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Evolution via imitation among like-minded individuals

Naoki Masuda

Department of Mathematical Informatics, The University of Tokyo, 7-3-1 Hongo, Bunkyo, Tokyo 113-8656, Japan

HIGHLIGHTS

- I study an evolutionary game model with idiosyncratic fitness.
- The model behaves differently from other models such as the bimatrix game.
- Polarization of strategies in different subpopulations often occurs.

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ABSTRACT

In social situations with which evolutionary game is concerned, individuals are considered to be heterogeneous in various aspects. In particular, they may differently perceive the same outcome of the game owing to heterogeneity in idiosyncratic preferences, fighting abilities, and positions in a social network. In such a population, an individual may imitate successful and similar others, where similarity refers to that in the idiosyncratic fitness function. I propose an evolutionary game model with two subpopulations on the basis of multipopulation replicator dynamics to describe such a situation. In the proposed model, pairs of players are involved in a two-person game as a well-mixed population, and imitation occurs within subpopulations in each of which players have the same payoff matrix. It is shown that the model does not allow any internal equilibrium such that the dynamics differs from that of other related models such as the bimatrix game. In particular, even a slight difference in the payoff matrix in the two subpopulations can make the opposite strategies to be stably selected in the two subpopulations in the snowdrift and coordination games.

1. Introduction

A basic assumption underlying many evolutionary and economic game theoretical models is that individuals are the same except for possible differences in the strategy that they select. In fact, a population of individuals involved in ecological or social interaction is considered to be heterogeneous. For example, different individuals may have different fighting abilities or endowments (Landau, 1951; Hammerstein, 1981; Maynard Smith, 1982; McNamara et al., 1999), occupy different positions in contact networks specifying the peers with whom the game is played (Szabó and Fáth, 2007; Jackson, 2008), or have different preferences over the objective outcome of the game. The last situation is succinctly represented by the Battle of the Sexes game in which a wife and husband prefer to go to watch opera and football, respectively, whereas their stronger priority is on going out together (Luce and Raiffa, 1957) (the Battle of the Sexes game here is different from the one that models conflicts between males and females concerning parental investment as described in Dawkins (1976), Schuster and Sigmund (1981), Maynard Smith (1982), Hofbauer and Sigmund (1988), and Hofbauer and Sigmund (1998). In behavioral game experiments, the heterogeneity of subjects is rather a norm than exceptions (e.g., Camerer, 2003). For example, some humans are cooperative in the public goods game and others are not (e.g., Fischbacher et al., 2001; Jacquet et al., 2012), and some punish non-cooperators more than others do (Fehr and Gächter, 2002; Dreber et al., 2008).

Evolution of strategies in such a heterogeneous population is the focus of the present paper. This question has been examined along several lines.

First, in theory of preference, it is assumed that individuals maximize their own idiosyncratic utilities that vary between individuals. The utility generally deviates from the fitness on which evolutionary pressure operates (e.g., Sandholm, 2001; Dekel et al., 2007; Alger and Weibull, 2012; Grund et al., 2013).

In fact, experimental evidence shows that individuals tend to imitate behavior of similar others in the context of diffusion of innovations (Rodgers, 2003) and health behavior (Centola, 2011). Also in the context of economic behavior described as games, individuals may preferentially imitate similar others because

E-mail address: masuda@mist.i.u-tokyo.ac.jp

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similar individuals are expected to be interested in maximizing similar objective functions. This type of behavior is not considered in previous preference models in which individuals can instantaneously maximize their own payoffs, and selection occurs on the basis of the fitness function common to the entire population. The model proposed in this study deals with evolutionary dynamics in which individuals in a heterogeneous population mimic successful and similar others. The similarity here refers to that in the idiosyncratic preference.

Second, evolution in heterogeneous populations has been investigated with the use of the evolutionary bimatrix game (Hofbauer and Sigmund, 1988, 1998; Weibull, 1995). A payoff bimatrix describes the payoff imparted to the two players in generally asymmetric roles. In its evolutionary dynamics, a population is divided into two subpopulations, pairs of individuals selected from the different subpopulations play the game, and selection occurs within each subpopulation. The population then has bipartite structure induced by the fixed role of individuals. However, the most generic population structure for investigating interplay of evolution via social learning and idiosyncratic preferences would be a well-mixed population without fixed roles of individuals.

Third, evolutionary game dynamics on heterogeneous social networks (Szabó and Fáth, 2007) is related to evolution in heterogeneous populations. In most of the studies on this topic, the payoff to an individual per generation is defined as the obtained payoff summed over all the neighboring individuals. Then, cooperation in social dilemma games is enhanced on heterogeneous networks (Santos and Pacheco, 2005; Durán and Mulet, 2005; Santos et al., 2006). In this framework, hubs (i.e., those with many neighbors) and non-hubs are likely to gain different payoffs mainly because of their positions in the contact network. In particular, if the payoff of a single game is assumed to be nonnegative, hubs tend to earn more than non-hubs simply because hubs have more neighbors than non-hubs by definition (Masuda, 2007). However, as long as the contact network is fixed, a non-hub player will not gain a large payoff by imitating the strategy of a successful hub neighbor. The number of neighbors serves as the resource of a player. Then, it may be more natural to assume that players imitate successful others with a similar number of neighbors.

Motivated by these examples, I examine evolutionary dynamics in which a player would imitate successful others having similar preferences or inhabiting similar environments. I divide the players into two subpopulations depending on the subjective perception of the result of the game; one may like a certain outcome of the game, and another may not like the same outcome. Imitation is assumed to occur within each subpopulation. However, the interaction occurs as a well-mixed population. I also assume that all the individuals have the same ability, i.e., no player is more likely to "win" the game than others.

2. Model

Consider a population comprising two subpopulations of players such that the payoff matrix depends on the subpopulation. The payoff is equivalent to the fitness in the present model. I call the game the subjective payoff game. Each player, independent of the subpopulation, selects either of the two strategies denoted by A and B. The case with a general number of strategies can be analogously formulated. The subjective payoff game and its replicator dynamics described in the following are a special case of the multipopulation game proposed before (Taylor, 1979; Schuster et al., 1981a) (for slightly different variants, see Maynard Smith, 1982; Hofbauer and Sigmund, 1988; Weibull, 1995).

