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Simulating corrective maintenance: Aggregating component level maintenance time uncertainty at the system level

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Abstract

The corrective maintenance process can be decomposed into failure and repair processes. Creating a model to capture the corrective maintenance process then requires an accurate estimate of the behavior of these constituent processes. For systems composed of many individual parts, information about failure and repair behavior is more likely to be available at the component level than the system level. Depending on the number of components that comprise the system, modeling each part may become computationally burdensome; in addition, some few components may account for a large portion of the overall system failures.

In such a situation, one solution to the modeling burden is aggregation: the mathematical assimilation of many component distributions into a single representative distribution for the group. This paper describes how aggregation may be performed for such a system and how an algorithm may be developed to automate the process. Next, it describes how to simulate an aggregated distribution using a pseudo-random number generator and finally demonstrates these concepts for a sample problem. The first section of the paper introduces corrective maintenance modeling and aggregation; the second section describes aggregation for corrective maintenance; the third explains how to simulate the aggregated distribution; the fourth demonstrates aggregation; and the fifth discusses limitations of the method and concludes.

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1. Introduction

Today's complex systems present a challenge for simulators because of the large number of components and the complex way in which they interact. Specifically, simulating maintenance processes for these systems is difficult because of the large number of parts and the stochastic nature of their failure and repair times. One potential solution to this difficulty is aggregation: a mathematical process by which many distributions are represented with a single distribution without significant loss of fidelity [1]. Though most methods presented in the literature focus on

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defining the shape of aggregate distributions [2], for simulation the distribution itself must be found. The desirability of aggregation may be especially great when the part failure distributions within the system follow a Pareto trend [3], where the failures of a few parts dominate the overall failure behavior of the system. However, some simulations may require aggregation even when this is not the case if the computational burden is significant.

In this paper we focus on systems with large numbers of components for which failure and maintenance information is available at the part level and not at the system level. In addition, the failure and maintenance information must be in the form of a stochastic distribution which is constant over time. Common cause failure is excluded to clarify the mathematical formulas that are developed. Systems which follow these characteristics are the most likely to require aggregation, as well as being conducive to such a computation.

The first piece required to simulate an aggregate distribution is a list of relative failure rates among all parts being aggregated. For the exponential distribution these rates are a simple function of the failure rate parameter for each part individually, but for other distributions the relative failure rates must be determined through simulation. Relative failure rates are discussed in more depth in Section 2. The next piece required to simulate aggregate parts is the aggregate function itself. Though this computation requires greater initial effort, over the lifetime of the simulation the computational burden is greatly reduced. Fortunately, the initial burden may be reduced by automating the process through computer algorithms. One such algorithm is presented in Section 2. The final aspect of the aggregation process that must be managed is the need to simulate the aggregated distribution, which may be a new distribution for which ready-made simulations will not exist. The inverse transform method [4] allows distributions to be simulated from a uniform pseudo-random number generator, but only for functions which are invertible. For others, numerical inversion is required. These are discussed in more detail in Section 3. In Section 4 we demonstrate this process for a small sample problem and suggest a scaled up version for the future.

2. Aggregating corrective maintenance

The factors that contribute to aggregation within a corrective maintenance perspective depend on the maintenance activities performed. One activity implicit in all corrective maintenance processes is a degradation or failure event in the system being maintained; however, the subsequent activities may differ between systems and even parts within a system. For the purpose of this paper we focus on a generalized maintenance process that is triggered by part failure or a subsequent activity and that takes a random amount of time to be completed. Specific processes that fall under the above process type include part removal, part repair and part installation [5]; though this list is by no means comprehensive.

The stochasticity of the assumed maintenance process requires some information to be gathered before simulation can occur. Whether from historical data or estimation, a distribution must be assigned to represent the time to complete the maintenance process. Because any corrective maintenance process is triggered by failure, the time to failure or degradation must also be assigned a distribution. And since the situation described in the introduction is one where data is known at the part level, separate distributions must be assigned to each individual part. Only once these distributions are defined may aggregation occur. The following sections discuss how to aggregate distributions, first in a general sense and then for specific families of distributions.

