A numerical study on the physics of flow over a flat plate with backward facing step

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Abstract

Some of the results of a numerical investigation of the flow field characteristics of a transonic compressible flow over a flat plate with backward facing step of different heights are presented. The study has been performed by solving Two-Dimensional Navier-Stokes equations. The system of governing equations has been solved, using an explicit Harten-Yee Non-MUSCL Modified flux type TVD scheme and a zero-equation algebraic turbulence model to calculate the eddy viscosity coefficient. The results presented in this paper are computed for fixed Mach number, M=0.8 and for different step heights, h=2.5, 5 and 7.5mm. The details on pressure, temperature and velocity field, together with recirculation length, expansion shock, reattached shock, interaction of shock wave are reported. The variations of flow characteristics due to change of step heights are also presented.

Key Word: Mach number; backward facing step; recirculation; shock wave.

1. Introduction

Numerical simulation is becoming a very important drive for the design and analysis of more complicated system. The advancement of computing urges of engineers to include high fidelity in computational fluid dynamics (CFD) in
the design and testing tools of new technological products and processes. Numerical simulation is now recognized to be a part of the computer aided engineering (CAE) spectrum of tools used extensively today in all engineering field. The separation and reattachment of flows due to the change in test section geometries are well known. It occurs in many practical engineering applications, both in internal flow systems such as diffusers, combustors channel with sudden expansion, and in external flows like flows around airfoils and buildings. In these situations, the flow experiences an adverse pressure gradient, i.e. the pressure increases in the direction of flow, which cause the boundary layer to separate from the solid surface. The flow subsequently reattaches downstream forming a recirculation bubble. In some application such as combustors, the presence of the recirculation and turbulence due to separation can help enhance the mixing of fuel and air. On the other hand, the separation in the pipe and duct flow causes loss of available energy. Thus understanding the flow separation and reattachment phenomena is important in engineering design.

The flow over a flat plate with facing step is very important in many engineering applications. In many cases, the flow field characteristic especially the flow separation is vital. A comprehensive review of the current understanding of the effect of backward facing step on high speed flow has been published [1-7]. Scherber and Smith [1] experimentally investigated the flow pressure structure of the flow field, due to changing the pressure, Mach number and step height. Popusco and Panait [2] conducted an experimental study to analyze the field velocity of a fully developed turbulent incompressible flow behind a backward facing step with a curve nose shape. Halupovich et al. [3] investigated numerically and experimentally the effects of the incoming boundary layer, Reynolds number and inlet Mach number on the flow. Saleel et al.[4] are facilitated a numerical investigation of the laminar forced fluid flow over a forward-backward facing steps through a 2-D channel by using immersed boundary method (IBM). Nadeem Hassan et al. [5] presented the details of peak pressure and temperature together with the length of separated region and precious location of detached shock in front of the step. Crouch et al. [6] reported on the boundary layer displacement thickness due to stepping. Chen et al. [7] revealed the flow structures, including supersonic laminar boundary layer, separation, reattachment, redeveloping turbulent boundary layer, expansion wave fan and reattachment shock in the transient flow field. Through a considerable number of researchers carried out their researches on flow over a flat plate, still it faces many unsolved problems. So more investigations are required to flow over a flat plate and overcome those problems. In present work a high speed flow field on flat plate with facing steps is considered. The physics of flow separation, flow transition from subsonic to supersonic and dynamical behavior of the flow field will be addressed.

### Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tr>
<td>L</td>
<td>Total length of the plate</td>
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<tr>
<td>H</td>
<td>Height of the calculating domain</td>
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<tr>
<td>h</td>
<td>Height of the step, 2.5, 5 and 7.5 mm</td>
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<tr>
<td>M</td>
<td>Mach number</td>
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### 2. Mathematical Description

The flow field is governed by the unsteady, two-dimensional full Navier-Stokes and species continuity equations. The body forces are neglected. With the conservation-law form, these equations can be expressed by

\[
\frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} = \frac{\partial F_v}{\partial x} + \frac{\partial G_v}{\partial y}
\]
Where \[ U = \begin{pmatrix} \rho \\ \rho u \\ \rho v \\ E \\ \rho Y_i \end{pmatrix}, \quad F = \begin{pmatrix} \rho u \\ \rho u^2 + p \\ \rho uv \\ (E + p)u \\ \rho Y_i u \end{pmatrix}, \quad G = \begin{pmatrix} \rho u \\ \rho uv \\ \rho u^2 + p \\ (E + p)v \\ \rho Y_i v \end{pmatrix}, \quad F_v = \begin{pmatrix} 0 \\ \sigma_x \\ \tau_{xy} \\ \sigma_x u + \tau_{xy} v - q_x \\ - \tilde{m}_x \end{pmatrix}, \]

