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Stringy unification of type IIA and IIB supergravities under $\mathcal{N} = 2$ $D = 10$ supersymmetric double field theory

Imtak Jeon^a, Kanghoon Lee^b, Jeong-Hyuck Park^{c,*}, Yoonji Suh^c

^a Harish-Chandra Research Institute, Chhatnag Road, Jhusi, Allahabad 211019, India

^b Blackett Laboratory, Imperial College, London SW7 2AZ, UK

^c Department of Physics, Sogang University, Seoul 121-742, Republic of Korea

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ABSTRACT

To the full order in fermions, we construct $D = 10$ type II supersymmetric double field theory. We spell the precise $\mathcal{N} = 2$ supersymmetry transformation rules as for 32 supercharges. The constructed action unifies type IIA and IIB supergravities in a manifestly covariant manner with respect to $\mathbf{O}(10, 10)$ T-duality and a pair of local Lorentz groups, or $\mathbf{Spin}(1, 9) \times \mathbf{Spin}(9, 1)$, besides the usual general covariance of supergravities or the generalized diffeomorphism. While the theory is unique, the solutions are twofold. Type IIA and IIB supergravities are identified as two different types of solutions rather than two different theories.

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1. Introduction

Strings perceive spacetime in a different way than particles do through Riemannian geometry. While the fundamental object in Riemannian geometry is the metric, string theory puts the Kalb–Ramond B -field and a scalar dilaton on an equal footing along with the metric, since they form a multiplet of T-duality [1–3], a genuine stringy property which is not present in ordinary particle theory.

Although type IIA and IIB supergravities provide low energy effective descriptions of closed superstrings, once formulated within the Riemannian setup, they appear unable to capture the full stringy structure like T-duality or to explain the appearance of enhanced symmetries after dimensional reductions [4,5]. String theory seems to urge us to look for a novel mathematical framework, such as Generalized Geometry [6–8] or Double Field Theory (DFT) [9–12] (see also [13,14] for relevant pioneering works).

While generalized geometry combines tangent and cotangent spaces giving a geometric meaning to the B -field [15,16], DFT doubles the spacetime dimension, from D to $D + D$ in order to manifest the $\mathbf{O}(D, D)$ T-duality group structure [13,14,17,18]. With an additional requirement of so-called *strong constraint* or *section condition*, DFT reduces to a known string theory effective action in D -dimension. The section condition means that all the DFT-fields live on a D -dimensional null hyperplane such that, the $\mathbf{O}(D, D)$

invariant d'Alembertian operator is trivial acting on arbitrary fields as well as their products,

$$\partial_A \partial^A = \mathcal{J}^{AB} \partial_A \partial_B \simeq 0, \quad \mathcal{J}^{AB} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \quad (1)$$

DFT unifies the B -field gauge symmetry and the diffeomorphism, as both are generated by generalized Lie derivative [6,19] (see also [20] for finite transformations),

$$\hat{\mathcal{L}}_X T_{A_1 \dots A_n} := X^B \partial_B T_{A_1 \dots A_n} + \omega_T \partial_B X^B T_{A_1 \dots A_n} + \sum_{i=1}^n (\partial_{A_i} X_B - \partial_B X_{A_i}) T_{A_1 \dots A_{i-1} B A_{i+1} \dots A_n}. \quad (2)$$

Further, recent study of the Scherk–Schwarz reduction in DFT has shown that, by relaxing the section condition (1)—and hence in a truly non-Riemannian set up—one may derive all the known gauged supergravities in lower than ten dimensions [21–25]. This seems to indicate the potential power of DFT and motivates further explorations.

In this work, we construct $\mathcal{N} = 2$ $D = 10$ supersymmetric double field theory (SDFT). We carry out the construction employing *genuine SDFT field-variables* which are subject to the section condition (1) and differ *a priori* from Riemannian, or supergravity variables. For example, ordinary zehnbains and various form-fields will never enter in our construction. We tend to believe that the usage of the genuine SDFT field-variables is quite crucial and it essentially ensures the following properties of the final results.

* Corresponding author.

E-mail address: park@sogang.ac.kr (J.-H. Park).

Table 1

Index for each symmetry representation and the corresponding “metric” to raise or lower the positions. For further details and a review on the formalism, we refer the reader to the appendix of [37].

Index	Representation	Raising and lowering indices
A, B, \dots	$\mathbf{O}(10, 10)$ and $\hat{\mathcal{L}}_X$ vector	\mathcal{J}_{AB}
p, q, \dots	$\mathbf{Pin}(1, 9)$ vector	$\eta_{pq} = \text{diag}(- + + \dots +)$
α, β, \dots	$\mathbf{Pin}(1, 9)$ spinor	$C_{+\alpha\beta}, (\gamma^p)^T = C_+ \gamma^p C_+^{-1}$
\bar{p}, \bar{q}, \dots	$\mathbf{Pin}(9, 1)$ vector	$\bar{\eta}_{\bar{p}\bar{q}} = \text{diag}(+ - - \dots -)$
$\bar{\alpha}, \bar{\beta}, \dots$	$\mathbf{Pin}(9, 1)$ spinor	$\bar{C}_{+\bar{\alpha}\bar{\beta}}, (\bar{\gamma}^{\bar{p}})^T = \bar{C}_+ \bar{\gamma}^{\bar{p}} \bar{C}_+^{-1}$

- Each term in the constructed Lagrangian is manifestly and simultaneously covariant with respect to $\mathbf{O}(10, 10)$ T-duality, a pair of local Lorentz groups, $\mathbf{Spin}(1, 9) \times \mathbf{Spin}(9, 1)$, and the DFT-diffeomorphism generated by $\hat{\mathcal{L}}_X$ in (2).
- The supersymmetric completion is fulfilled to the full order in fermions.
- Further, $\mathcal{N} = 2$ $D = 10$ SDFT unifies type IIA and IIB supergravities: While the theory is unique, the solutions are twofold, type IIA and type IIB.

Related key precedents include Refs. [26–29]. In [26,27], within the generalized geometry setup in terms of a pair of zehnbins and various form-fields, the type II supergravity was reformulated into a $\mathbf{Spin}(1, 9) \times \mathbf{Spin}(9, 1)$ covariant form (up to quadratic order in fermions). In [28,29], the bosonic part of type II SDFT was proposed which in particular put the R–R sector in an $\mathbf{O}(10, 10)$ spinorial representation, as in [30,31]. In our case, the R–R sector is in a $\mathbf{Spin}(1, 9) \times \mathbf{Spin}(9, 1)$ bi-fundamental spinorial representation, e.g. ‘ $C^\alpha_{\bar{\alpha}}$ ’. Table 1 summarizes our index gymnastics.