2.1. Aggregating with any distributions

In this most general case, each part fails according to some random distribution and each takes a randomly distributed amount of time (t_i) to complete a subsequent maintenance process. Also, as mentioned in the introduction, these distributions do not change with time. Thus it can be generally said that the maintenance process for aggregate part $part_{1,2,\ldots n}$ composed of parts $part_1$ through $part_n$ will last the amount of time

$$t_{1,2,\ldots,n} = \begin{cases} t_1 \text{ if } part_1 \text{ fails} \\ \ldots \\ t_n \text{ if } part_n \text{ fails} \end{cases} = \begin{cases} t_1 \text{ with probability } P(part_1 \text{ fails}) \\ \ldots \\ t_n \text{ with probability } P(part_n \text{ fails}) \end{cases} \tag{1}$$

Equation (1) as written is a probability mass function (PMF) for the time to complete the maintenance process for an aggregate part; because it represents all possible values that $t_{1,2,\ldots,n}$ can adopt, the probabilities must sum to one [6]. The relative probabilities of failure for the set of parts being aggregated, obtained from the time to failure distribution of each part, meet this criterion. However, a closed-form solution for the relative failure probabilities may only be obtained for the exponential distribution; for any others, simulation is required.

Because the values of the piecewise function in Equation (1) are themselves distributions, the function should be expanded. Depending on the form of the t_i's, this expanded function may remain in the form of a PMF, or more probably will be revealed as a probability distribution function (PDF). If the component distributions are finitely bounded, the PDF will be piecewise continuous and the ranges over which the individual t_i's are distributed may overlap. This indicates that there are multiple ways for a given time to failure to occur: for example, some maintenance process completion time t_ξ could occur either when $Part_1$ fails and distribution t_1 returns ξ (event $t_{1\xi}$) or when $Part_2$ fails and distribution t_2 returns ξ (event $t_{2\xi}$). In mathematical terms, event $t_{i\xi}$ is the intersection of $Part_i$ failure and distribution i returning ξ [6], and since these events are independent,

$$P(t_{i\xi}) = P(Part\ i\ fails)P(t_i=\xi)$$

Also in mathematical terms, t_ξ is the union of $t_{1\xi}$ and $t_{2\xi}$ [6]. Since we excluded common-cause failures in the introduction, part failures are disjoint, which means that $t_{1\xi}$ and $t_{2\xi}$ are disjoint and that the probability of the union is the sum of the probabilities of the disjoint events. This concept is extended to more than two parts in Equation (2).

$$P(t_\xi) = \sum_{i=1}^{n} P(t_{i\xi}) = \sum_{i=1}^{n} P(Part\ i\ fails)P(t_i = \xi) \tag{2}$$

Thus when expanding Equation (1) for finitely bounded distributions, any region of the function that has overlapping time ranges from multiple t_i's should be combined according to Equation (2). The infinite distributions are simply a special case of this example with a single region, and should also be combined as per Equation (2).

This description of an aggregation process is intentionally general. In the following section, simplifications available for specific distributions of part failure time and time to complete the maintenance process are presented, as well as options for other distributions.

2.2. Aggregating with specific distributions

The first aspect of the aggregation process that must be computed is the set of relative failure probabilities for the parts being aggregated. For most distribution types, no closed-form solution exists for these probabilities, though simulation can be used to find the values. However, as mentioned in Section 2.1, the exponential distribution does permit a closed-form solution to be computed. This takes the form

$$P(part\ j\ fails) = P_{Fj} = \frac{\lambda_j}{\sum_i \lambda_i} \tag{3}$$

where λ_i is the rate parameter of each individual exponential distribution. The computation of this formula is derived in [6] as the probability that $Part_j$ fails before any other parts. To use this formula as the relative probability that $Part_j$ fails in the long term, each part would need to return to the original failure distribution. For most distributions, this would require that all parts be repaired when one part in the aggregation breaks. However, because of the memoryless property of the exponential distribution [7], the assumption that the part distributions are reset after one part fails is computationally valid even without fixing all parts and the result in Equation (3) does not change.

The second aspect of the aggregation process that must be computed is the expanded version of Equation (1). Fortunately this expansion can be carried out for a wide range of distribution types, as most distributions have a functional form that can be multiplied by constants and added to other functions symbolically. The most trivial of these are the uniform and triangular distributions since these are piecewise polynomial functions. An additional advantage carried by these two distributions will be demonstrated in Section 3 when simulating aggregate distributions is addressed. Though less trivial, shifted beta, exponential, truncated normal and gamma distributions can all be aggregated in a similar manner. Finally, a user-defined distribution for time to complete the maintenance process may be aggregated so long as it can be multiplied by a constant and added to another function. Example forms of an expanded Equation (1) are shown for each of these distribution types in the following sections.