The population is infinite, well-mixed, and consists of a fraction p $(0<p<1)$ of type X players and a fraction $1-p$ of type Y players. The subjective payoff matrices that an X player and a Y player perceive as row player are defined by

$$\begin{array}{c} \\ A \\ B \end{array}\begin{array}{c} \begin{array}{cc} A & B \end{array} \\ \begin{pmatrix} a_X & b_X \\ c_X & d_X \end{pmatrix} \end{array} \quad \text{and} \quad \begin{array}{c} \\ A \\ B \end{array}\begin{array}{c} \begin{array}{cc} A & B \end{array} \\ \begin{pmatrix} a_Y & b_Y \\ c_Y & d_Y \end{pmatrix} \end{array}, \tag{1}$$

respectively. It should be noted that the payoff that an X player, for example, perceives depends on the opponent's strategy (i.e., A or B) but not on the opponent's type (i.e., X or Y). The use of the two payoff matrices represents different idiosyncrasies in preferences in the two subpopulations. Alternatively, the payoff matrix differs by subpopulations because X and Y players have different tendencies to transform the result of the one-shot game (i.e., one of the four consequences composed of a pair of A and B) into the fitness. For example, X and Y players may benefit the most from mutual A and mutual B, respectively.

The fractions of X and Y players that select strategy A are denoted by x and y, respectively. The fractions of X and Y players that select strategy B are equal to $1-x$ and $1-y$, respectively. The payoffs to an X player with strategies A and B are given by

$$\pi_{X,A} = a_X[px+(1-p)y]+b_X[p(1-x)+(1-p)(1-y)] \tag{2}$$

and

$$\pi_{X,B} = c_X[px+(1-p)y]+d_X[p(1-x)+(1-p)(1-y)], \tag{3}$$

respectively. The payoff to a Y player is defined with X replaced by Y in Eqs. (2) and (3).

I assume that in the evolutionary dynamics, the players can only copy the strategies of peers in the same subpopulation. This assumption reflects the premise that the payoff in the present model is subjective such that the only comparison that makes sense is that between the players in the same subpopulation. The replicator dynamics of the subjective payoff game is then defined by

$$\begin{aligned} \dot{x} &= x[\pi_{X,A}-(x\pi_{X,A}+(1-x)\pi_{X,B})] = x(1-x)\{(a_X-c_X)[px+(1-p)y] \\ &\quad +(b_X-d_X)[p(1-x)+(1-p)(1-y)]\} \end{aligned} \tag{4}$$

and

$$\dot{y} = y(1-y)\{(a_Y-c_Y)[px+(1-p)y]+(b_Y-d_Y)[p(1-x)+(1-p)(1-y)]\}, \tag{5}$$

where $\dot{x}$ and $\dot{y}$ represent the time derivatives.

3. General results

3.1. Absence of internal equilibrium

If (x, y) is an internal equilibrium (i.e., $0<x,y<1$) of the replicator dynamics given by Eqs. (4) and (5), $(a_X-c_X)[px+(1-p)y]+(b_X-d_X)[p(1-x)+(1-p)(1-y)] = (a_Y-c_Y)[px+(1-p)y]+(b_Y-d_Y)[p(1-x)+(1-p)(1-y)] = 0$ must be satisfied. However, this is impossible unless a degenerate condition $(a_X-c_X)(b_Y-d_Y)=(a_Y-c_Y)(b_X-d_X)$ is satisfied. Therefore, for a generic pair of payoff matrices, the replicator dynamics does not have an internal equilibrium.

Three remarks are in order. First, the absence of internal equilibrium implies that the present dynamics does not allow limit cycles (Hofbauer and Sigmund, 1988, 1998). Second, the present result contrasts with that for a two-subpopulation dynamics in which the perceived payoff matrix depends on the opponent's subpopulation as well as on the focal player's subpopulation. In the latter case, an internal equilibrium or limit cycle can exist (Schuster et al., 1981a). Third, the present conclusion is different from that for the bimatrix game. In the bimatrix game, each player in subpopulation X exclusively interacts with each player in subpopulation Y. Then, an internal equilibrium can exist, whereas, when it exists, it is either a saddle or a neutrally stable point surrounded by periodic orbits (Pohley and Thomas, 1979; Selten, 1980; Schuster et al., 1981b; Maynard Smith, 1982; Hofbauer and Sigmund, 1988, 1998).

3.2. Invariance under the transformation of payoff matrices

The replicator dynamics without population structure, which is referred to as the ordinary replicator dynamics in the following, is invariant under some transformations of the payoff matrix. The dynamics given by Eqs. (4) and (5) is also invariant under some payoff transformations.

First, trajectories of the ordinary replicator equation are invariant under the addition of a common constant to all the entries of the payoff matrix. Similarly, replacing the payoff matrices given by Eq. (1) by

$$\begin{pmatrix} a_X+h_X & b_X+h_X \\ c_X+h_X & d_X+h_X \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} a_Y+h_Y & b_Y+h_Y \\ c_Y+h_Y & d_Y+h_Y \end{pmatrix}, \tag{6}$$

where h_X and h_Y are arbitrary constants, does not alter the dynamics.

Second, trajectories of the ordinary replicator equation are invariant under multiplication of all the entries of the payoff matrix by a common positive constant. It only changes the time scale. In the present model, replacing Eq. (1) by

$$\begin{pmatrix} ka_X & kb_X \\ kc_X & kd_X \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} ka_Y & kb_Y \\ kc_Y & kd_Y \end{pmatrix}, \tag{7}$$

where $k>0$, does not alter the dynamics. It should be noted that the multiplicative factor for the two payoff matrices has to be the same for the dynamics to be conserved.

Third, in the ordinary replicator equation, adding a common constant to any column of the payoff matrix does not alter a trajectory. In the present model, replacing Eq. (1) by

$$\begin{pmatrix} a_X+h_{X,A} & b_X+h_{X,B} \\ c_X+h_{X,A} & d_X+h_{X,B} \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} a_Y+h_{Y,A} & b_Y+h_{Y,B} \\ c_Y+h_{Y,A} & d_Y+h_{Y,B} \end{pmatrix} \tag{8}$$

does not alter the trajectory for arbitrary $h_{X,A}$, $h_{X,B}$, $h_{Y,A}$, and $h_{Y,B}$. This invariance is a generalization of the first invariance. It is also equivalent to the invariance relationship found for a more general model (Schuster and Sigmund, 1981).

