $l_{ij}$. According to the result of testing based on prospect theory, the best parameters value are $\chi=0.61$, $\delta=0.69$, which reflect decision maker behaviour preference (Yang et al., 2009). Computing scene weight indicates that people tend to overestimate low probability events, underestimate high probability events and are insensitive to intermediate probability change.

Prospect value $EV_{ij}$ of route $l_{ij}$ is

$$EV_{ij} = \sum_{j=1}^{\infty} v_{ij}' \pi(p_{ij}')$$

(4)

Obviously the bigger the prospect value, the closer to decision maker’s psychology. The route of the biggest prospect value is the first choice. Therefore the initial route is

$$l_{ij} = \{l_{ij} | \max(EV_{ij}), j = 1, 2, \cdots, n\}$$

(5)

### 4.3. Micro driving rules

Traveller begins to drive after initial route is selected, and observes the following rules throughout the whole running process until the destination.

Rule1: vehicle acceleration rule. If $v < v_{\text{max}}$, then $v = v + 1$, in which $v$ is vehicle speed, $v_{\text{max}}$ is the maximum speed limit.

Rule2: safety control rule. If $v > gap$, then $v = gap$, in which $gap$ is the distance between the vehicle and the one in front on the same driveway.
Rule3: random deceleration rule. If $v > 0$, then $v = v - 1$ by probability $p$, in which $p$ is probability of vehicle random deceleration, namely randomization probability.

Rule4: vehicle movement rule. $x = x + v$, in which $x$ is the location of vehicle.

4.4. Adjusting on the way

Traveller drive through section $d_{r-lc1}$ according to route $l_{r-lc1}$ selected at intersection $i - 1$. Traveller may not continue original route when he reaches intersection $i$. Traveller will choose again route from intersection $i$ to the destination based on the current situation. Feasible route set at intersection $i$ is $L_i = \{l_{i1}, l_{i2}, \cdots, l_{in}\}$, scene state set of route $l_i$ is $T_i = \{t_{i1}, t_{i2}, \cdots, t_{in}\}$. The situation of section $d_{r-lc1}$ has great influence on the change of reference point. Let scene of section $d_{r-lc1}$ is $t_{r-lc1}^i$, reference point of intersection $i$ is

$$k_j = k_{i-1} - t_{i-lc1}^r$$

Value of scene $t_{i}^r$ for reference point $k_i$ is

$$v_{i}^r = \begin{cases} 
(k_j - t_{i}^r)^\lambda & k_j - t_{i}^r \geq 0 \\
-\lambda(k_j - t_{i}^r) & k_j - t_{i}^r < 0 
\end{cases}$$

Prospect value $EV_i$ of route $l_i$ is

$$EV_i = \sum_{j=1}^{n} v_{i}^r \pi(p_{ij}^r)$$

Selected route $l_i$ at intersection $i$ is

$$l_i = \{l_j \mid \max(EV_j), j = 1, 2, \cdots, n\}$$

Driving route is again chosen according to above method every reaching an intersection until the destination. The actual route at last is $\{d_{u1}, d_{u2}, \cdots, d_{u1}\}$, in which $u$ is the number of intersection throughout travel process. The actual total travel time $t$ is

$$t = \{\sum_{j=0}^{u} t_{i}^r \mid x \in \{1, 2, 3, \cdots, w_{i}\}\}$$

To sum up, traveller dynamic route choice process based on prospect theory as follows:

Step1: Computing gain or loss and value of all scene $t_{i}^r$, $x = 1, 2, 3, \cdots, w_i$, $j = 1, 2, 3, \cdots n$ at starting location for reference point $k_i$ according to formula(1), (2) respectively;

Step2: Computing scene weights $\pi(p_{ij}^r)$ of feasible routes for scene $t_{i}^r$ according to formula (3);

Step3: Computing prospect value $EV_{ij}$ of route $l_{ij}$ according to formula (4), and selecting the initial route according to formula (5);
Step 4: Driving according to initial route, and observing micro driving rules (Rule1-Rule4) in the whole travel process.
Step 5: Modifying the current reference point according to formula (6) while reaching an intersection.
Step 6: Making sure the current feasible route set, computing again prospect values of the routes, and selecting the most satisfied route according to formula (7)-(9) to continue driving.
Step 7: Repeating step 5-step 6 until reaching the destination, and computing the actual total travel time \( t \) according to formula (10) at last.

5. Example Testing

Nodes (1→3) in Fig. 2 are defined as OD testing to verify reasonableness and reality of the method above. All travellers start at node 1 and end at node 3. Parameters of all sections are shown in Table 1.