2.2.1. Infinitely bounded distributions

Because the exponential and gamma distributions are defined from 0 to ∞ and the normal distribution is defined from -∞ to ∞, all functions in Equation (1) will have the same bounds if composed of these distribution types. Therefore, though the functional form of the expanded Equation (1) may be more complex than for finitely bounded distributions, a piecewise definition is not required. This creates significant advantages in the initial steps of aggregation, though it may introduce difficulties in the simulation portion of the process, which will be addressed in Section 3.

Assuming that all mission process time distributions are exponential [8], Equation (1) takes the following form:

$$f(t) = \begin{cases} t_1 \sim Exp(\lambda_1)\ with\ probability\ P_{F1} \\ \ldots \\ t_n \sim Exp(\lambda_n)\ with\ probability\ P_{Fn} \end{cases} = \sum_{i=1}^{n} P_{Fi} \lambda_i e^{-\lambda_i t}$$

Similarly, the form of Equation (1) for a set of gamma distributions [8] is

$$f(t) = \begin{cases} t_1 \sim \Gamma(\lambda_1)\ with\ probability\ P_{F1} \\ \ldots \\ t_n \sim \Gamma(\lambda_n)\ with\ probability\ P_{Fn} \end{cases} = \sum_{i=1}^{n} \frac{P_{Fi}}{\Gamma(\alpha_i)} \lambda_i^{\alpha_{ii}} t^{\alpha_i - 1} e^{-\lambda_i t}$$

and the form of Equation (1) for a set of normal distributions [8] is

$$f(t) = \begin{cases} t_1 \sim N(\mu_1, \sigma_1^2)\ with\ probability\ P_{F1} \\ \ldots \\ t_n \sim N(\mu_n, \sigma_n^2)\ with\ probability\ P_{Fn} \end{cases} = \sum_{i=1}^{n} \frac{P_{Fi}}{\sqrt{2\pi}\sigma_i} e^{-\frac{(t-\mu_i)^2}{2\sigma^2}}$$

2.2.2. Finitely bounded distributions

When the component distributions of an aggregation are bounded, the expansion of Equation (1) becomes complicated. Because it is possible for some or all of the component distributions to overlap, it is necessary to iterate through the set of bounds for all distributions and order them. In each pair of ordered bounds, all functions active over the region must treated as in Equation (2). This process is shown for a general piecewise distribution in Equations (4) and (5), where the PDF formula for any bounded distribution can be substituted for f(t).

Assuming that all mission process time distributions are of the same form (i.e. all uniform, all triangular, etc.), Equation (1) takes the following form without accounting for overlaps:

$$f(t) = \begin{cases} t_1 \sim Dist(\{parameters\}_1)\ with\ probability\ P_{F1} \\ \ldots \\ t_n \sim Dist(\{parameters\}_n)\ with\ probability\ P_{Fn} \end{cases} = \begin{cases} P_{F1} f(t_1)\ for\ a_1 < t < b_1 \\ \ldots \\ P_{Fn} f(t_n)\ for\ a_n < t < b_n \\ 0 \quad else \end{cases} \tag{4}$$

To resolve the overlaps, the bounds must be ordered (denoted in Equation (5) by Roman numerals) and any functions active within each pair of bounds must be summed.

$$f(t)=\begin{cases}\sum_{i=I}^{II} P_{Fi} f(t_i) \text{ for } I<t<II \\ \dots \\ \sum_{i=R}^{R+I} P_{Fi} f(t_i) \text{ for } R<t<R+I \\ 0 \quad \text{else}\end{cases} \tag{5}$$

The PDF's for uniform, triangular and shifted beta (beta4) distributions can be found in [8] and [9]. It is important to note that, while the uniform and triangular distributions simplify when summed, the shifted beta will not. It is for this reason that simulating the first two is simple, as will be discussed in Section 3.

2.3. Algorithm for creating finitely bounded aggregations

When aggregating a large number of finitely bounded distributions, the potential for a large number of bound pairs due to complex overlaps grows. In these cases, it may be undesirable to perform the aggregation by hand; a computer algorithm for automating this process is presented in Fig 1.