3.3. Condition for ESS

Let us calculate the conditions for the combination of pure strategies in each subpopulation to be evolutionarily stable strategies (ESSs). Some definitions of ESS for multipopulation games exist (Taylor, 1979; Schuster et al., 1981a; Cressman, 1992, 1996; Cressman et al., 2001), and it seems that consensus on the definition of the ESS in the case of multiple subpopulations has not been reached (Weibull, 1995). Here I adhere to the definition given in Taylor (1979) (also see Schuster et al., 1981a), which was proposed for general two-subpopulation games in which intra-subpopulation and inter-subpopulation interactions yield a different payoff to a focal player.

I start with stating the definition of the ESS by obeying Taylor (1979). Consider a general two-subpopulation matrix game such that there are m and n pure strategies in subpopulations X and Y, respectively. A mixed strategy in X and Y is parametrized as $\boldsymbol{x}=(x_1\ x_2\ \cdots\ x_m)^\top$ and $\boldsymbol{y}=(y_1\ y_2\ \cdots\ y_n)^\top$, where x_i $(1\le i\le m)$ and y_i $(1\le i\le n)$ are the probabilities that the mixed strategy in X and Y takes the ith strategy, respectively, $\sum_{i=1}^m x_i=\sum_{i=1}^n y_i=1$, and $\top$ denotes the transposition. Consider a population of resident players taking strategies $\boldsymbol{x}$ and $\boldsymbol{y}$ in subpopulations X and Y, respectively. I assume that the payoff to a player adopting mixed strategy $\boldsymbol{x}'$ in subpopulation X embedded in this resident population is given by

$$\boldsymbol{x}'^\top (R_{XX}\boldsymbol{x}+R_{XY}\boldsymbol{y}). \tag{9}$$

Similarly, assume that the payoff to mixed strategy $\boldsymbol{y}'$ in subpopulation Y embedded in the same resident population is given by

$$\boldsymbol{y}'^\top (R_{YX}\boldsymbol{x}+R_{YY}\boldsymbol{y}). \tag{10}$$

Strategy $(\boldsymbol{x},\boldsymbol{y})$ is ESS if for any $(\boldsymbol{x}',\boldsymbol{y}')\neq(\boldsymbol{x},\boldsymbol{y})$,

$$\begin{aligned} &\boldsymbol{x}'^\top (R_{XX}\boldsymbol{x}+R_{XY}\boldsymbol{y})+\boldsymbol{y}'^\top (R_{YX}\boldsymbol{x}+R_{YY}\boldsymbol{y}) \\ &\le \boldsymbol{x}^\top (R_{XX}\boldsymbol{x}+R_{XY}\boldsymbol{y})+\boldsymbol{y}^\top (R_{YX}\boldsymbol{x}+R_{YY}\boldsymbol{y}). \end{aligned} \tag{11}$$

When the equality holds in Eq. (11), it is also required that an additional condition given by

$$\begin{aligned} &\boldsymbol{x}'^\top (R_{XX}\boldsymbol{x}'+R_{XY}\boldsymbol{y}')+\boldsymbol{y}'^\top (R_{YX}\boldsymbol{x}'+R_{YY}\boldsymbol{y}') \\ &< \boldsymbol{x}^\top (R_{XX}\boldsymbol{x}'+R_{XY}\boldsymbol{y}')+\boldsymbol{y}^\top (R_{YX}\boldsymbol{x}'+R_{YY}\boldsymbol{y}') \end{aligned} \tag{12}$$

is satisfied.

In the case of the subjective payoff game, I obtain $m=n=2$,

$$R_{XX}=p\begin{pmatrix} a_X & b_X \\ c_X & d_X \end{pmatrix}, \tag{13}$$

$$R_{XY}=(1-p)\begin{pmatrix} a_X & b_X \\ c_X & d_X \end{pmatrix}, \tag{14}$$

$$R_{YX}=p\begin{pmatrix} a_Y & b_Y \\ c_Y & d_Y \end{pmatrix}, \tag{15}$$

$$R_{YY}=(1-p)\begin{pmatrix} a_Y & b_Y \\ c_Y & d_Y \end{pmatrix}, \tag{16}$$

$x_1=x$, $x_2=1-x$, $y_1=y$, and $y_2=1-y$. Therefore, Eqs. (11) and (12) are reduced to

$$\begin{aligned} &[(x-x')(a_X-c_X\ \ b_X-d_X)+(y-y')(a_Y-c_Y\ \ b_Y-d_Y)] \\ &\cdot\left[p\begin{pmatrix} x \\ 1-x \end{pmatrix}+(1-p)\begin{pmatrix} y \\ 1-y \end{pmatrix}\right]\ge 0 \end{aligned} \tag{17}$$

and

$$\begin{aligned} &[(x-x')(a_X-c_X\ \ b_X-d_X)+(y-y')(a_Y-c_Y\ \ b_Y-d_Y)] \\ &\cdot\left[p\begin{pmatrix} x' \\ 1-x' \end{pmatrix}+(1-p)\begin{pmatrix} y' \\ 1-y' \end{pmatrix}\right]> 0 \end{aligned} \tag{18}$$

respectively.

3.4. Pure strategy ESSs

In this section, let us identify the pure strategy ESSs of the subjective payoff game. First, suppose that the population in which all players in both subpopulations adopt strategy A is evolutionarily stable. By substituting $x=y=1$ in Eq. (17), I obtain

$$(1-x')(a_X-c_X)+(1-y')(a_Y-c_Y)\ge 0. \tag{19}$$

Because Eq. (19) must hold true for $0\le x'<1$ and $y'=1$, a necessary condition reads $a_X\ge c_X$. If $a_X\ge c_X$ is satisfied with equality, Eq. (18) for the same $0\le x'<1$ and $y'=1$, i.e.,

$$(1-x')(a_X-c_X\ \ b_X-d_X)\left[p\begin{pmatrix} x' \\ 1-x' \end{pmatrix}+(1-p)\begin{pmatrix} 1 \\ 0 \end{pmatrix}\right]>0 \tag{20}$$

must be satisfied. By substituting $a_X = c_X$ in Eq. (20), I obtain $b_X > d_X$. The necessary conditions obtained so far are summarized as

$$a_X > c_X \quad \text{or } (a_X = c_X \text{ and } b_X > d_X). \tag{21}$$

These conditions are the same as the ESS conditions for the structureless population. By considering the mutant parametrized by $x' = 1$ and $0 \le y' < 1$, I similarly obtain the necessary conditions for subpopulation Y as

$$a_Y > c_Y \quad \text{or } (a_Y = c_Y \text{ and } b_Y > d_Y). \tag{22}$$

On the other hand, if Eqs. (21) and (22) are satisfied, Eqs. (17) and (18) are satisfied for any $(x', y') \ne (1,1)$. Therefore, Eqs. (21) and (22) provide the necessary and sufficient conditions for strategy A to be evolutionarily stable. In conclusion, A is evolutionarily stable for the entire population if A is evolutionarily stable in each subpopulation in the ordinary sense. Similarly, B is an ESS of the subjective payoff game if B is evolutionarily stable in each subpopulation.