\[ G_x = \begin{pmatrix} 0 \\ \sigma_y \\ \tau_{xy} u + \sigma_{yy} - q_y \\ - \tilde{m}_y \end{pmatrix}, \quad P = \sum_{i=1}^{n_s} \rho_i R_i T = \sum_{i=1}^{n_s} \rho_i \frac{R}{W_i} T \]

\[ E = \sum_{i=1}^{n_s} \rho_i h_i - \sum_{i=1}^{n_s} \rho_i \frac{R}{W_i} T + \frac{\rho}{2} (u^2 + v^2) = \sum_{i=1}^{n_s} \rho_i C_{pi} T - \sum_{i=1}^{n_s} \rho_i \frac{R}{W_i} T + \frac{\rho}{2} (u^2 + v^2) \]

\[ \sigma_x = \lambda \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) + 2\mu \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right), \quad \sigma_y = \lambda \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) + 2\mu \left( \frac{\partial v}{\partial y} \right), \quad \tau_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial y}, \quad \lambda = -\frac{2}{3} \mu \]

3. Flow Field Description and Numerical Parameters

The geometric configuration of the calculation domain is shown in Fig. 1. The position of the step is at \( L = 0 \). Throughout the study, the grid system consists of 194 nodes in the longitudinal direction and 121 nodes in the transverse direction. The temperature and pressure of the incoming stream are 302K and 126 kPa, respectively.

![Fig.1 Calculation Domain](image)

4. Results and Discussion

The main objective of this study is to investigate the variation of different flow field characteristics of a transonic flow with the change of the height of step. The investigation has been done by varying the height of the step as 2.5, 5 and 7.5mm at constant Mach number of 0.8.

4.1 Characteristic of different shocks
Figures 2(a-c) illustrate the Mach contour for different step heights. Mack contours are characterized by the presence of shocks in the flow. The two shock regions are visible in the flow field, namely expansion shock and reattachment shock. Expansion shock appears to emanate from the vicinity of the top of the step and reattached shock appears at the reattachment region, which is located below the expansion shock region. By analyzing the figures, it is found that, the expansion shock rotates clockwise, moves downward and the angle of shock with main flow direction decreases with the increase of step height. Due to the change of step height, the width of the expansion shock and the degree of expansion also increase. An interaction of two shocks (expansion shock and reattachment shock) is found in the flow field and its position also rotates with the increase of step height.

4.2 Dynamical behaviour of the flow field
Figures 3(a-c) show the velocity contour for different step heights. The velocity contours are characterized by the
sudden rise of velocity in the expansion shock region. At the left bottom corner of the velocity field recirculation appears and the zero value of the velocity contour is considered as recirculation zone. The length of this zone along the bottom wall is considered as recirculation length. With the increase of step height, the length of recirculation increases, which is found 0.0222, 0.0264 and 0.039m, respectively. The strength of recirculation also increases, as the step height increases. The stream wise vorticity is stronger near the step and reduces in strength further downstream. The maximum strength of recirculation is found at about one-fourth of the recirculation length. In the flow field, it is found that after the interaction of two shocks a part of flow is reflected to the downward direction which exists up to the exit of the boundary.

In Fig. 4, it is found that the velocity is negative along the recirculation zone, after recirculation the velocity refits. This structure of velocity indicates the redevelopment of boundary layer after recirculation.

4.3 Effect on Pressure and Temperature in the flow Field
Figures 5(a-c) represent pressure contour for different step heights. The pressure contours show a sudden drop of pressure in the expansion shock region and the minimum pressure is found in the recirculation zone. In the expansion shock region, with the increase of step heights, the pressure gradually decreases due to the result of overexpansion. Immediate behind the expansion shock, the reattachment shock evolves where the pressure again increases.

From the temperature contour as shown in Fig. 6(a-c), it is found that there is a sudden drop of temperature in the
expansion shock region and again the temperature gradually increases at reattachment shock region. In the expansion shock region, it is found that the temperature decreases, as the height of the step increases. There is a sudden increase of temperature near the step because of recirculation zone, which tends to accumulate the recirculated particles near the left-bottom corner causing the increasing of temperature. The maximum temperature in this region can be found around 940K. There is no remarkable change of maximum temperature in the flow field, with the change of step height.

5. Conclusion

A study of transonic flow over a backward facing step is carried out by using an explicit Harten-Yee Non- MUSCL Modified flux type TVD scheme. The effects of step height on the dynamical behaviour of flow field have been reported. Using the Mach contours, expansion and reattachment shock region have been analyzed. The degree of expansion at expansion shock region, the width of the expansion shock and the length as well as strength of recirculation have increased, with the increase of step height. The increment rate increases with step height. Sudden drop of pressure and temperature in expansion shock region can be observed. At the end of the recirculation, the reattachment shock follows the expansion shock where both the pressure and temperature again increase. Maximum temperature and minimum pressure of the flow field are found in recirculation zone, which is almost same for different step heights.

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References