3. Simulating an aggregated distribution

Using an aggregated distribution for a set of parts rather than simulating each individual part removes computational burden from a simulation by reducing the number of computations that must be performed to achieve the same effect. However, by creating a custom distribution the use of existing algorithms to simulate a distribution is virtually impossible. This requires that the simulation designer create a custom transformation that will allow a pseudo-random number generator to be used to simulate the desired distribution. This is possible due to the inverse transform method [3] shown in Equation (6).

$$\begin{aligned} &X \sim f(x) \\ &F(x)=\int_{-\infty}^{\infty} f(x)dx \\ &U \sim U(0,1) \\ &X = F^{-1}(U) \end{aligned} \tag{6}$$

Given the need to integrate and invert the distribution's function, the benefits of uniform and triangular distributions mentioned in Section 2 become clear. Because these functions are piecewise constant and piecewise linear, respectively, their integration and inversion is a well-defined function of their parameters. This makes aggregations with these forms easy to automate and simulate. The process for doing so is discussed in Section 3.1, and the process for dealing with more complex aggregations in Section 3.2.

3.1. Simulating uniform and triangular aggregations

The functional form of a set of aggregated uniform distributions is piecewise constant, so this represents the simplest of the aggregated functions to integrate and invert.

$$f(t)=\begin{cases} c_I \text{ for } I<t<II \\ \dots \\ c_R \text{ for } R<t<R+1 \end{cases}, \quad c_{Lower}=\sum_{i=Lower}^{Upper} \frac{P_{Fi}}{b_i - a_i}$$