2. Field content

We postulate the fundamental fields of type II SDFT to be strictly, from [32–37],

$$d, V_{Ap}, \bar{V}_{A\bar{p}}, C^\alpha_{\bar{\alpha}}, \psi_{\bar{p}}^\alpha, \rho^\alpha, \psi'_p{}^{\bar{\alpha}}, \rho'^{\bar{\alpha}}. \quad (3)$$

We wish to stress that, for the sake of the full covariance and the (relatively) compact way of full order supersymmetric completion, it is crucial to set the fundamental fields to be precisely those above. Although some of them may be parametrized in terms of Riemannian zehnbins and form-fields, the parametrization is not unique, may render “non-geometric” interpretations, and will certainly becloud the whole symmetry structure listed in Table 1.

Firstly for the NS–NS sector, the DFT-dilaton, d , gives rise to a scalar density with weight one, e^{-2d} [10]. The DFT-vielbeins, $V_{Ap}, \bar{V}_{A\bar{p}}$, satisfy the following four defining properties [34,35]:

$$\begin{aligned} V_{Ap} V^A{}_q &= \eta_{pq}, & \bar{V}_{A\bar{p}} \bar{V}^A{}_{\bar{q}} &= \bar{\eta}_{\bar{p}\bar{q}}, \\ V_{Ap} \bar{V}^A{}_{\bar{q}} &= 0, & V_{Ap} V_B{}^p + \bar{V}_{A\bar{p}} \bar{V}_B{}^{\bar{p}} &= \mathcal{J}_{AB}. \end{aligned} \quad (4)$$

In particular, they generate a pair of orthogonal and complete projections,

$$P_{AB} = V_A{}^p V_{Bp}, \quad \bar{P}_{AB} = \bar{V}_A{}^{\bar{p}} \bar{V}_{B\bar{p}}, \quad (5)$$

satisfying

$$\begin{aligned} P_A{}^B P_B{}^C &= P_A{}^C, & \bar{P}_A{}^{\bar{B}} \bar{P}_{\bar{B}}{}^{\bar{C}} &= \bar{P}_A{}^{\bar{C}}, \\ P_A{}^B \bar{P}_{\bar{B}}{}^{\bar{C}} &= 0, & P_A{}^B + \bar{P}_A{}^{\bar{B}} &= \delta_A{}^B. \end{aligned} \quad (6)$$

The DFT-vielbeins, $V_{Ap}, \bar{V}_{A\bar{p}}$, are $\mathbf{O}(D, D)$ vectors as the index structure indicates. They are the only field variables in (3) which are $\mathbf{O}(D, D)$ non-singlet. As a solution to (4), they can be

parametrized in terms of ordinary zehnbins and B -field, in various ways up to $\mathbf{O}(D, D)$ rotations and field redefinitions [37]. Yet, in order to maintain the clear manifestation of the $\mathbf{O}(D, D)$ covariance, it is necessary to work with the parametrization-independent and $\mathbf{O}(D, D)$ covariant DFT-vielbeins, i.e. V_{Ap} and $\bar{V}_{A\bar{p}}$, rather than the Riemannian variables, i.e. ordinary zehnbins and B -field.

For fermions, the gravitinos and the DFT-dilatinos are *not* twenty, but ten-dimensional Majorana–Weyl spinors, as in [26,27],

$$\begin{aligned} \gamma^{(11)} \psi_{\bar{p}} &= \mathbf{c} \psi_{\bar{p}}, & \gamma^{(11)} \rho &= -\mathbf{c} \rho, \\ \bar{\gamma}^{(11)} \psi'_p &= \mathbf{c}' \psi'_p, & \bar{\gamma}^{(11)} \rho' &= -\mathbf{c}' \rho', \end{aligned} \quad (7)$$

where \mathbf{c} and \mathbf{c}' are arbitrary independent two sign factors, $\mathbf{c}^2 = \mathbf{c}'^2 = 1$. Yet, *a priori* all the possible four different sign choices are equivalent up to $\mathbf{Pin}(1, 9) \times \mathbf{Pin}(9, 1)$ rotations. That is to say, $\mathcal{N} = 2$ $D = 10$ SDFT is *chiral* with respect to *both* $\mathbf{Pin}(1, 9)$ and $\mathbf{Pin}(9, 1)$, and the theory is unique. Hence, without loss of generality, we may safely set $\mathbf{c} \equiv \mathbf{c}' \equiv +1$. Later we shall see that, while the theory is unique the solutions are twofold and can be identified as type IIA or IIB supergravity backgrounds.

We also have $\mathcal{N} = 2$ supersymmetry parameters, $\varepsilon, \varepsilon'$, which carry the same chirality as the gravitinos, such that $\gamma^{(11)} \varepsilon = \mathbf{c} \varepsilon, \bar{\gamma}^{(11)} \varepsilon' = \mathbf{c}' \varepsilon'$.

Lastly for the R–R sector, we set the R–R potential, $C^\alpha_{\bar{\alpha}}$, to be in the bi-fundamental spinorial representation of $\mathbf{Pin}(1, 9) \times \mathbf{Pin}(9, 1)$ [26,27,37] rather than an $\mathbf{O}(10, 10)$ spinorial one [28,29]. It possesses the chirality,

$$\gamma^{(11)} C \bar{\gamma}^{(11)} = \mathbf{c} \mathbf{c}' C. \quad (8)$$

3. Derivatives

Another essential ingredient is so-called *master semi-covariant derivative* from [35],

$$\mathcal{D}_A = \partial_A + \Gamma_A + \Phi_A + \bar{\Phi}_A, \quad (9)$$

which contains generically three kinds of connections: Γ_A for the DFT-diffeomorphism or the generalized Lie derivative (2), Φ_A for $\mathbf{Spin}(1, 9)$ and $\bar{\Phi}_A$ for $\mathbf{Spin}(9, 1)$ local Lorentz symmetries. Contracted with the projections (6) or the DFT-vielbeins properly, it can produce various fully covariant derivatives, and hence the name, ‘semi-covariant’ [34,35,37].