Next, assume that the population in which all the players in subpopulations X and Y adopt A and B, respectively, is evolutionarily stable. By substituting $x=1$ and $y=0$ in Eq. (17), I obtain

$$(1-x')[(a_X - c_X)p + (b_X - d_X)(1-p)] + y'[(a_Y - c_Y)p + (b_Y - d_Y)(1-p)] \ge 0. \tag{23}$$

Because Eq. (23) must hold true for $0 \le x' < 1$ and $y' = 0$, a necessary condition reads

$$a_X p + b_X(1-p) \ge c_X p + d_X(1-p). \tag{24}$$

If Eq. (24) is satisfied with equality, Eq. (18) for the same $0 \le x' < 1$ and $y' = 0$, i.e.,

$$(1-x')(a_X - c_X \;\; b_X - d_X)\left[p\begin{pmatrix} x' \\ 1-x' \end{pmatrix} + (1-p)\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right] > 0, \tag{25}$$

must be satisfied. By substituting $a_X p + b_X(1-p) = c_X p + d_X(1-p)$ in Eq. (25), I obtain

$$b_X > d_X. \tag{26}$$

Similarly, by considering the mutant parametrized by $x' = 1$ and $0 \le y' < 1$, I obtain

$$a_Y p + b_Y(1-p) \le c_Y p + d_Y(1-p). \tag{27}$$

When Eq. (27) is satisfied with equality, Eq. (18) and $a_Y p + b_Y(1-p) = c_Y p + d_Y(1-p)$ lead to

$$a_Y < c_Y. \tag{28}$$

The population given by (x,y)=(1,0) is an ESS if Eqs. (24) and (27) are satisfied, Eq. (26) holds true when Eq. (24) is satisfied with equality, and Eq. (28) holds true when Eq. (27) is satisfied with equality. It should be noted that the conditions given by Eqs. (24) and (27) depend on p.

3.5. Non-equivalence to the bimatrix game

In this section, I show that the replicator equation of the subjective payoff game cannot be mapped to the replicator equation of a bimatrix game. It should be noted that the following arguments can be readily generalized to the case of an arbitrary number of strategies.

In the bimatrix game in the well-mixed population, we consider all possible pairs of a player in subpopulation X and a player in subpopulation Y. The two selected players are involved in a two person game, which is generally asymmetric. The payoff bimatrix is given by

$$\begin{array}{cc} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{pmatrix} (\tilde{a}_X, \tilde{a}_Y) & (\tilde{b}_X, \tilde{c}_Y) \\ (\tilde{c}_X, \tilde{b}_Y) & (\tilde{d}_X, \tilde{d}_Y) \end{pmatrix} \end{array}, \tag{29}$$

where the first and second elements in each entry of the bimatrix represent the payoffs imparted to an X player and a Y player, respectively. The row and column players correspond to subpopulations X and Y, respectively. The payoff to an X player with strategies A and B is equal to $\tilde{a}_X y + \tilde{b}_X(1-y)$ and $\tilde{c}_X y + \tilde{d}_X(1-y)$, respectively. Then, the replicator equation for subpopulation X is given by

$$\dot{x} = x(1-x)[(\tilde{a}_X - \tilde{c}_X)y + (\tilde{b}_X - \tilde{d}_X)(1-y)]. \tag{30}$$

Similarly,

$$\dot{y} = y(1-y)[(\tilde{a}_Y - \tilde{c}_Y)x + (\tilde{b}_Y - \tilde{d}_Y)(1-x)]. \tag{31}$$

If the dynamics given by Eqs. (30) and (31) is equivalent to that for the subjective payoff game Eqs. (4) and (5), the comparison of Eqs. (4) and (30) yields $p(a_X - b_X - c_X + d_X) = 0$ because the right-hand side of Eq. (4) must be independent of x except for the multiplication factor $x(1-x)$. Because $p=0$ implies a structureless population, $a_X - b_X - c_X + d_X = 0$ holds true. Under this condition, Eq. (4) is reduced to $\dot{x} = x(1-x)(b_X - d_X)$. Then, $\tilde{a}_X - \tilde{b}_X - \tilde{c}_X + \tilde{d}_X = 0$ and $\tilde{b}_X - \tilde{d}_X = b_X - d_X$ must hold true. Similarly, $a_Y - b_Y - c_Y + d_Y = \tilde{a}_Y - \tilde{b}_Y - \tilde{c}_Y + \tilde{d}_Y = 0$ and $\tilde{b}_Y - \tilde{d}_Y = b_Y - d_Y$ must hold true. Except for this degenerate case, the two dynamics are not mapped from one to the other.

4. Examples

4.1. Snowdrift game

Consider the snowdrift game, also called the chicken game or the hawk-dove game, which represents a social dilemma situation (Sugden, 1986; Hauert and Doebeli, 2004). The snowdrift game in two subpopulations in which the payoff matrix for a player depends on the opponent's subpopulation as well as the focal player's subpopulation is analyzed in Auger et al. (2001). In the case without population structure, a standard payoff matrix for the snowdrift game is given by

$$\begin{array}{cc} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{pmatrix} \beta - \frac{1}{2} & \beta - 1 \\ \beta & 0 \end{pmatrix} \end{array}, \tag{32}$$

where $\beta > 1$. Strategies A and B correspond to cooperation and defection, respectively. If the opponent cooperates, it is better to defect. Otherwise, it is better to cooperate. The mixed population with a fraction of A players given by $x^* = (2\beta - 2)/(2\beta - 1)$ is the unique ESS.