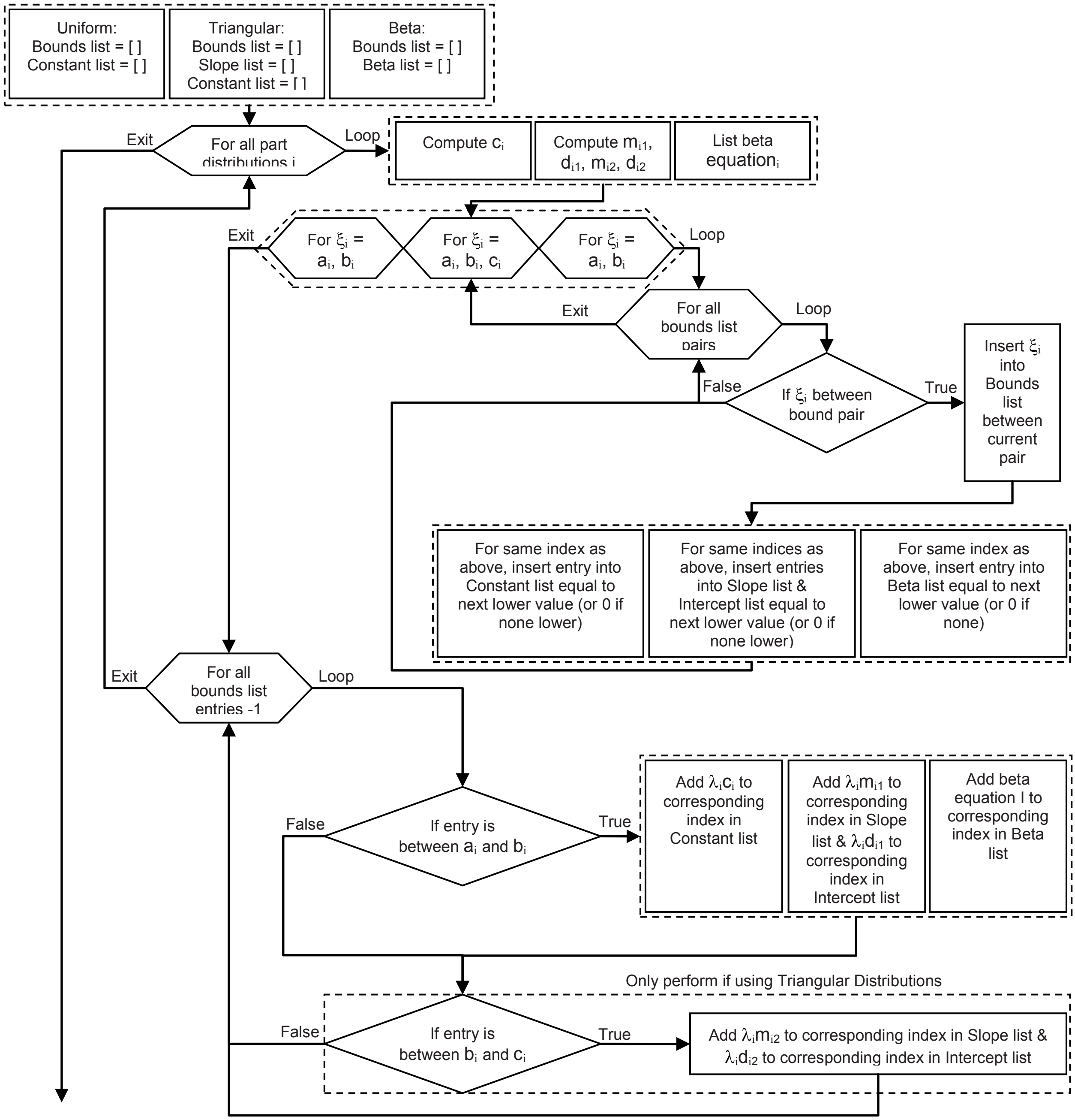

Fig. 1. Algorithm for creating finitely bounded distributions

Each pair of bounds in the piecewise function along with the knowledge that F(t) is bounded between 0 and 1 provides the boundary conditions for the individual integrated segments.

$$F(t) = \begin{cases} c_I t + C \ \ for\ I < t < II \\ \dots \\ c_R t + C \ \ for\ R < t < R+1 \end{cases} = \begin{cases} c_I t - c_I I \ \ for\ I < t < II \\ \dots \\ c_R t - c_R R + \sum_{i=I}^{R-I} c_i \left((i+I) - i \right) \ for\ R < t < R+1 \end{cases}$$

Finally, each segment is inverted and the transformed bounds computed from F(t).

$$F^{-1}(t)=\begin{cases}\dfrac{u+c_I I}{c_I} \; for \; 0<u<c_I(II-I)\\ \ldots \\ \dfrac{u+c_R R-\sum_{i=I}^{R-I} c_i((i+I)-i)}{c_R} \; for \; \sum_{i=I}^{R-I} c_i((i+I)-i)<u<1\end{cases}$$

The transformation of an aggregation of triangular distributions is similarly simple to integrate (j is 0 if the slope of the segment being added in the pair of bounds is positive and 1 if the slope is negative).

$$f(t)=\begin{cases}m_I t+d_I \; for \; I<t<II\\ \ldots \\ m_R t+d_R \; for \; R<t<R+1\end{cases}, m_{Lower}=\sum_{i=Lower}^{Upper} P_{Fi}\left(\frac{(-1)^j 2}{c_i-a_i}\right)\left(\frac{1}{b_i-a_i}\right), d_L=\sum_{i=Lower}^{Upper} -P_{Fi}\left(\frac{(-1)^j 2}{c_i-a_i}\right)\left(\frac{a_i}{b_i-a_i}\right)$$

$$F(t)=\begin{cases}\dfrac{m_I}{2}t^2+d_I t-\dfrac{m_I}{2}I^2-d_I I \; for \; I<t<II\\ \ldots \\ \dfrac{m_R}{2}t^2+d_R t-\dfrac{m_I}{2}I^2-d_I I+\sum_{i=I}^{R-I}\left[\dfrac{m_i}{2}\left((i+I)^2-i^2\right)+d_i((i+I)-i)\right] \; for \; R<t<R+1\end{cases}$$

The inversion process, however, is more complex as it depends whether the slope of the original piecewise function segment was positive, negative, or zero.

$$F^{-1}(t)=\begin{cases}\begin{cases}\dfrac{u+d_I I}{d_I} & if \; m_I=0\\ \dfrac{-d_I}{2m_I}+(-1)^j\sqrt{\dfrac{d_I{}^2+2m_I\left(\dfrac{m_I}{2}I^2+d_I I\right)+2u}{m_I}} & if \; m_I\neq 0\end{cases} & for \; 0<u<m_I\left(II^2-I^2\right)+d_I(II-I)\\ \ldots \\ \begin{cases}\dfrac{u+d_R R-\xi}{d_R} & if \; m_R=0\\ \dfrac{-d_R}{2m_R}(-1)^j\sqrt{\dfrac{d_R{}^2+2m_R\left(\dfrac{m_R}{2}R^2+d_R R-\xi\right)+2u}{m_R}} & if \; m_R\neq 0\end{cases} & for \; \xi<u<1\\ where \; \xi=\sum_{i=I}^{R-I}\left[\dfrac{m_i}{2}\left((i+I)^2-i^2\right)+d_i((i+I)-i)\right] \; and \; j=0 \; if \; m_i>0, \; j=1 \; if \; m_i<0\end{cases}$$