By definition, the master derivative (9) is required to be compatible with all the constants in Table 1 (“metrics” and gamma matrices), and further to annihilate the whole NS–NS sector,

$$\mathcal{D}_A d = 0, \quad \mathcal{D}_A V_{Bp} = 0, \quad \mathcal{D}_A \bar{V}_{B\bar{p}} = 0. \quad (10)$$

The connections are then related to each other through

$$\begin{aligned} \Phi_{Apq} &= V^B{}_p \nabla_A V_{Bq}, & \bar{\Phi}_{A\bar{p}\bar{q}} &= \bar{V}^{\bar{B}}{}_{\bar{p}} \nabla_A \bar{V}_{B\bar{q}}, \\ \Gamma_{ABC} &= V_B{}^p D_A V_{Cp} + \bar{V}_B{}^{\bar{p}} D_A \bar{V}_{C\bar{p}}, \end{aligned} \quad (11)$$

where we put $\nabla_A = \partial_A + \Gamma_A$ and $D_A = \partial_A + \Phi_A + \bar{\Phi}_A$.

Especially, as the DFT analogy of the Riemannian–Christoffel connection, the *torsionless connection*, Γ_A^0 , can be uniquely singled out [34,37] (cf. [38]):

$$\begin{aligned} \Gamma_{CAB}^0 &= 2(P \partial_C P \bar{P})_{[AB]} + 2(\bar{P}_{[A}{}^D \bar{P}_{B]}{}^E - P_{[A}{}^D P_{B]}{}^E) \partial_D P_{CE} \\ &\quad - \frac{4}{9} (\bar{P}_{C[A} \bar{P}_{B]}{}^D + P_{C[A} P_{B]}{}^D) (\partial_D d + (P \partial^E P \bar{P})_{[ED]}), \end{aligned} \quad (12)$$

such that a generic torsionful DFT-diffeomorphism connection assumes the following general form:

$$\Gamma_{CAB} = \Gamma_{CAB}^0 + \Delta_{C[pq]} V_A^p V_B^q + \bar{\Delta}_{C[\bar{p}\bar{q}]} \bar{V}_A^{\bar{p}} \bar{V}_B^{\bar{q}}, \quad (13)$$

where $\Delta_{C[pq]}$ and $\bar{\Delta}_{C[\bar{p}\bar{q}]}$ correspond to torsions. Explicitly we shall employ four different kinds of torsions: (21) for the curvature, (22) for the fermionic kinetic terms, (23) for the supersymmetry, and (29) for the equations of motion.

The R–R field strength, $\mathcal{F}^\alpha_{\bar{\alpha}}$, is defined from [37],

$$\mathcal{F} := \mathcal{D}_+^0 \mathcal{C}, \quad (14)$$

where \mathcal{D}_+^0 corresponds to one of the two fully covariant and nilpotent differential operators, \mathcal{D}_\pm^0 , which are set by the torsionless connection (12), and may act on an arbitrary $\mathbf{Pin}(1, 9) \times \mathbf{Pin}(9, 1)$ bi-fundamental field, $\mathcal{T}^\alpha_{\bar{\beta}}$:

$$\mathcal{D}_\pm^0 \mathcal{T} := \gamma^p \mathcal{D}_p^0 \mathcal{T} \pm \gamma^{(11)} \mathcal{D}_p^0 \mathcal{T} \bar{\gamma}^{\bar{p}}, \quad (\mathcal{D}_\pm^0)^2 \mathcal{T} \simeq 0, \quad (15)$$

where we put¹ $\mathcal{D}_p^0 = V^A{}_p \mathcal{D}_A^0$ and $\mathcal{D}_{\bar{p}}^0 = \bar{V}^{\bar{A}}{}_{\bar{p}} \mathcal{D}_{\bar{A}}^0$.

4. Curvature

The final ingredient we shall employ is the semi-covariant DFT-curvature, S_{ABCD} , from [34],

$$S_{ABCD} := \frac{1}{2} (R_{ABCD} + R_{CDAB} - \Gamma^E{}_{AB} \Gamma_{ECD}), \quad (16)$$

which is defined through the standard (yet never-covariant) field strength of the DFT-diffeomorphism connection (13),

$$R_{CDAB} = \partial_A \Gamma_{BCD} - \partial_B \Gamma_{ACD} + \Gamma_{AC}{}^E \Gamma_{BED} - \Gamma_{BC}{}^E \Gamma_{AED}. \quad (17)$$

Again, with the help of the projections, it can produce fully covariant curvatures, such as Ricci (28) and scalar,

$$(P^{AB} P^{CD} - \bar{P}^{AB} \bar{P}^{CD}) S_{ACBD}. \quad (18)$$

5. The Lagrangian and supersymmetry

The Lagrangian of $\mathcal{N} = 2$ $D = 10$ SDFT we construct in this work is the following,

$$\begin{aligned} \mathcal{L}_{\text{Type II}} = e^{-2d} & \left[\frac{1}{8} (P^{AB} P^{CD} - \bar{P}^{AB} \bar{P}^{CD}) S_{ACBD} + \frac{1}{2} \text{Tr}(\mathcal{F} \bar{\mathcal{F}}) \right. \\ & - i \bar{\rho} \mathcal{F} \rho' + i \bar{\psi}_{\bar{p}} \gamma_q \mathcal{F} \bar{\gamma}^{\bar{p}} \psi'^q + i \frac{1}{2} \bar{\rho} \gamma^p \mathcal{D}_p^* \rho - i \bar{\psi}^{\bar{p}} \mathcal{D}_{\bar{p}}^* \rho \\ & - i \frac{1}{2} \bar{\psi}^{\bar{p}} \gamma^q \mathcal{D}_q^* \psi_{\bar{p}} - i \frac{1}{2} \bar{\rho}' \bar{\gamma}^{\bar{p}} \mathcal{D}_{\bar{p}}'^* \rho' + i \bar{\psi}'^p \mathcal{D}_p'^* \rho' \\ & \left. + i \frac{1}{2} \bar{\psi}'^p \bar{\gamma}^{\bar{q}} \mathcal{D}_{\bar{q}}'^* \psi'_p \right]. \quad (19) \end{aligned}$$

As they are contracted with the DFT-vielbeins properly, each term in the Lagrangian is fully covariant with respect to $\mathbf{O}(10, 10)$ T-duality, $\mathbf{Spin}(1, 9) \times \mathbf{Spin}(9, 1)$ local Lorentz symmetry and the DFT-diffeomorphism. With the charge conjugation of the R–R field strength, $\bar{\mathcal{F}} = \bar{C}_+^{-1} \mathcal{F}^T C_+$, the trace, $\text{Tr}(\mathcal{F} \bar{\mathcal{F}})$ in (19) is over the $\mathbf{Spin}(1, 9)$ spinorial indices.

