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AN ATTACK ON GAUSS, PUBLISHED BY LEGENDRE IN 1820

BY STEPHEN M. STIGLER
UNIVERSITY OF WISCONSIN, MADISON 53706 [1]

SUMMARIES

The question of priority in the discovery of the method of least squares reached a climax when Legendre published an attack on Gauss in 1820. The background of the dispute is sketched, and this little known attack is presented in translation.

Cette note discute la dispute entre Gauss et Legendre sur la priorité de la méthode des moindres carrés, et une traduction en anglais d'une note de Legendre est presentée.

Disputes over questions of priority have been common in the history of mathematics, and on several occasions these disputes have engaged the greatest mathematicians in acrimonious debate. The question of the priority in the discovery of the method of least squares provides an interesting example, as it involved two of the most famous mathematicians of the time--Legendre and Gauss--and it focused attention on the problems that can arise in deciding priority between a published account of a discovery and an informal verbal account. Plackett [1972] has given an excellent description of the dispute; the purpose of this note is to present an additional and apparently little known document which sheds additional light on one of the protagonist's attitude toward the other.

The basic facts are these. In 1805 Legendre published his *Nouvelles méthodes pour la determination des orbites des comètes* which presented (and named) the method of least squares, without any attempt to tie it to probability. In early 1809 Gauss published his *Theoria Motus Corporum Coelestium* which also presented the method and linked it with the normal distribution, adding: "On the other hand our principle, which we have made use of since the year 1795, has lately been published by LEGENDRE ..." [Gauss 1963, 270]. The evidence presented by Plackett [1972] strongly indicates that the two discoveries were independently made, and that Gauss was telling the truth and had mentioned his method to a few others, including Olbers and Lindenau, before 1805.

Another mathematician, the American Robert Adrain, is sometimes given as a third independent discoverer of least squares. In 1808 or 1809 Adrain solved a problem in surveying he had presented in his own journal, the *Analyst* (published in Philadelphia) [2], by deriving the normal distribution as a distribution of errors, and from it the method of least squares [Coolidge 1926]. There seems to be no reason to doubt that Adrain's derivation of the normal distribution was arrived at independently of Gauss--their methods are totally dissimilar (Adrain's is much inferior), and the works appeared at about the same time on different continents. However, it is far less certain that he had not seen Legendre's work. Babb [1926] tells us that the original 1805 edition [3] of Legendre's book in the original paper cover was in Adrain's personal library, and Coolidge [1926] documents an instance where Adrain borrowed from a contemporary without citation. In any case, Adrain's work remained nearly totally obscure for over half a century, and had no influence on the development of the subject.

The debate on the priority for the method of least squares has therefore centered on Legendre and Gauss, and it was not to be left to historians of mathematics. Legendre was nettled by Gauss's use of the phrase "our principle" ("Principium nostrum" in the original Latin) in disregard of the convention that priority be settled by date of publication, and wrote to Gauss shortly after Gauss's book appeared, in May of 1809: "It was with pleasure that I saw that in the course of your meditations you had hit on the same method which I had called Méthode des moindres quarrés in my memoir on comets.... I confess to you that I do attach some value to this little find. I will therefore not conceal from you, Sir, that I felt some regret to see that in citing my memoir p. 221 you say *principium nostrum quo jam inde ab anno 1795 usi sumus* etc. There is no discovery that one cannot claim for oneself by saying that one had found the same thing some years previously; but if one does not supply the evidence by citing the place where one has published it, this assertion becomes pointless and serves only to do a disservice to the true author of the discovery. In Mathematics it often happens that one discovers the same things that have been discovered by others and which are well known; this has happened to me a number of times, but I have never mentioned it and I have never called *principium nostrum* a principle which someone else had published before me. You have treasures enough of your own, Sir, to have no need to envy anyone; and I am perfectly satisfied, besides, that I have reason to complain of the expression only and by no means of the intention." [quoted from Plackett 1972]. Legendre's pique at Gauss seems, as we shall see, to have been exacerbated by his feeling that this was not the first occasion Gauss had given Legendre insufficient recognition. In 1785 Legendre had presented and partly proved the beautiful law of quadratic reciprocity of number theory; in 1801

Gauss presented this theorem in *Disquisitiones Arithmeticae* as *the fundamental theorem* and gave the first full proof. Only two pages later did he allude somewhat vaguely to Legendre's contribution.