3.2. Simulating complex aggregations

The complex aggregations referred to in these sections are those which are formed from long sums of distribution functions, and which cannot be simplified. These include the shifted beta, truncated normal, gamma and exponential distributions as well as user-defined distributions which form unsimplifiable sums. When integrated, each of these distribution types forms a long sum of cumulative distribution functions which makes closed-form inversion impossible. In order to simulate such a distribution, numerical inversion methods must be applied. Though none are discussed here, detailed methods are presented in [10,11].

4. Aggregation Demonstration and Planned Case Study

A small sample problem for aggregating two parts (A and B) is presented in this section to demonstrate the principles described in this paper. For clarity, the distributions chosen are those that allow a closed-form solution to be obtained. Both parts' times to failure are exponentially distributed with λ_A = 0.01/operational hour and λ_B = 0.0025/operational hour. Both parts' time to complete the maintenance process are triangularly distributed with a_A = 60, b_A=90, c_A = 120, a_B = 30, b_B = 50 and c_B= 70.

The first step is to compute the relative probabilities of failure for the parts as in Equations (3).

$$P(part\ A\ fails) = \frac{\lambda_A}{\lambda_A + \lambda_B} = \frac{0.01}{0.01 + 00025} = 0.8$$

$$P(part\ B\ fails) = \frac{\lambda_B}{\lambda_A + \lambda_B} = \frac{0.0025}{0.01 + 0.0025} = 0.2$$

Next, the aggregation is computed as in Equation (5). A graphical representation of this aggregate distribution is shown in Fig 2.

$$f(t) = \begin{cases} t_1 \sim T(60,90,120)\ with\ probability\ 0.8 \\ t_n \sim T(30,50,70)\ with\ probability\ 0.2 \end{cases} = \begin{cases} \frac{1.6}{120-60}\left[\frac{t}{90-60} - \frac{60}{90-60}\right] for\ 60 < t < 90 \\ \frac{-1.6}{120-60}\left[\frac{t}{120-90} - \frac{120}{120-90}\right] for\ 90 < t < 120 \\ \frac{0.4}{70-30}\left[\frac{t}{50-30} - \frac{30}{50-30}\right] for\ 30 < t < 50 \\ \frac{-0.4}{70-30}\left[\frac{t}{70-50} - \frac{70}{70-50}\right] for\ 50 < t < 70 \\ 0 \quad else \end{cases}$$

$$f(t) = \begin{cases} \frac{0.4}{70-30}\left[\frac{t}{50-30} - \frac{30}{50-30}\right] & = 5E-4t - 0.015\ for\ 30 < t < 50 \\ \frac{-0.4}{70-30}\left[\frac{t}{70-50} - \frac{70}{70-50}\right] & = -5E-4t + 0.035\ for\ 50 < t < 60 \\ \frac{-0.4}{70-30}\left[\frac{t}{70-50} - \frac{70}{70-50}\right] + \frac{1.6}{120-60}\left[\frac{t}{90-60} - \frac{60}{90-60}\right] & = 3.\bar{8}E-4t - 0.018\bar{3} for\ 60 < t < 70 \\ \frac{1.6}{120-60}\left[\frac{t}{90-60} - \frac{60}{90-60}\right] & = 8.\bar{8}E-4t - 0.05\bar{3}\ for\ 70 < t < 90 \\ \frac{-1.6}{120-60}\left[\frac{t}{120-90} - \frac{120}{120-90}\right] & = -8.\bar{8}E-4t + 0.10\bar{6} for\ 90 < t < 120 \\ & 0 \quad else \end{cases}$$

Finally, the inverse transform method is used to enable simulation of the aggregate distribution.