¹ Strictly speaking, due to the presence of $\gamma^{(11)}$ in (15), the R–R field strength, $\mathcal{F} = \mathcal{D}_+^0 \mathcal{C}$, is covariant—up to the flipping of the chirality—with respect to, not $\mathbf{Pin}(1, 9) \times \mathbf{Pin}(9, 1)$ but $\mathbf{Spin}(1, 9) \times \mathbf{Pin}(9, 1)$. For the opposite equivalent choice, see Eq. (2.25) in [37].

The $\mathcal{N} = 2$ supersymmetry transformation rules are

$$\begin{aligned} \delta_\varepsilon d &= -i \frac{1}{2} (\bar{\varepsilon} \rho + \bar{\varepsilon}' \rho'), \\ \delta_\varepsilon V_{Ap} &= i \bar{V}_A^{\bar{q}} (\bar{\varepsilon}' \bar{\gamma}_{\bar{q}} \psi'_p - \bar{\varepsilon} \gamma_p \psi_{\bar{q}}), \\ \delta_\varepsilon \bar{V}_{A\bar{p}} &= i V_A^q (\bar{\varepsilon} \gamma_q \psi_{\bar{p}} - \bar{\varepsilon}' \bar{\gamma}_{\bar{p}} \psi'_q), \\ \delta_\varepsilon \mathcal{C} &= i \frac{1}{2} (\gamma^p \varepsilon \bar{\psi}'_p - \varepsilon \bar{\rho}' - \psi_{\bar{p}} \bar{\varepsilon}' \bar{\gamma}^{\bar{p}} + \rho \bar{\varepsilon}') + \mathcal{C} \delta_\varepsilon d \\ &\quad - \frac{1}{2} (\bar{V}^{\bar{A}}{}_{\bar{q}} \delta_\varepsilon V_{Ap}) \gamma^{(d+1)} \gamma^p \mathcal{C} \bar{\gamma}^{\bar{q}}, \\ \delta_\varepsilon \rho &= -\gamma^p \hat{\mathcal{D}}_p \varepsilon + i \frac{1}{2} \gamma^p \varepsilon \bar{\psi}'_p \rho' - i \gamma^p \psi_{\bar{q}} \bar{\varepsilon}' \bar{\gamma}_{\bar{q}} \psi'_p, \\ \delta_\varepsilon \rho' &= -\bar{\gamma}^{\bar{p}} \hat{\mathcal{D}}_{\bar{p}} \varepsilon' + i \frac{1}{2} \bar{\gamma}^{\bar{p}} \varepsilon' \bar{\psi}_{\bar{p}} \rho - i \bar{\gamma}^{\bar{q}} \psi'_p \bar{\varepsilon} \gamma^p \psi_{\bar{q}}, \\ \delta_\varepsilon \psi_{\bar{p}} &= \hat{\mathcal{D}}_{\bar{p}} \varepsilon + \left(\mathcal{F} - i \frac{1}{2} \gamma^q \rho \bar{\psi}'_q + i \frac{1}{2} \psi^{\bar{q}} \bar{\rho}' \bar{\gamma}_{\bar{q}} \right) \bar{\gamma}_{\bar{p}} \varepsilon' + i \frac{1}{4} \varepsilon \bar{\psi}_{\bar{p}} \rho \\ &\quad + i \frac{1}{2} \psi_{\bar{p}} \bar{\varepsilon} \rho, \\ \delta_\varepsilon \psi'_p &= \hat{\mathcal{D}}_p \varepsilon' + \left(\bar{\mathcal{F}} - i \frac{1}{2} \bar{\gamma}^{\bar{q}} \rho' \bar{\psi}_{\bar{q}} + i \frac{1}{2} \psi'^q \bar{\rho} \gamma_q \right) \gamma_p \varepsilon + i \frac{1}{4} \varepsilon' \bar{\psi}'_p \rho' \\ &\quad + i \frac{1}{2} \psi'_p \bar{\varepsilon}' \rho'. \quad (20) \end{aligned}$$

6. Torsions

Presenting our main results above, (19) and (20), we have organized all the higher order fermionic terms into various torsions. Firstly, with (16), the DFT-curvature, S_{ABCD} , in the Lagrangian is given by the connection,

$$\begin{aligned} \Gamma_{ABC} &= \Gamma_{ABC}^0 + i \frac{1}{3} \bar{\rho} \gamma_{ABC} \rho - 2i \bar{\rho} \gamma_{BC} \psi_A - i \frac{1}{3} \bar{\psi}^{\bar{p}} \gamma_{ABC} \psi_{\bar{p}} \\ &\quad + 4i \bar{\psi}_B \gamma_A \psi_C + i \frac{1}{3} \bar{\rho}' \bar{\gamma}_{ABC} \rho' - 2i \bar{\rho}' \bar{\gamma}_{BC} \psi'_A \\ &\quad - i \frac{1}{3} \bar{\psi}'^p \gamma_{ABC} \psi'_p + 4i \bar{\psi}'_B \bar{\gamma}_A \psi'_C. \quad (21) \end{aligned}$$

Secondly, the master derivatives in the fermionic kinetic terms are twofold: \mathcal{D}_A^* for the unprimed fermions and \mathcal{D}'_A for the primed fermions. They are set by the following twin connections,

$$\begin{aligned} \Gamma_{ABC}^* &= \Gamma_{ABC} - i \frac{11}{96} \bar{\rho} \gamma_{ABC} \rho + i \frac{5}{4} \bar{\rho} \gamma_{BC} \psi_A + i \frac{5}{24} \bar{\psi}^{\bar{p}} \gamma_{ABC} \psi_{\bar{p}} \\ &\quad - 2i \bar{\psi}_B \gamma_A \psi_C + i \frac{5}{2} \bar{\rho}' \bar{\gamma}_{BC} \psi'_A, \\ \Gamma_{ABC}' &= \Gamma_{ABC} - i \frac{11}{96} \bar{\rho}' \bar{\gamma}_{ABC} \rho' + i \frac{5}{4} \bar{\rho}' \bar{\gamma}_{BC} \psi'_A \\ &\quad + i \frac{5}{24} \bar{\psi}'^p \bar{\gamma}_{ABC} \psi'_p - 2i \bar{\psi}'_B \bar{\gamma}_A \psi'_C + i \frac{5}{2} \bar{\rho} \gamma_{BC} \psi_A. \quad (22) \end{aligned}$$

