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## Thermal relaxation of charm in hadronic matter

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## ARTICLE INFO

## Article history:

Received 1 April 2011

Received in revised form 7 June 2011

Accepted 9 June 2011

Available online 15 June 2011

Editor: W. Haxton

## Keywords:

Charm transport

Hot hadronic matter

Ultrarelativistic heavy-ion collisions

## ABSTRACT

The thermal relaxation rate of open-charm ( $D$ ) mesons in hot and dense hadronic matter is calculated using empirical elastic scattering amplitudes.  $D$ -meson interactions with thermal pions are approximated by  $D^*$  resonances, while scattering off other hadrons ( $K$ ,  $\eta$ ,  $\rho$ ,  $\omega$ ,  $K^*$ ,  $N$ ,  $\Delta$ ) is evaluated using vacuum scattering amplitudes as available in the literature based on effective Lagrangians and constrained by realistic spectroscopy. The thermal relaxation time of  $D$ -mesons in a hot  $\pi$  gas is found to be around 25–50 fm/c for temperatures  $T = 150$ –180 MeV, which reduces to 10–25 fm/c in a hadron-resonance gas. The latter values, argued to be conservative estimates, imply significant modifications of  $D$ -meson spectra in heavy-ion collisions. Close to the critical temperature ( $T_c$ ), the spatial diffusion coefficient ( $D_s$ ) is surprisingly similar to recent calculations for charm quarks in the Quark–Gluon Plasma using non-perturbative  $T$ -matrix interactions. This suggests a possibly continuous minimum structure of  $D_s$  around  $T_c$ .

Published by Elsevier B.V.

## 1. Introduction

After the initial discovery of a new state of matter in high-energy nuclear collisions at the Relativistic Heavy Ion Collider (RHIC), the focus is now shifting to quantifying the properties of what is believed to constitute a strongly coupled Quark–Gluon Plasma (sQGP). Basic quantities characterizing the medium are its thermal spectral functions and transport properties. In heavy-ion collisions (HICs), the former are most directly studied in the electromagnetic (vector) channel via the thermal emission of lepton pairs (cf. Ref. [1] for a recent review). The latter, however, are best studied using observables with small but controlled deviations from thermal equilibrium. Thus, heavy quarks (charm ( $c$ ) and bottom ( $b$ )), whose thermal equilibration time is expected to be of the order of the QGP lifetime in HICs, are a promising tool to quantify flavor transport, and eventually deduce general properties of the QGP as formed in these reactions [2].

The large masses of  $c$  and  $b$  quarks ( $m_{c,b}$ ) enable us to assess the modifications of their momentum spectra in HICs via a diffusion process in an evolving background medium as formulated, e.g., within a Fokker–Planck equation [3] (typical early temperatures of the medium produced at RHIC,  $T \simeq 250$  MeV [4], are well below  $m_{c,b} \simeq 1.5, 4.5$  GeV). A reliable determination of the heavy-quark (HQ) transport coefficients in the QGP depends on several

components. First and foremost these are microscopic calculations of the thermal HQ relaxation rate in the QGP [3,5–14] (see Ref. [2] for a review). Second, the coefficients need to be implemented into a realistic bulk medium evolution (see, e.g., Ref. [15] for a recent discussion). Third, heavy-flavor (HF) interactions in evolution phases other than the QGP have to be evaluated, i.e., in the so-called pre-equilibrium phase as well as in the hadronic phase. The former is of a relatively short duration,  $\Delta\tau_{\text{pre}} \lesssim 1$  fm/c, and is sometimes mimicked by reducing the formation time of the QGP. The duration of the hadronic phase is substantially longer,  $\Delta\tau_{\text{had}} \simeq 5$ –10 fm/c. Its relevance for HF phenomenology is further augmented by the fact that the hadronic medium inherits the full momentum anisotropy from the QGP, believed to be close to the finally observed one. Thus, even a rather weak coupling of HF hadrons to hadronic matter can lead to noticeable contributions to their elliptic flow. Furthermore, if the QGP realizes a minimum in its viscosity-to-entropy-density ratio,  $\eta/s$ , close to the (pseudo-)critical temperature,  $T_c \simeq 170$  MeV, a hadronic liquid close to  $T_c$  should possess similar properties. This is usually referred to as a “quark–hadron duality”, as suggested, e.g., in calculations of thermal dilepton emission rates [1].