$$F(t)=\begin{cases}0 \ for\ t<30\\ 2.5E-4t^2-0.015t+0.225 \ for\ 30<t<50\\ -2.5E-4t^2+0.035t-1.025 \ for\ 50<t<60\\ 1.9\bar{4}E-4t^2-0.018\bar{3}t+0.575 \ for\ 60<t<70\\ 4.\bar{4}E-4t^2-0.05\bar{3}t+1.8 \ for\ 70<t<90\\ -4.\bar{4}E-4t^2+0.10\bar{6}t-5.4 \ for\ 90<t<120\\ 1 \ for\ 120<t\end{cases},\quad F^{-1}(t)=\begin{cases}30+20\sqrt{10u} \ for\ 0<u<0.1\\ 70-20\sqrt{2-10u} \ for\ 0.1<u<0.175\\ \frac{330}{7}+\frac{60}{7}\sqrt{-10+70u} \ for\ 0.175<u<\frac{11}{45}\\ 60+15\sqrt{-2+10u} \ for\ \frac{11}{45}<u<0.6\\ 120-15\sqrt{10-10u} \ for\ 0.6<u<1\end{cases} \quad (7)$$

To show that this process has in fact enabled the simulation of an aggregated distribution, Fig. 2 (b) shows a histogram of 10,000 samples taken from the distribution defined in Equation (7). The histogram shows visually that the inverse transform method is an accurate way to simulate the desired distribution using only a uniformly distributed random variable.

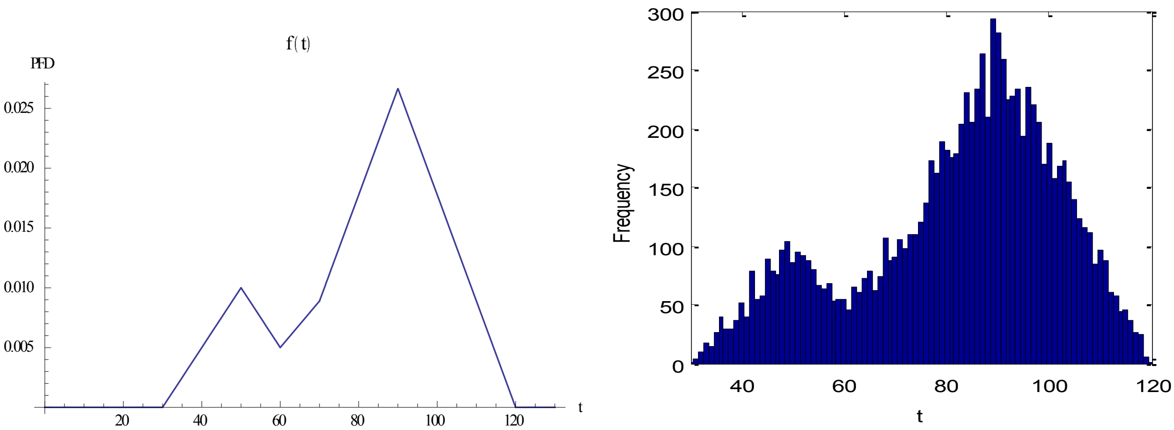

Fig. 2. (a) Aggregate distribution ; (b) Histogram of the simulated aggregate distribution

The example presented above is of limited scope and its applicability to a systems engineering context is not entirely clear. For this reason a case study is planned to expand on the ideas presented. The case study data is derived from a thesis discussing the simulation of a logistical process for military aircraft using the newly developed Autonomic Logistics System [12], and provides both exponential failure rates and triangularly distributed repair times which are ideal for the method presented in this paper. The case study will examine how the four parts presented in the thesis may be aggregated, and will extend this idea to even larger numbers of parts to show the value to be gained from simulating aggregated part distributions, specifically in the improvement of simulation time for non-aggregated and aggregated parts. The planned case study will constitute the focus of this paper's presentation at the CSER conference.

5. Conclusions

For specific kinds of systems, aggregation can be a powerful tool to reduce computational burden without corresponding loss of fidelity. Aggregation is especially powerful for those systems whose parts fail according to the exponential distribution and whose maintenance processes are distributed in a uniform or triangular fashion, since in this case the aggregation is mathematically equivalent to the non-aggregated parts, as was demonstrated through the development of Equations (1) through (5).

The greatest disadvantage of the method presented in this paper is the relatively limited set of distributions for which an aggregation is a closed-form solution. Although the loss in fidelity associated with distributions not in this set is not significantly great, the need to use numerical methods for estimation introduces undesirable computational burden. Depending on the algorithms used to perform these numerical methods and the number of parts being aggregated, this method may not represent a significant improvement over the original simulation. One further limiting assumption made is that part failures are disjoint; this may not be a valid assumption for many systems. Though the method in this paper would not apply in the wider case, it could easily be expanded to include disjoint failures if the joint failure probabilities of all pairs, triples, etc. of parts were known.

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