Similarly, for the supersymmetry transformations (20), we take

$$\begin{aligned} \hat{\Gamma}_{ABC} &= \Gamma_{ABC} - i \frac{17}{48} \bar{\rho} \gamma_{ABC} \rho + i \frac{5}{2} \bar{\rho} \gamma_{BC} \psi_A + i \frac{1}{4} \bar{\psi}^{\bar{p}} \gamma_{ABC} \psi_{\bar{p}} \\ &\quad - 3i \bar{\psi}_B \bar{\gamma}_A \psi'_C, \\ \hat{\Gamma}'_{ABC} &= \Gamma_{ABC} - i \frac{17}{48} \bar{\rho}' \bar{\gamma}_{ABC} \rho' + i \frac{5}{2} \bar{\rho}' \bar{\gamma}_{BC} \psi'_A + i \frac{1}{4} \bar{\psi}'^p \bar{\gamma}_{ABC} \psi'_p \\ &\quad - 3i \bar{\psi}'_B \bar{\gamma}_A \psi_C. \quad (23) \end{aligned}$$

The connection, Γ_{ABC} given in (21) and also appearing in (22), (23), has been fixed by requiring the 1.5 formalism to work, see (25). The additional parts of the connections in (22) and (23)

are then uniquely determined from the full order supersymmetric completion.

7. Self-duality and equations of motion

The type II SDFT Lagrangian (19) is pseudo: An additional self-duality relation needs to be imposed by hand on the R-R field strength combined with fermions,

$$\tilde{\mathcal{F}}_- := (1 - \gamma^{(11)}) \left(\mathcal{F} - i \frac{1}{2} \rho \tilde{\rho}' + i \frac{1}{2} \gamma^p \psi_{\tilde{q}} \tilde{\psi}'_p \tilde{\gamma}^{\tilde{q}} \right) \equiv 0. \quad (24)$$

Under arbitrary infinitesimal variations of all the fields, the Lagrangian transforms, up to total derivatives,

$$\begin{aligned} \delta \mathcal{L}_{\text{Type II}} &\simeq -2\delta d \times \mathcal{L}_{\text{Type II}} \\ &+ \delta \Gamma_{ABC} \times 0 \\ &+ \frac{1}{2} e^{-2d} \delta V^{Bp} \tilde{V}_B^{\tilde{q}} [\tilde{S}_{p\tilde{q}} + \text{Tr}(\mathcal{F} \tilde{\gamma}_{\tilde{q}} \tilde{\mathcal{F}} \gamma_p)] \\ &- i e^{-2d} \delta \tilde{\psi}^{\tilde{p}} (\tilde{D}_{\tilde{p}} \rho + \gamma^p \tilde{D}_p \psi_{\tilde{p}} - \gamma^p \mathcal{F} \tilde{\gamma}_{\tilde{p}} \psi'_p) \\ &+ i e^{-2d} \delta \tilde{\rho}' (\gamma^p \tilde{D}_p \rho - \tilde{D}_{\tilde{p}} \psi^{\tilde{p}} - \mathcal{F} \rho') \\ &+ i e^{-2d} \delta \tilde{\psi}'^p (\tilde{D}'_p \rho' + \tilde{\gamma}^{\tilde{p}} \tilde{D}'_{\tilde{p}} \psi'_p - \tilde{\gamma}^{\tilde{p}} \tilde{\mathcal{F}} \gamma_p \psi_{\tilde{p}}) \\ &- i e^{-2d} \delta \tilde{\rho}' (\tilde{\gamma}^{\tilde{p}} \tilde{D}'_{\tilde{p}} \rho' - \tilde{D}'_p \psi'^p - \tilde{\mathcal{F}} \rho) \\ &+ e^{-2d} \text{Tr} \left[\tilde{\mathcal{F}}_- \left(\delta d \tilde{\mathcal{F}} - \frac{1}{2} \delta V^{Ap} \tilde{V}_A^{\tilde{q}} \tilde{\gamma}_{\tilde{q}} \tilde{\mathcal{F}} \gamma_p \right) \right. \\ &\left. - \mathcal{D}^0_- \tilde{\mathcal{F}}_- \tilde{\delta \mathcal{C}} \right]. \quad (25) \end{aligned}$$

Each line then corresponds to the equation of motion of $\mathcal{N} = 2$ $D = 10$ SDFT. In particular, the on-shell Lagrangian vanishes, $\mathcal{L}_{\text{Type II}} = 0$, and the DFT-generalization of the Einstein equation follows

$$\tilde{S}_{p\tilde{q}} + \text{Tr}(\mathcal{F} \tilde{\gamma}_{\tilde{q}} \tilde{\mathcal{F}} \gamma_p) = 0. \quad (26)$$

The self-duality (24) implies the equation of motion for the R-R potential, $\mathcal{D}^0_- \tilde{\mathcal{F}}_- = 0$. Further, as in the $\mathcal{N} = 1$ SDFT [36], the 1.5 formalism, ' $\delta \Gamma_{ABC} \times 0$ ', nicely works here with the connection spelled in (21).

Writing (25), we set some shorthand notations: For the arbitrary variations of the fields,

$$\begin{aligned} \delta \tilde{\rho} &:= \delta \rho - \frac{1}{4} \delta V_{Bq} \tilde{\rho} \gamma^{Bq}, \\ \delta \tilde{\psi}^{\tilde{p}} &:= \delta \psi^{\tilde{p}} - \delta \tilde{V}^{B\tilde{p}} \tilde{\psi}_B - \frac{1}{4} \delta V_{Bq} \tilde{\psi}^{\tilde{p}} \gamma^{Bq}, \\ \delta \tilde{\rho}' &:= \delta \rho' - \frac{1}{4} \delta \tilde{V}_{B\tilde{q}} \tilde{\rho}' \tilde{\gamma}^{B\tilde{q}}, \\ \delta \tilde{\psi}'^p &:= \delta \psi'^p - \delta V^{Bp} \tilde{\psi}'_B - \frac{1}{4} \delta \tilde{V}_{B\tilde{q}} \tilde{\psi}'^p \tilde{\gamma}^{B\tilde{q}}, \\ \delta \tilde{\mathcal{C}} &:= \delta \mathcal{C} - \mathcal{C} \delta d + \frac{1}{4} \delta V_{Ap} \gamma^{Ap} \mathcal{C} - \frac{1}{4} \delta \tilde{V}_{A\tilde{p}} \tilde{\mathcal{C}} \tilde{\gamma}^{A\tilde{p}} \\ &+ \frac{1}{2} \delta V_{Ap} \gamma^{(11)} \gamma^p \mathcal{C} \tilde{\gamma}^A, \quad (27) \end{aligned}$$