Charm diffusion in hadronic matter has received little attention to date (see Ref. [16] for a very recent estimate using heavy-meson chiral perturbation theory). Its potential relevance has been noted in Ref. [2] based on calculations of  $D$ -meson spectral functions in nuclear matter using effective hadron Lagrangians [17,18], as well as in a hot pion gas [19]. In the present Letter, we augment these works to evaluate charm diffusion in hadronic matter. Since the

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latter features many resonances at temperatures approaching  $T_c$ , we not only utilize  $D\pi$  and  $DN$  interactions but also scattering amplitudes off excited hadrons ( $K$ ,  $\eta$ ,  $\rho$ ,  $\omega$ ,  $K^*$ ,  $\Delta$ ), as constructed in the literature using effective Lagrangians and constrained by charm-resonance spectroscopy. In this sense we provide a lower estimate of the diffusion coefficient, based on existing elastic  $D$ -hadron amplitudes.

Our Letter is organized as follows. In Section 2 we “reconstruct” microscopic models for  $D\pi$  scattering in a hot pion gas (via  $s$ -channel resonances; Section 2.1), for  $D$  scattering off strange and vector mesons (Section 2.2), and off baryons (Section 2.3), by parameterizing pertinent scattering amplitudes. In Section 3 these are applied to calculate thermal  $D$ -meson relaxation rates and diffusion coefficients in hot hadronic matter at vanishing chemical potential, first in a pion gas (Section 3.1) and then in a resonance gas (Section 3.2), and finally including chemical potentials as appropriate for heavy-ion collisions at RHIC (Section 3.3). Conclusions are given in Section 4.

## 2. $D$ -meson scattering and width in hot hadronic matter

In this section we recapitulate basic elements of the  $D$ -hadron scattering amplitudes and apply them to calculate pertinent  $D$ -meson widths in hadronic matter. We only employ known amplitudes from the literature for the most abundant hadrons in a hot medium and combine them into a lower-limit estimate for the  $D$ -meson width (and  $D$ -meson relaxation rate in Section 3). We first focus on pion-gas effects, followed by interactions with strange and vector mesons, as well as hot nuclear matter.

### 2.1. Hot pion gas

Following Ref. [19]  $D$  interactions in the pion gas are dominated by its chiral partner,  $D_0^*(2308)$ , by the vector  $D^*(2010)$  and by the tensor meson  $D_2^*(2460)$ . Phenomenological control over their interaction vertices has become possible due to new observations of  $D$ -meson resonances by the BELLE Collaboration [20]. Especially, the large width of the  $D_0^*(2308)$ ,  $\Gamma_0^{\text{tot}} = 276 \pm 99$  MeV, attributed to  $S$ -wave pion decay, leads to a large  $D\pi D_0^*$  coupling constant. With the constituent-quark model assignment of isospin  $I = 1/2$  for  $D$ -states, the pertinent forward scattering amplitudes have been parameterized in Breit-Wigner form as

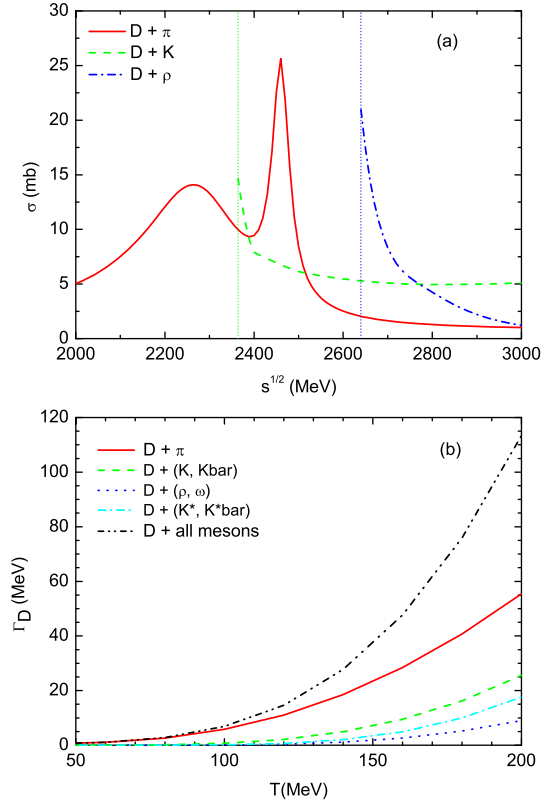
$$\mathcal{M}(s, \Theta = 0) = \sum_{j=0}^2 \frac{8\pi\sqrt{s}}{k} \frac{(2j+1)\sqrt{s}\Gamma_j^{D\pi}}{s - M_j^2 + i\sqrt{s}\Gamma_j^{\text{tot}}}, \quad (1)$$