Legendre did not drop the matter after his 1809 letter, but pressed his claim for priority by having his 1805 account of least squares reprinted in the 1810 volume of the *Memoires* of l'Institut de France (published 1814). His feeling toward Gauss may have intensified in 1817 when Gauss wrote in *Göttingische gelehrte Anzeigen*, regarding the law of quadratic reciprocity, "It is characteristic of higher arithmetic that many of its most beautiful theorems can be discovered by induction with the greatest of ease but have proofs that lie anywhere but near at hand and are often found only after many fruitless investigations with the aid of deep analysis and lucky combinations, ... the finding of new proofs for known truths is often at least as important as the discovery itself." [quoted from May 1972]. The law of quadratic reciprocity is a standard topic in histories of mathematics [e.g. Boyer 1968, 550-551], where there seems to be no dispute in the attribution of its first modern statement to Legendre and its first full proof to Gauss [4], but one can imagine Legendre's anger if he read Gauss's insensitive trivialization of Legendre's accomplishment.

Legendre took the opportunity of the publication in 1820 of a Second Supplement to his 1805 book to present an attack on Gauss. This Supplement is dated "10 Aout 1820", and at its end one finds appended (p. 79-80) a "Note par M***". It is not certain that Legendre wrote the note himself, although the tone and several phrases are strikingly similar to his 1809 letter to Gauss. In any event he was responsible for its publication. In reading the following translation of this note it should be kept in mind that Gauss was born in 1777.

NOTE BY M***

At the beginning of the preceding work, page 4, the author spoke of his Method of Least Squares: *it is enough to recall that he published it, for the first time, in 1805, at the end of a memoir on comets which bears the date of the month of* March *of that year. However, as a very celebrated geometer has not hesitated to appropriate this method to himself in a work printed in 1809, we believe it is our duty to pause a moment and consider this claim, so that all impartial readers will be able henceforth to call this method by the name they judge to be suitable. Here is how it is stated: After having explained this method or principle as it pertained to him, this geometer added: "This principle of* ours *which we have made use of since the year 1795, has lately been published by Legendre in a work* New Methods, *etc." If this is not conclusive, it is at least* *quite clear, and above all quite convenient. With this system, the history of sciences will be written much more easily; a discovery will no longer be assigned to he who has made it, but no matter! It will be assigned always to whomever found it convenient to claim it without right, despite its dating from a more remote time. In good faith, is such a system admissible? It had not occurred to us until now, how in such circumstances the history of mathematics requires trust on every page; we had regarded the propriety of a discovery as invariably assured to the one who, for the first time, brought it to light, and any claim to the contrary could expose him to suspicions of an injurious nature, requiring the support of precise and authentic documentation. In addition, we would have willingly spoken of the episode of 1809 as of a totally new and different kind, if we had not found that in 1801 the same geometer made another attempt of this type, in a way this earlier behavior was even more imperfect. We have seen published in the Memoires de l'Academie des Sciences for 1785 a theorem (the law of reciprocity) which led to much genuine progress in number theory, without the inventer, we admit, having proved it rigorously in all cases. Now, how did the geometer of whom we speak express himself on this subject* sixteen *years later? He first proved this theorem, which he called* fundamental *because of its importance, then he added: "The fundamental theorem must certainly be regarded as one of the most elegant of its type. No one has thus far presented it in as simple a form as we have done above." We find, however, two pages further on "Legendre in his excellent tract in Mem. Acad. des Sci. 1785 arrived at a theorem which is basically the same as the fundamental theorem." Thus we find an acknowledgement which agrees but little with what precedes it, and a quite singular theorem which, though published in 1785, had been presented by no one in 1801! But at least this time he does not go further, and claim it for himself* ten years earlier, *as he did in 1809: "This theorem of* ours *which we have made use of since 1775 ..." Whatever may further come of this system of literary propriety, it will perhaps not be useless to recall in conclusion, that the famous series of Lagrange, for the reversion of series, while not proved by its author, has never ceased to bear the name of* Theorem of Lagrange, *and no one has dreamed of disputing his priority.* [5]

NOTES

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2. The issue in question is the fourth and last of the volume for 1808, but the printed apologies to the readers for the delay in publication would make early 1809 a much more likely time for the actual publication of this issue.

3. The title page of Legendre's book was reprinted to read 1806 when a supplement was added in that year; this would seem to indicate that Adrain's copy indeed came from the 1805 printing.

4. It is frequently stated [e.g. Boyer 1968, 532] that the statement of the theorem (without proof) is implicit in earlier work of Euler, although this seems to have not been Gauss's view [Gauss 1963, 104].

5. The quotations from Gauss (which are given in Latin in the supplement) can be found in Gauss [1963, 270] and Gauss [1966, 104-5].

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