and for the Ricci curvature,

$$\begin{aligned} \tilde{S}_{p\tilde{q}} &:= V^A_p \tilde{V}^B_{\tilde{q}} S^C_{ACB} + 2i \tilde{\psi}_{\tilde{q}} \tilde{D}_p \rho - i \tilde{\psi}^{\tilde{p}} \gamma_p \tilde{D}_{\tilde{q}} \psi_{\tilde{p}} + 2i \tilde{\psi}'_p \tilde{D}'_{\tilde{q}} \rho' \\ &- i \tilde{\psi}'^q \tilde{\gamma}_{\tilde{q}} \tilde{D}'_p \psi'_q + i \tilde{\rho} \gamma_p \tilde{D}_{\tilde{q}} \rho + i \tilde{\rho}' \tilde{\gamma}_{\tilde{q}} \tilde{D}'_p \rho'. \quad (28) \end{aligned}$$

We also set the derivatives, $\tilde{D}_A, \tilde{D}'_A$ appearing in (25), by

$$\begin{aligned} \tilde{T}_{ABC} &= \Gamma_{ABC} - i \frac{23}{54} \tilde{\rho} \gamma_{ABC} \rho + i \frac{23}{27} \tilde{\rho} \gamma_{BC} \psi_A + i \frac{23}{54} \tilde{\psi}^{\tilde{p}} \gamma_{ABC} \psi_{\tilde{p}} \\ &- i \frac{73}{18} \tilde{\psi}_B \gamma_A \psi_C - i \frac{5}{4} \tilde{\rho}' \tilde{\gamma}_{ABC} \rho' + i \frac{5}{2} \tilde{\rho}' \tilde{\gamma}_{BC} \psi'_A \\ &+ i \frac{5}{4} \tilde{\psi}'^p \tilde{\gamma}_{ABC} \psi'_p - 5i \tilde{\psi}'_B \tilde{\gamma}_A \psi'_C, \\ \tilde{T}'_{ABC} &= \Gamma_{ABC} - i \frac{23}{54} \tilde{\rho}' \tilde{\gamma}_{ABC} \rho' + i \frac{23}{27} \tilde{\rho}' \tilde{\gamma}_{BC} \psi'_A + i \frac{23}{54} \tilde{\psi}'^p \tilde{\gamma}_{ABC} \psi'_p \\ &- i \frac{73}{18} \tilde{\psi}'_B \tilde{\gamma}_A \psi'_C - i \frac{5}{4} \tilde{\rho} \gamma_{ABC} \rho + i \frac{5}{2} \tilde{\rho} \gamma_{BC} \psi_A \\ &+ i \frac{5}{4} \tilde{\psi}^{\tilde{p}} \tilde{\gamma}_{ABC} \psi_{\tilde{p}} - 5i \tilde{\psi}_B \gamma_A \psi_C, \quad (29) \end{aligned}$$

which are designed to serve as common connections for all the equations of motion, see Appendix of [53].

Under the $\mathcal{N} = 2$ supersymmetry (20), disregarding total derivatives, the Lagrangian transforms concisely,

$$\delta_\varepsilon \mathcal{L}_{\text{Type II}} \simeq -\frac{1}{8} e^{-2d} \tilde{V}^A_{\tilde{q}} \delta_\varepsilon V_{Ap} \text{Tr}(\gamma^p \tilde{\mathcal{F}}_- \tilde{\gamma}^{\tilde{q}} \tilde{\mathcal{F}}_-). \quad (30)$$

This verifies, to the full order in fermions, the supersymmetric invariance of the type II SDFT action modulo the self-duality (24), see Appendix of [53] for details. For a nontrivial consistency check, the supersymmetric variation of the self-duality relation (24) is, to the full order precisely, closed by the equations of motion for fermions, especially the gravitinos (25),

$$\begin{aligned} \delta_\varepsilon \tilde{\mathcal{F}}_- &= -i (\tilde{D}_{\tilde{p}} \rho + \gamma^p \tilde{D}_p \psi_{\tilde{p}} - \gamma^p \mathcal{F} \tilde{\gamma}_{\tilde{p}} \psi'_p) \tilde{\varepsilon}'^{\tilde{p}} \\ &- i \gamma^p \varepsilon (\tilde{D}'_p \rho' + \tilde{D}'_{\tilde{p}} \tilde{\psi}'_p \tilde{\gamma}^{\tilde{p}} - \tilde{\psi}'_p \gamma_p \mathcal{F} \tilde{\gamma}^{\tilde{p}}). \quad (31) \end{aligned}$$

8. Unification

As stressed before, one of the characteristic features in our construction of $\mathcal{N} = 2$ $D = 10$ SDFT is the usage of the covariant fundamental fields, identified in (3). However, the relation to an ordinary supergravity can be established only after we solve the defining algebraic relations of the DFT-vielbeins (4) and parametrize the solution in terms of zehnbins and B -field: Up to $\mathbf{O}(10, 10)$ rotations and field redefinitions, the generic solution reads [34,37]

$$V_{Ap} = \frac{1}{\sqrt{2}} \begin{pmatrix} (e^{-1})_p{}^\mu \\ (B+e)_{vp} \end{pmatrix}, \quad \tilde{V}_{A\tilde{p}} = \frac{1}{\sqrt{2}} \begin{pmatrix} (\tilde{e}^{-1})_{\tilde{p}}{}^\mu \\ (B+\tilde{e})_{v\tilde{p}} \end{pmatrix}, \quad (32)$$

where e_{μ}^p and $\tilde{e}_{\nu}^{\tilde{p}}$ are two copies of zehnbins which must constitute a common spacetime metric,

$$e_{\mu}^p e_{\nu}^q \eta_{pq} = -\tilde{e}_{\mu}^{\tilde{p}} \tilde{e}_{\nu}^{\tilde{q}} \tilde{\eta}_{\tilde{p}\tilde{q}} = g_{\mu\nu}. \quad (33)$$