where  $\sqrt{s}$  and  $k$  denote the center-of-mass energy and 3-momentum in the  $D\pi$  collision, respectively, and  $j$  is the resonance spin. The total resonance decay width  $\Gamma_{\text{tot}}$ , is assumed to be saturated by the partial one,  $\Gamma_{0,1}^{D\pi}$ , for  $D^*$  and  $D_0^*$ , while for the narrow state  $D_2^*$  ( $\Gamma_2^{\text{tot}} = 45.6 \pm 12.5$  MeV) a branching fraction estimated by the particle data group is employed [21]. The resulting total  $D\pi$  cross section is displayed in the upper panel of Fig. 1.

The  $D$ -meson self-energy in a pion gas can now be obtained by the standard procedure of closing the in- and outgoing pion lines of the forward  $D\pi$  amplitude with a thermal pion propagator. In the narrow-width approximation for the pion propagator, the  $D$ -meson self-energy takes the form

$$\Pi(p_D; T) = \int \frac{d^3p_\pi}{(2\pi)^3 2E_\pi} f_B(E_\pi; T) \mathcal{M}(s, \Theta = 0), \quad (2)$$

where  $f_B$  is the thermal Bose function and  $E_\pi$  the on-shell pion energy. Alternatively, one can obtain the collision rate (or on-shell



**Fig. 1.** (a) Total elastic cross sections for  $D$ - $\pi$  (solid line),  $S$ -wave  $D$ - $K$  (dashed line) and  $D$ - $\rho$  (dash-dotted line) scattering based on the invariant amplitudes as discussed in the text; vertical lines indicate the respective thresholds. (b) Total  $D$ -meson scattering width in a meson gas (dash double dotted line), consisting of contributions from Goldstone bosons and vector mesons.

width),  $\Gamma = -\text{Im} \Pi(p_D^2 = m_D^2, T)/m_D$  from the Boltzmann equation as

$$\Gamma(p_D; T) = \frac{\gamma_\pi}{2E_D(p_D)} \int \frac{1}{(2\pi)^9} \frac{d^3q}{2E_\pi(q)} \frac{d^3p'}{2E_D(p')} \frac{d^3q'}{2E_\pi(q')} \times f_B(E_\pi(q); T) |\mathcal{M}|^2 (2\pi)^4 \delta^4(p + q - p' - q'), \quad (3)$$

where  $\gamma_\pi = 3$  denotes the spin-isospin degeneracy factor of the in-medium particle (pion), and  $p, q$  and  $p', q'$  are the momenta of in- and outgoing particles, respectively. Eqs. (2) and (3) are related via the optical theorem (we have checked their consistency). In the following we employ the latter since it is close to the form of the thermal relaxation rate discussed in Section 3 below.

The scattering width of a  $D$ -meson at rest in a pion gas is displayed in the lower panel of Fig. 1. For  $T = 150$ – $180$  MeV we find  $\Gamma_D = 20$ – $40$  MeV, where the chiral partner of the  $D$  provides the largest contribution through  $S$ -wave  $D\pi$  scattering. For constant resonance widths, we find close agreement with the results plotted in Fig. 2 of Ref. [19], while for energy-momentum dependent widths (as quoted in Ref. [19]) our results displayed in Fig. 1 turn out to be slightly smaller (by ca. 20%).

Chiral effective Lagrangians have been applied to  $D$ -meson interactions with Goldstone bosons in Refs. [22–24]. Once the parameters are constrained by empirical information, the resulting scattering amplitudes are very similar and closely agree with the resonance ansatz of Ref. [19] as employed here. Chiral Lagrangians also predict non-resonant interactions. In the (repulsive)  $S$ -wave  $I = 3/2$   $D\pi$  channel the scattering length has been computed as  $a_{D\pi}^{I=3/2} \equiv -f(s_{\text{thr}}) = -0.1$  fm in unitarized chiral perturbation theory ( $\chi$ PT) [25] vs.  $a_{D\pi}^{I=3/2} = -0.15$  fm in  $\chi$ PT to NNLO [26]. We

employ the former to construct the  $I = 3/2$  scattering amplitude as  $\mathcal{M}(s) = 8\pi\sqrt{s}f(s)$ .

## 2.2. Strange and vector mesons

In a hot meson gas, the next abundances species after the pions are the strangeness carrying Goldstone bosons and the light and strange vector mesons.