![Fig. 2. Topology structure of road network](image)

Table 1. Parameters of all sections

<table>
<thead>
<tr>
<th>No of section</th>
<th>Free through time/min</th>
<th>Probability of congestion</th>
<th>Congestion through time/min</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>0.4</td>
<td>20</td>
</tr>
<tr>
<td>2</td>
<td>30</td>
<td>0</td>
<td>30</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
<td>0</td>
<td>15</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td>0.4</td>
<td>20</td>
</tr>
</tbody>
</table>

Three routes are feasible as follows according to the data of Table 1:

Route 1: \{section 1, section 3\}. Through time is 25 min, and the probability is 0.4 when traffic is free; through time is 35 min, and the probability is 0.6 when traffic congestion occurs.

Route 2: \{section 1, section 4\}. Through time is 20 min and the probability is 0.16 when traffic is free; through time are 30 min or 40 min, and the probability are 0.6 and 0.36 respectively when traffic congestion occurs.

Route 3: \{section 2\}. Through time is 30 min, and the probability is 1 when traffic is free.

Firstly, travellers choose initial routes by computing prospect values of all routes under different reference points (reference points values from 20 min to 40 min here in accordance with data). The results are shown as Fig. 3. The Fig. 3 show that travellers whose reference point \( K \in [20, 25] \) will choose Route 1 to drive, however travellers whose reference point \( K \in [26, 40] \) will choose Route 3 to drive and no one will choose Route 2. IF adjusting on the way is not considered, travellers would be driving until node 3 according to initial routes, and there are no vehicle on the section 4 that is not correspond to the fact.

Next we consider adjusting on the way. When travellers reach node 2, section 3 or section 4 is again chosen. The choice depends on the situation of section 1 travellers just passed. If the situation is free, travellers’ reference points change into \( K-10 \), that is \( K \in [10, 15] \); if the situation is congested, travellers’ reference points change into \( K-20 \), that is \( K \in [0, 5] \). Travellers’ choice results are seen from Fig. 4. As is shown in the Fig. 4 travellers whose reference point \( K \in [0, 5] \) will choose Section 4 to drive, however travellers whose reference point \( K \in [10, 15] \) will choose Section 3. We draw a conclusion that travellers will change route into section 4 if they meet congestion on the section 1. Otherwise travellers will continuous to drive on the section 3 according to initial route. The conclusion is closer to actual road situation and is consistent with human thinking habits.
6. Application Simulation

The above method is applied into the traveller simulation of the traffic network in the Fig. 5. Fig. 5 is a common network topology structure abstracted from actual traffic network. The traffic network consists of 9 nodes and 12 sections, the length of which is described as cellular numbers. Cellular length is the minimum headway and each cellular only hold a vehicle. Suppose all sections length are 20cellulars. Nodes (1→9) is OD, namely all travellers enter into simulation area from node1 and leave it from node9. Parameters of all sections are shown in Table 2.

![Fig. 5. Topology structure of traffic network2](image)

Vehicles enter into simulation area randomly, generating stochastic reference points within 100min (maximum travel time is 100min from node1 to node9). Travellers adjust routes every reaching an intersection. Situation of traffic network is shown in the Fig.6 when simulation time Tick=400. Vehicles distribution from section1 to section12 is seen from top to bottom successively. Black spots are vehicles, driving from left to right on the sections. Vehicles at the right end of sections choose again next section, finally reaching node9 through section10 or
section12, and then leaving the network. As can be seen from Fig. 6, vehicles will be distributed at all sections after different travellers select and adjust routes based on personal psychological expectation.

Table 2. Parameters of all sections

<table>
<thead>
<tr>
<th>No</th>
<th>Free through time/min</th>
<th>Congestion through time/min</th>
<th>Probability of congestion</th>
<th>No</th>
<th>Free through time/min</th>
<th>Congestion through time/min</th>
<th>Probability of congestion</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>20</td>
<td>0.4</td>
<td>7</td>
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<td>0.6</td>
<td>10</td>
<td>10</td>
<td>20</td>
<td>0.4</td>
</tr>
<tr>
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<td>20</td>
<td>0.4</td>
<td>11</td>
<td>5</td>
<td>10</td>
<td>0.6</td>
</tr>
<tr>
<td>6</td>
<td>15</td>
<td>30</td>
<td>0.2</td>
<td>12</td>
<td>10</td>
<td>20</td>
<td>0.4</td>
</tr>
</tbody>
</table>

The monitoring results of density changes of all sections continuously are shown in Fig. 7. Obviously densities of section1, section4, section3, section12, section 10 and section9, especially section1 and section4, are greater than that of sections else, which means many travellers can select these sections to drive. Therefore it is easy to occurring congestion on these sections. We need pay attention to these sections and take affective measures to relieve traffic congestion.

7. Conclusion

A new method of traveller dynamic route choice considering adjusting on the way is presented in this paper. We can see that travellers tend to change route while meeting congestion and sometimes choose risky route due to the urgency of time from examples, which is consistent with human thinking habits and actual situation. The proposed
method is closer to actual travel behaviour and reflects true traffic state. It is beneficial to predict traveller behaviours and changes of traffic network situation, and can work as supplement to traffic network planning and traffic intelligent control.

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