Consider the case in which the β value depends on the subpopulation. Denote by β_X and β_Y the subpopulation-dependent β value such that

$$\begin{pmatrix} a_X & b_X \\ c_X & d_X \end{pmatrix} = \begin{pmatrix} \beta_X - \frac{1}{2} & \beta_X - 1 \\ \beta_X & 0 \end{pmatrix} \tag{33}$$

and

$$\begin{pmatrix} a_Y & b_Y \\ c_Y & d_Y \end{pmatrix} = \begin{pmatrix} \beta_Y - \frac{1}{2} & \beta_Y - 1 \\ \beta_Y & 0 \end{pmatrix}. \tag{34}$$

Without loss of generality, I assume that $\beta_X > \beta_Y > 1$.

Equations (4) and (5) read, respectively,

$$\dot{x} = x(1-x)\left\{(\beta_X - 1) - \left(\beta_X - \frac{1}{2}\right)[px + (1-p)y]\right\}, \tag{35}$$

$$\dot{y} = y(1-y)\left\{(\beta_Y - 1) - \left(\beta_Y - \frac{1}{2}\right)[px + (1-p)y]\right\}. \tag{36}$$

Therefore, I obtain

$$\begin{cases} \dot{x} > 0,\ \dot{y} > 0 & \text{if } px + (1-p)y < \frac{\beta_Y - 1}{\beta_Y - 1/2}, \\ \dot{x} > 0,\ \dot{y} < 0 & \text{if } \frac{\beta_Y - 1}{\beta_Y - 1/2} < px + (1-p)y < \frac{\beta_X - 1}{\beta_X - 1/2}, \\ \dot{x} < 0,\ \dot{y} < 0 & \text{if } px + (1-p)y > \frac{\beta_X - 1}{\beta_X - 1/2}. \end{cases} \tag{37}$$

Equation (37) implies that the replicator dynamics has the unique stable equilibrium whose location depends on the p value. If $0 < p < (\beta_Y - 1)/(\beta_Y - 1/2)$, the stable equilibrium is located at

$$(x^*, y^*) = \left(1, -\frac{p}{1-p} + \frac{\beta_Y - 1}{(1-p)(\beta_Y - 1/2)}\right). \tag{38}$$

An example of this case is shown in Fig. 1(a). This equilibrium is an ESS, which is shown in Appendix with the use of the ESS criterion established in Taylor (1979). If $(\beta_Y - 1)/(\beta_Y - 1/2) \le p \le (\beta_X - 1)/(\beta_X - 1/2)$, the stable equilibrium is located at

$$(x^*, y^*) = (1, 0). \tag{39}$$

An example of this case is shown in Fig. 1(b). In this case, the equilibrium is an ESS because Eqs. (24), (26), (27), and (28) are satisfied. Finally, if $(\beta_X - 1)/(\beta_X - 1/2) < p < 1$, the stable equilibrium is located at

$$(x^*, y^*) = \left(\frac{\beta_X - 1}{p(\beta_X - 1/2)}, 0\right). \tag{40}$$

An example of this case is shown in Fig. 1(c). This equilibrium is also an ESS (see Appendix for the proof). In all three cases, X, i.e., the subpopulation with the larger β value, has a larger fraction of A players than Y does.

In particular, at least one subpopulation is monomorphic for any p; A monopolizes subpopulation X, or B monopolizes subpopulation Y. Even a slight difference in the payoff matrix (i.e., β value) in the two subpopulations yields polarization of the strategies. Similar polarization also occurs in the bimatrix snowdrift game. However, the internal equilibrium exists but is a saddle in the case of the bimatrix game (Hofbauer and Sigmund, 1998) such that the mechanism is different from that for the subjective payoff snowdrift game.

4.2. Coordination game

Consider a coordination game in which the players in different subpopulations have different preferences to one strategy over the other. Specifically, let us set the payoff matrix for subpopulations X and Y to

$$\begin{pmatrix} a_X & b_X \\ c_X & d_X \end{pmatrix} = \begin{pmatrix} 1+\alpha_X & 0 \\ 0 & 1-\alpha_X \end{pmatrix} \tag{41}$$

and

$$\begin{pmatrix} a_Y & b_Y \\ c_Y & d_Y \end{pmatrix} = \begin{pmatrix} 1+\alpha_Y & 0 \\ 0 & 1-\alpha_Y \end{pmatrix}, \tag{42}$$

respectively, where $-1 < \alpha_Y < 0 < \alpha_X < 1$. Players in X and Y prefer strategies A and B, respectively. Eqs. (41) and (42) can be regarded as a payoff bimatrix of the Battle of the Sexes game (Luce and Raiffa, 1957). However, in the bimatrix game, the population is composed of two subpopulations corresponding to the roles in the game. In contrast, the players in the present game do not have roles and interact in a well-mixed population.

Given Eqs. (41) and (42), Eqs. (4) and (5) read, respectively,

$$\dot{x} = x(1-x)[p(2x-1) + (1-p)(2y-1) + \alpha_X], \tag{43}$$

$$\dot{y} = y(1-y)[p(2x-1) + (1-p)(2y-1) + \alpha_Y]. \tag{44}$$

Near $x = y = 1$, $p(2x-1) + (1-p)(2y-1) + \alpha_X > p(2x-1) + (1-p)(2y-1) + \alpha_Y > 0$ is satisfied. Therefore, $(x^*, y^*) = (1, 1)$ is a stable equilibrium of the replicator equations of the subjective payoff coordination game. Because Eqs. (21) and (22) are satisfied with Eqs. (41) and (42), $(x^*, y^*) = (1, 1)$ is an ESS. Similarly, because $p(2x-1) + (1-p)(2y-1) + \alpha_Y < p(2x-1) + (1-p)(2y-1) + \alpha_X < 0$ near $x = y = 0$, $(x^*, y^*) = (0, 0)$ is a stable equilibrium. Because Eqs. (41) and (42) yield $b_X < d_X$ and $b_Y < d_Y$, $(x^*, y^*) = (0, 0)$ is another ESS. These results are consistent with those for the coordination game without population structure; the two strategies are bistable.