We also set $B_{\mu p} = B_{\mu\nu} (e^{-1})_p{}^\nu$ and $B_{\mu \tilde{p}} = B_{\mu\nu} (\tilde{e}^{-1})_{\tilde{p}}{}^\nu$. The pair of zehnbins directly reflects the double local Lorentz groups, $\mathbf{Spin}(1, 9) \times \mathbf{Spin}(9, 1)$. It follows that $(e^{-1}\tilde{e})_p{}^{\tilde{p}}$ is a Lorentz rotation,

$$(e^{-1}\tilde{e})_p{}^{\tilde{p}} (e^{-1}\tilde{e})_{\tilde{q}}{}^q \tilde{\eta}_{\tilde{p}\tilde{q}} = -\eta_{pq}, \quad (34)$$

and further that there is a spinorial representation of this Lorentz rotation which relates² $\tilde{\gamma}^{\tilde{p}}$ to $\gamma^{(11)} \gamma^p$,

$$S_e \tilde{\gamma}^{\tilde{p}} S_e^{-1} = \gamma^{(11)} \gamma^p (e^{-1}\tilde{e})_p{}^{\tilde{p}}. \quad (35)$$

² Since, as seen from Table 1, our convention assumes the signature of η_{pq} for $\mathbf{Spin}(1, 9)$ to be opposite to that of $\tilde{\eta}_{\tilde{p}\tilde{q}}$ for $\mathbf{Spin}(9, 1)$, the spinorial representation, S_e , relates $\tilde{\gamma}^{\tilde{p}}$ to $\gamma^{(11)} \gamma^p$ rather than γ^p . Note the minus sign,

$\{\gamma^{(11)} \gamma^p, \gamma^{(11)} \gamma^q\} = -2\eta^{pq}$.

Now we may consider ‘fixing’ the two zehnbins equal to each other,

$$e_{\mu}^p \equiv \bar{e}_{\mu}^{\bar{p}}, \quad (36)$$

using a $\mathbf{Pin}(9, 1)$ local Lorentz rotation which effectively “unwinds” $(e^{-1}\bar{e})_p^{\bar{p}}$ and S_e such that they become trivial *i.e.* identities. This rotation may, or may not, flip the chirality as

$$\mathbf{c}' \longrightarrow \det(e^{-1}\bar{e})\mathbf{c}', \quad (37)$$

since (35) implies [37]

$$S_e \bar{\gamma}^{(11)} S_e^{-1} = -\det(e^{-1}\bar{e})\gamma^{(11)}. \quad (38)$$

Namely, the chirality remains the same if $\det(e^{-1}\bar{e}) = +1$, while it changes the sign if $\det(e^{-1}\bar{e}) = -1$. Therefore, it depends on each specific background or each individual solution of the theory whether the chirality changes or not. That is to say, formulated in terms of the covariant fields, *i.e.* V_{Ap} , $\bar{V}_{A\bar{p}}$, $C^{\alpha}_{\bar{\alpha}}$, *etc.* the $\mathcal{N} = 2$ $D = 10$ SDFT is simply a chiral theory with respect to the pair of local Lorentz groups. All the possible chirality choices are equivalent and hence the theory is *unique*. We may safely put $\mathbf{c} \equiv \mathbf{c}' \equiv +1$ without loss of generality. However, the theory contains two ‘types’ of solutions. All the solutions are classified into two groups,

$$\begin{aligned} \mathbf{c}\mathbf{c}' \det(e^{-1}\bar{e}) &= +1 : \text{type IIA,} \\ \mathbf{c}\mathbf{c}' \det(e^{-1}\bar{e}) &= -1 : \text{type IIB.} \end{aligned} \quad (39)$$

Conversely, making full use of the above $\mathbf{Pin}(9, 1)$ rotation, any solution in type IIA and type IIB supergravities can be mapped to a solution of $\mathcal{N} = 2$ $D = 10$ SDFT of fixed chirality *e.g.* $\mathbf{c} \equiv \mathbf{c}' \equiv +1$. *The single unique $\mathcal{N} = 2$ $D = 10$ SDFT unifies type IIA and IIB supergravities.*

9. Comments

After the fixing, $e_{\mu}^p \equiv \bar{e}_{\mu}^{\bar{p}}$ (36), the pair of local Lorentz groups, $\mathbf{Spin}(1, 9) \times \mathbf{Spin}(9, 1)$, is broken to its diagonal subgroup, $\mathbf{Spin}(1, 9)_D$, which acts on both $\mathbf{Pin}(1, 9)$ and $\mathbf{Pin}(9, 1)$ indices simultaneously. This allows us to expand $C^{\alpha}_{\bar{\alpha}}$ in terms of odd (type IIA) or even (type IIB) p -forms [37], and eventually reduces the $\mathcal{N} = 2$ $D = 10$ SDFT to the so-called ‘democratic supergravity’ formulated, up to quadratic order in fermions, in [41] (*cf.* [39,40]).

The diagonal “gauge” fixing (36) inevitably modifies the $\mathbf{O}(10, 10)$ T-duality transformation rule to call for a compensating $\mathbf{Pin}(9, 1)$ local Lorentz rotation [37], such that the fermions and the R–R sector are *no longer* $\mathbf{O}(10, 10)$ singlets. In particular, the R–R sector can be mapped to the $\mathbf{O}(10, 10)$ spinor in [28–31]. Moreover, the modified $\mathbf{O}(10, 10)$ T-duality transformation, or more precisely the compensating $\mathbf{Pin}(9, 1)$ local Lorentz rotation, may flip the chirality of the theory, resulting in the usual exchange of IIA and IIB.

However, *a priori* T-duality is not a Noether symmetry. It becomes so only if it acts on an isometry direction. Hence, as is well known, within the supergravity setup the equivalence between IIA and IIB can be established only when the background admits an isometry. This is compared to the ‘background independent’ unification of the two supergravities by $\mathcal{N} = 2$ $D = 10$ SDFT, discussed in this work.

Turning off both the primed fermions and the R–R sector truncates the $\mathcal{N} = 2$ SDFT to the previously constructed $\mathcal{N} = 1$ SDFT [36], to the full order in fermions consistently (*cf.* [42]). The uplift of type II SDFT to \mathcal{M} -theory, or the extension of $\mathbf{O}(10, 10)$

T-duality to E_{11} U-duality, remains as a challenging future work, *cf.* [43–52].