The subjective payoff version of the coordination game yields two phenomena that are absent in the same game without

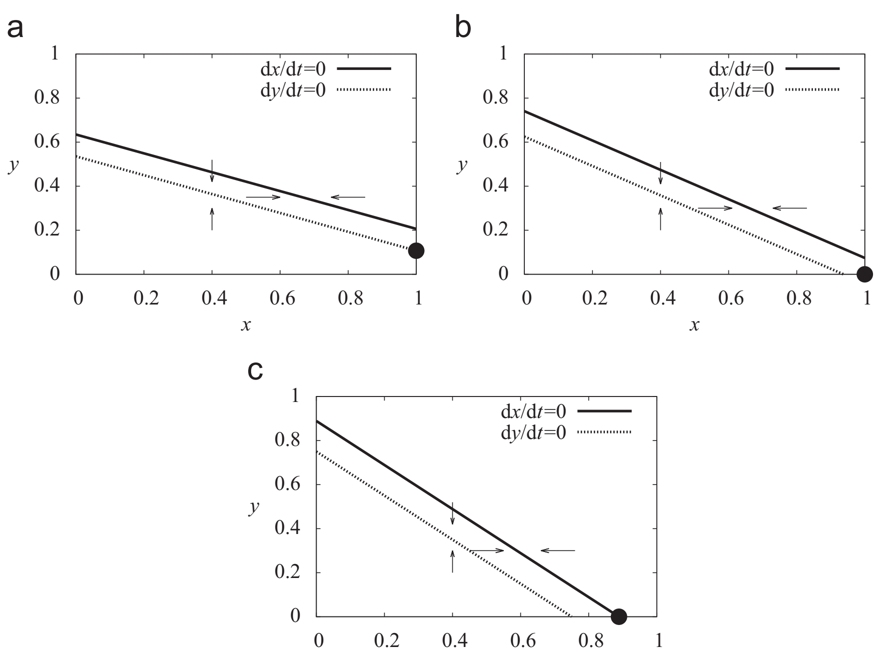

Fig. 1. The isoclines and direction field for the replicator dynamics of the subjective payoff snowdrift game with $\beta_X = 1.4$ and $\beta_Y = 1.3$. The filled circles indicate the ESSs. (a) $p = 0.3$. (b) $p = 0.4$. (c) $p = 0.5$.

population structure. First, the polarized configuration in which all the X players adopt A and all the Y players adopt B is an stable equilibrium if

$$\frac{1-\alpha_X}{2} < p < \frac{1-\alpha_Y}{2}. \quad (45)$$

When Eq. (45) is satisfied, Eqs. (24) and (27) are satisfied with inequality such that this population is a pure-strategy ESS. Equation (45) is satisfied if p is close to 1/2 or if α_X or $-\alpha_Y(>0)$, i.e., the asymmetry in the liking of the two actions, is large. As an example, the attractive basins of the three equilibria for p=0.5, $\alpha_X = 0.3$, and $\alpha_Y = -0.2$ are shown in Fig. 2(a). The final configuration of the population depends on the initial condition. It should be noted that stable coexistence of the opposite pure strategies does not occur in the bimatrix coordination game (i.e., Battle of the Sexes game).

Second, the fraction of players employing a strategy in a subpopulation can non-monotonically change in time. Some non-monotonic trajectories starting from different initial conditions are shown in Fig. 2(b) with p=0.7, $\alpha_X = 0.2$, and $\alpha_Y = -0.2$. $\dot{x} > 0$ holds true to the right of the thick solid line in Fig. 2(b). If the initial condition is located slightly right to this line, y first decreases because $\dot{y} < 0$ holds true to the left of the thick dotted line. If $\dot{x}(>0)$ is not large, the trajectory eventually crosses the thick solid line ($\dot{x} = 0$) such that x starts to decrease. If the initial $\dot{x}$ value is large enough, the trajectory eventually crosses the thick dotted line ($\dot{y} = 0$) such that y starts to increase. In both cases, the trajectory shows non-monotonic behavior. Such a non-monotonic behavior does not occur in the coordination game without population structure.

In extensions of the Ising model (Galam, 1997) and the voter model (Masuda et al., 2010; Masuda and Redner, 2011), which are non-game population dynamics, idiosyncratic preferences of individuals lead to coexistence of different states, where states are equivalent to strategies in games. The present results are consistent with these results in that idiosyncratic preferences let multiple states coexist when unanimity necessarily occurs in the absence of idiosyncrasy.

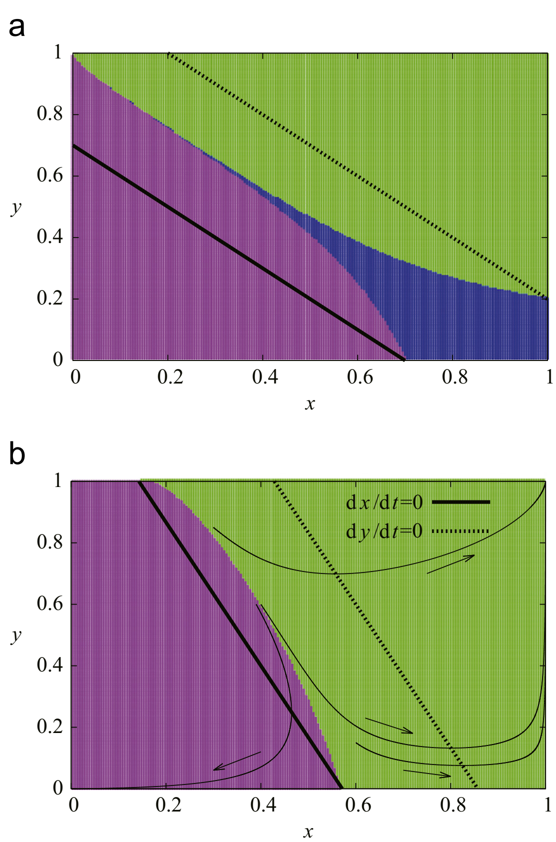

Fig. 2. Replicator dynamics of the subjective payoff coordination game. The magenta, green, and blue regions represent the attractive basins for $(x^*, y^*) = (0,0)$, (1,1), and (1,0), respectively. The thick solid lines represent $\dot{x} = 0$, i.e., $y = -px/(1-p) + (1-\alpha_X)/2(1-p)$. The thick dotted lines represent $\dot{y} = 0$, i.e., $y = -px/(1-p) + (1-\alpha_Y)/2(1-p)$. (a) p=0.5, $\alpha_X = 0.3$, and $\alpha_Y = -0.2$. (b) p=0.7, $\alpha_X = 0.2$, and $\alpha_Y = -0.2$. The thin solid curves in (b) represent trajectories converging to $(x^*, y^*) = (0,0)$ or (1,1). For calculating the attractive basins and individual trajectories, the Euler scheme with $dt = 0.005$ was used. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

4.3. Iterated prisoner's dilemma

In this section, I examine the possibility of cooperation in the iterated prisoner's dilemma (IPD) in which the unconditional cooperation (C) and unconditional defection (D) are strategies A and B, respectively. I do not assume error in action implementation and do assume that a next round of the game between the same pair of players occurs with probability w $(0 < w < 1)$. The following results also hold true if C is replaced by the tit-for-tat (TFT) or the so-called GRIM strategy. TFT starts with cooperation and selects the previous action (i.e., cooperate or defect) selected by the opponent. GRIM strategy starts with cooperation and switches to permanent defection once the opponent ever defects. The invariance of the following results holds true because the payoff matrix for the IPD, i.e., Eq. (47) below, does not change when C is replaced by TFT or GRIM.

Consider a standard payoff matrix for the single-shot prisoner's dilemma given by

$$\begin{array}{c} \\ \text{C} \\ \text{D} \end{array} \begin{array}{c} \begin{array}{cc} \text{C} & \text{D} \end{array} \\ \begin{pmatrix} b-c & -c \\ b & 0 \end{pmatrix} \end{array}, \quad (46)$$

where b and c are the benefit and the cost of cooperation and satisfy $b > c > 0$. The expected payoff matrix for the IPD in the structureless population is given by

$$\begin{array}{c} \\ \text{C} \\ \text{D} \end{array} \begin{array}{c} \begin{array}{cc} \text{C} & \text{D} \end{array} \\ \begin{pmatrix} \frac{b-c}{1-w} & -c \\ b & 0 \end{pmatrix} \end{array}. \quad (47)$$

If

$$w > w_{\text{crit}} \equiv \frac{c}{b}, \quad (48)$$

the prisoner's dilemma is effectively transformed into a coordination game such that mutual cooperation by C and mutual defection by D are bistable (Axelrod, 1984; Nowak, 2006).

Let us consider the situation in which two subpopulations possess different discount factors w_X and w_Y. In other words, assume

$$\begin{pmatrix} a_X & b_X \\ c_X & d_X \end{pmatrix} = \begin{pmatrix} \frac{b-c}{1-w_X} & -c \\ b & 0 \end{pmatrix} \quad (49)$$

and

$$\begin{pmatrix} a_Y & b_Y \\ c_Y & d_Y \end{pmatrix} = \begin{pmatrix} \frac{b-c}{1-w_Y} & -c \\ b & 0 \end{pmatrix}. \quad (50)$$

Because the duration of IPD is the same for the two players, I interpret that X players put more emphasis on long-term benefits than Y players. Specifically, I assume

$$w_Y < \frac{c}{b} < w_X. \quad (51)$$

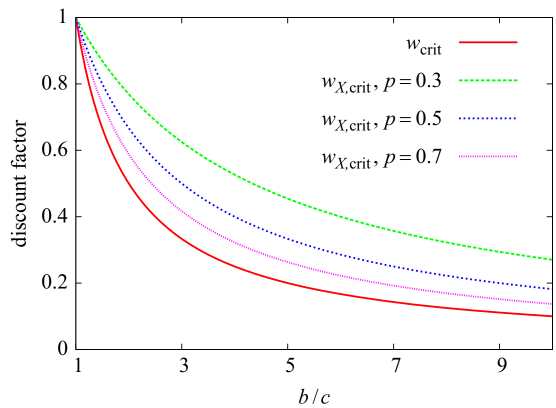

Fig. 3. Threshold discount factors above which C is locally stable in the IPD. In the subjective payoff game, I set $p=0.3$, $p=0.5$, and $p=0.7$.

Given Eq. (51), Eq. (5) implies $\dot{y} < 0$ for all $0 \le x,\ y \le 1$. Therefore, D eventually occupies subpopulation Y. I examine the possibility that cooperation occurs in subpopulation X. On the line $y=0$, Eq. (4) is reduced to

$$\dot{x} = x(1-x)\left[\frac{(b-c)w_X p x}{1-w_X} - c\right]. \tag{52}$$

Therefore, the population in which D dominates in both subpopulations, i.e., $(x^*, y^*) = (0, 0)$, is always a stable equilibrium of the replicator equations of the subjective payoff IPD game. It is an ESS because Eqs. (49) and (50) imply $b_X < d_X$ and $b_Y < d_Y$, respectively.

Equation (52) implies that the combination of C in subpopulation X and D in subpopulation Y is a stable equilibrium if

$$w_X > w_{X,\mathrm{crit}} \equiv \frac{1}{\left(\frac{b}{c} - 1\right)p + 1}. \tag{53}$$

Because Eqs. (24) and (27) are satisfied with inequality when Eq. (53) holds true, this population is an ESS. A large p (i.e., large fraction of X players) and a large benefit-to-cost ratio b/c lessen $w_{X,\mathrm{crit}}$ such that cooperation would occur.

The threshold discount factors w_{crit} and $w_{X,\mathrm{crit}}$ are compared in Fig. 3 for some p values. The figure indicates that for a wide range of p, the condition for cooperation in the subjective payoff case is not very severe relative to the case without population structure. In particular, both w_{crit} and $w_{X,\mathrm{crit}}$ tend to unity in the limit $b/c \to 1$. As $b/c \to \infty$, it follows that $w_{X,\mathrm{crit}}/w_{\mathrm{crit}} \to 1/p$ and both w_{crit} and $w_{X,\mathrm{crit}}$ converge to 0. When $p = 1/2$, condition (53) coincides with the condition for the risk dominance of C over D in the structureless population, i.e., $w > 2c/(b+c)$.

5. Discussion

I proposed the so-called subjective payoff game and its replicator dynamics. The model is mathematically a special case of the previously analyzed model with two subpopulations (Taylor, 1979; Schuster et al., 1981a). However, the present model is motivated by the possibility that different players may perceive the same result of the game to transform it to the fitness in different manners. The model shows polarization in the snowdrift and coordination games, non-monotonic time courses in the coordination game, and a wide margin of cooperation in the IPD. Extension of the present model to the case of more than two strategies and more than two subpopulations is straightforward. Generalizing the present results for such extended models warrants future work.