The appendices of the arXiv version of this work [53] contain some details of the computations for (25) and (30).

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References

- [1] T.H. Buscher, Phys. Lett. B 159 (1985) 127.
- [2] T.H. Buscher, Phys. Lett. B 194 (1987) 59.
- [3] T.H. Buscher, Phys. Lett. B 201 (1988) 466.
- [4] A. Giveon, E. Rabinovici, G. Veneziano, Nucl. Phys. B 322 (1989) 167.
- [5] K.A. Meissner, G. Veneziano, Phys. Lett. B 267 (1991) 33.
- [6] M. Gualtieri, Ph.D. thesis, Oxford University, 2003, arXiv:math/0401221.
- [7] N. Hitchin, Quart. J. Math. Oxford Ser. 54 (2003) 281.
- [8] N. Hitchin, arXiv:1008.0973 [math.DG].
- [9] C. Hull, B. Zwiebach, JHEP 0909 (2009) 099.
- [10] C. Hull, B. Zwiebach, JHEP 0909 (2009) 090.
- [11] O. Hohm, C. Hull, B. Zwiebach, JHEP 1007 (2010) 016, arXiv:1003.5027 [hep-th].
- [12] O. Hohm, C. Hull, B. Zwiebach, JHEP 1008 (2010) 008, arXiv:1006.4823 [hep-th].
- [13] W. Siegel, Phys. Rev. D 47 (1993) 5453.
- [14] W. Siegel, Phys. Rev. D 48 (1993) 2826.
- [15] M. Grana, R. Minasian, M. Petrini, D. Waldram, JHEP 0904 (2009) 075, arXiv:0807.4527 [hep-th].
- [16] P. Koerber, Fortsch. Phys. 59 (2011) 169.
- [17] A.A. Tseytlin, Phys. Lett. B 242 (1990) 163.
- [18] A.A. Tseytlin, Nucl. Phys. B 350 (1991) 395.
- [19] T. Courant, Trans. Amer. Math. Soc. 319 (1990) 631.
- [20] O. Hohm, B. Zwiebach, arXiv:1207.4198 [hep-th].
- [21] D. Geissbuhler, JHEP 1111 (2011) 116, arXiv:1109.4280 [hep-th].
- [22] G. Aldazabal, W. Baron, D. Marques, C. Nunez, JHEP 1111 (2011) 052, arXiv:1109.0290 [hep-th]; G. Aldazabal, W. Baron, D. Marques, C. Nunez, JHEP 1111 (2011) 109 (Erratum).
- [23] M. Grana, D. Marques, JHEP 1204 (2012) 020, arXiv:1201.2924 [hep-th].
- [24] G. Dibitetto, J.J. Fernandez-Melgarejo, D. Marques, D. Roest, Fortsch. Phys. 60 (2012) 1123, arXiv:1203.6562 [hep-th].
- [25] G. Dibitetto, arXiv:1210.2301 [hep-th].
- [26] A. Coimbra, C. Strickland-Constable, D. Waldram, JHEP 1111 (2011) 091, arXiv:1107.1733 [hep-th].
- [27] A. Coimbra, C. Strickland-Constable, D. Waldram, Fortsch. Phys. 60 (2012) 982, arXiv:1202.3170 [hep-th].
- [28] O. Hohm, S.K. Kwak, B. Zwiebach, Phys. Rev. Lett. 107 (2011) 171603, arXiv:1106.5452 [hep-th].
- [29] O. Hohm, S.K. Kwak, B. Zwiebach, JHEP 1109 (2011) 013, arXiv:1107.0008 [hep-th].
- [30] M. Fukuma, T. Oota, H. Tanaka, Prog. Theor. Phys. 103 (2000) 425, arXiv:hep-th/9907132.
- [31] S.F. Hassan, Nucl. Phys. B 583 (2000) 431.
- [32] I. Jeon, K. Lee, J.-H. Park, JHEP 1104 (2011) 014.
- [33] I. Jeon, K. Lee, J.-H. Park, Phys. Lett. B 701 (2011) 260, arXiv:1102.0419 [hep-th].
- [34] I. Jeon, K. Lee, J.-H. Park, Phys. Rev. D 84 (2011) 044022, arXiv:1105.6294 [hep-th].
- [35] I. Jeon, K. Lee, J.-H. Park, JHEP 1111 (2011) 025.
- [36] I. Jeon, K. Lee, J.-H. Park, Phys. Rev. D Rapid comm. 85 (2012) 081501, arXiv:1112.0069 [hep-th].
- [37] I. Jeon, K. Lee, J.-H. Park, JHEP 1209 (2012) 079.
- [38] O. Hohm, B. Zwiebach, JHEP 1205 (2012) 126.
- [39] P.K. Townsend, in: M.J. Duff (Ed.), The World in Eleven Dimensions, 1995, pp. 375–389, arXiv:hep-th/9507048.
- [40] M.B. Green, C.M. Hull, P.K. Townsend, Phys. Lett. B 382 (1996) 65, arXiv:hep-th/9604119.
- [41] E. Bergshoeff, R. Kallosh, T. Ortin, D. Roest, A. Van Proeyen, Class. Quant. Grav. 18 (2001) 3359.
- [42] O. Hohm, S.K. Kwak, JHEP 1203 (2012) 080.
- [43] D.S. Berman, M.J. Perry, JHEP 1106 (2011) 074.
- [44] D.C. Thompson, JHEP 1108 (2011) 125.
- [45] D.S. Berman, H. Godazgar, M. Godazgar, M.J. Perry, JHEP 1201 (2012) 012.

- [46] D.S. Berman, H. Godazgar, M.J. Perry, P. West, JHEP 1202 (2012) 108, arXiv:1111.0459 [hep-th].
- [47] A. Coimbra, C. Strickland-Constable, D. Waldram, arXiv:1112.3989 [hep-th].
- [48] P.C. West, Class. Quant. Grav. 18 (2001) 4443.
- [49] P. West, Phys. Lett. B 696 (2011) 403.
- [50] A. Rocen, P. West, arXiv:1012.2744 [hep-th].
- [51] P. West, JHEP 1202 (2012) 018.
- [52] I.A. Bandos, N. Berkovits, D.P. Sorokin, Nucl. Phys. B 522 (1998) 214, arXiv:hep-th/9711055.
- [53] I. Jeon, K. Lee, J.-H. Park, Y. Suh, arXiv:1210.5078 [hep-th].