The replicator dynamics of the subjective payoff game is different from that of the bimatrix game (Section 3.5). In addition, the subjective payoff game cannot be mapped to a model with strategy-dependent interaction rates, which does not have multiple subpopulations within each of which imitation occurs (Taylor and Nowak, 2006). The subjective payoff game is also different from those in which interaction is confined in single subpopulations, such as group selection models (Wilson, 1975; West et al., 2007), island model (Taylor, 1992), and evolutionary set theory (Tarnita et al., 2009).

The present model is distinct from previous models of evolution of preference (e.g., Sandholm, 2001; Dekel et al., 2007; Alger and Weibull, 2012; Grund et al., 2013) and the so-called subjective game (Kalai and Lehrer, 1995; Matsushima, 1997; Oechssler and Schipper, 2003). Both in these and present models, the preference, or the subjective payoff, is assumed to be consistent within each individual (Gintis, 2009). In these previous models, the utility that a player maximizes and the fitness on which the selection pressure operates are different. A player in such a model is rational enough to be able to personally maximize the player's idiosyncratic utility. In the present model, as in standard evolutionary models, a player is subjected to bounded rationality and tends to imitate successful others (i.e., social learning). The difference from standard evolutionary models is that, in the present model, each player limits the set of possible parents from whom the strategy is copied to those with the same idiosyncratic payoff. In this way, the player can pursue both maximization of fitness via social learning and consistency with the player's idiosyncratic preference.

The subjective payoff game does not allow internal equilibria regardless of the stability. This result has implications in games in which internal equilibria play an important role in structureless populations. In the snowdrift game, a mixture of the two strategies is stable under ordinary replicator dynamics. In contrast, in the subjective payoff game, a slight difference in the payoff matrices perceived by the two subpopulations leads to polarization such that the two subpopulations tend to select the opposite strategies.

The rock–scissors–paper game comprises three strategies that cyclically dominate one another. It is straightforward to show that there is no internal equilibrium in the subjective payoff game with a general number of strategies. Therefore, the subjective payoff variant of the rock–scissors–paper game lacks the internal equilibrium of any kind and limit cycles. Such a game behaves very differently from the same game played in the structureless population (Hofbauer and Sigmund, 1988, 1998; Nowak, 2006), bimatrix population (Hofbauer and Sigmund, 1988, 1998; Sato et al., 2002), and two subpopulations with different social learning rates (Masuda, 2008); these models allow a unique internal equilibrium. It may be interesting to examine the rock–scissors–paper game under the current framework.

I assumed that players imitate others in the same subpopulation. In fact, there may be competition of update rules between such players and those that imitate from the entire population. Nevertheless, at least near pure stable equilibria, the population is considered to be resistant against invasion by mutants that imitate from the entire population. To explain why, let us suppose that X and Y players select A and B in the equilibrium. A mutant that imitates from the entire population and attempts to invade subpopulation X would sometimes select B because Y players select B. Because A, not B, is the best response in this population, such a mutant is considered not able to invade the subpopulation of resident players. Therefore, the imitation rule considered in the present study is considered to have evolutionary stability, at least in this case.

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Appendix

In this section, I show that the equilibria given by Eqs. (38) and (40) are ESSs of the subjective payoff snowdrift game. To this end, I use a matrix criterion for the ESS (Taylor, 1979) accommodated to the case of the two-strategy game.

Assume that a population given by (x^*, y^*) satisfies

$$(x' \; 1-x')\left[R_{XX}\begin{pmatrix} x^* \\ 1-x^* \end{pmatrix} + R_{XY}\begin{pmatrix} y^* \\ 1-y^* \end{pmatrix}\right] \le (x^* \; 1-x^*)\left[R_{XX}\begin{pmatrix} x^* \\ 1-x^* \end{pmatrix} + R_{XY}\begin{pmatrix} y^* \\ 1-y^* \end{pmatrix}\right] \tag{54}$$

and

$$(y' \; 1-y')\left[R_{YX}\begin{pmatrix} x^* \\ 1-x^* \end{pmatrix} + R_{YY}\begin{pmatrix} y^* \\ 1-y^* \end{pmatrix}\right] \le (y^* \; 1-y^*)\left[R_{YX}\begin{pmatrix} x^* \\ 1-x^* \end{pmatrix} + R_{YY}\begin{pmatrix} y^* \\ 1-y^* \end{pmatrix}\right] \tag{55}$$

for $(x', y') = (0,0)$, $(1,0)$, $(0,1)$, and $(1,1)$. I also assume that Eqs. (54) and (55) are satisfied with equality for $(x', y') = (0,0)$ if $x^* < 1$ and $y^* < 1$, for $(x', y') = (1,0)$ if $x^* > 0$ and $y^* < 1$, for $(x', y') = (0,1)$ if $x^* < 1$ and $y^* > 0$, and for $(x', y') = (1,1)$ if $x^* > 0$ and $y^* > 0$. The criterion dictates that (x^*, y^*) is an ESS if and only if

$$(x_1 \; -x_1 \; y_1 \; -y_1)\begin{pmatrix} R_{XX} & R_{XY} \\ R_{YX} & R_{YY} \end{pmatrix}\begin{pmatrix} x_1 \\ -x_1 \\ y_1 \\ -y_1 \end{pmatrix} < 0. \tag{56}$$

Here, $x_1 \neq 0$ ($y_1 \neq 0$) if and only if the payoff of a pure A player in subpopulation X (Y) and that of a pure B player in subpopulation X (Y) are the same in the equilibrium of interest.

Under the snowdrift game, the assumptions for (x^*, y^*) are satisfied when (x^*, y^*) is given by Eq. (38) or (40). Substitution of Eqs. (13), (14), (15), (16), (33), and (34) in Eq. (56) yields

$$[px_1 + (1-p)y_1] \cdot \left[x_1\left(\frac{1}{2} - \beta_X\right) + y_1\left(\frac{1}{2} - \beta_Y\right)\right] < 0. \tag{57}$$

For the equilibrium given by Eq. (38) to be an ESS, Eq. (57) must be satisfied for $x_1 = 0$ and $y_1 \neq 0$. For the equilibrium given by Eq. (40) to be an ESS, Eq. (57) must be satisfied for $x_1 \neq 0$ and $y_1 = 0$. In fact, Eq. (57) is satisfied in both cases. Therefore, the two equilibria are ESSs.

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