For  $D(K, \bar{K})$  interactions we directly employ the results of the (unitarized) chiral effective theory of Ref. [22] (by parameterizing the amplitudes in Figs. 1 and 2 in there). In the  $S$ -wave  $DK$  channel, with isospin-strangeness  $(I, S) = (0, 1)$ , the results are constrained by the (loosely bound)  $D_{s0}^*(2317)$  state (the analogue of  $D_0^*(2308)$  in  $D\pi$ ), and again closely agree with Refs. [23,24]. The application to the  $S$ -wave  $DK$   $(I, S) = (1, 1)$  and  $D\bar{K}$   $(I, S) = (0, -1)$  produces Feshbach-type resonances (tetra-quarks) right at threshold ( $E_{DK}^{\text{thr}} = 2360$  MeV). For  $D\bar{K}$  in the  $(I, S) = (1, -1)$  channel the Born amplitude is predicted to be repulsive (analogous to  $I = 3/2$   $D\pi$ ); the scattering length has been calculated as  $a_{D\bar{K}}^{I=1} = -0.22$  fm in unitarized  $\chi$ PT [25] vs.  $a_{D\bar{K}}^{I=1} = -0.33$  fm in  $\chi$ PT at NNLO [26]. Analogous to  $I = 3/2$   $D\pi$ , we employ the former to construct the pertinent scattering amplitude.

For  $D\eta$  scattering we also adopt Ref. [22], where the  $S$ -wave  $(I, S) = (1/2, 0)$  amplitude is governed by a narrow state of mass 2413 MeV just below threshold.

The evaluation of  $DV$  scattering ( $V = \rho, \omega, K^*$ ) requires to go beyond the chiral Lagrangian. This has been done in Ref. [27] starting from  $SU(4)$  flavor symmetry and then implementing chiral breaking terms. This framework, properly unitarized, recovers the resonance poles computed with the chiral Lagrangians, but extends to axialvector states coupling to  $S$ -wave  $DV$  interactions (in particular  $D_1(2420)$ ,  $D_1'(2427)$ ,  $D_{s1}(2460)$ ,  $D_{s1}(2536)$ ). A convenient Breit-Wigner parametrization of the elastic coupling of  $DV$  to the dynamically generated resonances has been quoted as

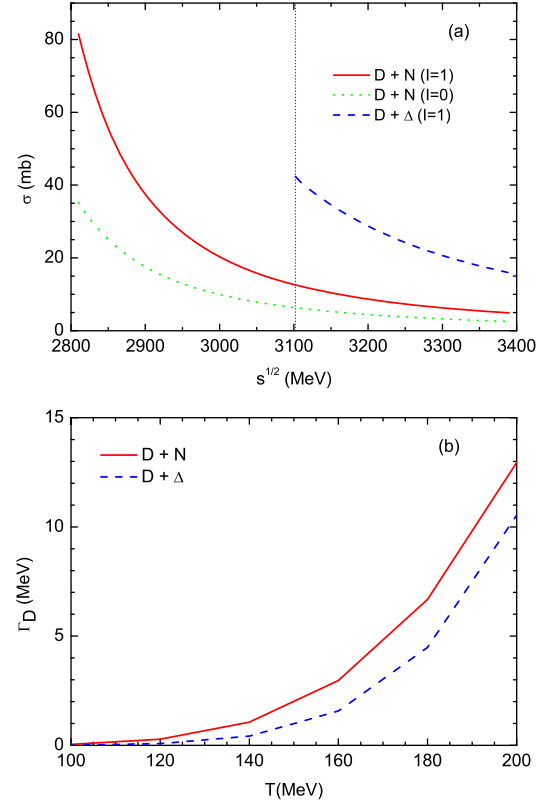
$$\mathcal{M}(s) = \frac{|g_{DV}|^2}{s - s_R}, \quad (4)$$

where  $g_{DV}$  is the dimensionful coupling constant and  $s_R$  the complex resonance-pole position. We include the three  $I = 1/2$  resonance couplings to  $D\rho$  and  $D\omega$  from Table 7 in Ref. [27], two  $I = 0$  and two  $I = 1$  resonances with  $DK^*$  coupling (Tables 5 and 4 in [27], respectively) and one  $I = 0$  state coupling to  $D\bar{K}^*$  (Table 8 in [27]). As a representative, the isospin  $I = 1/2$   $D\rho$  cross section is shown in the upper panel of Fig. 1.

The lower panel of Fig. 1 shows the temperature dependence of mesonic contributions to the  $D$ -meson scattering width as calculated from the above amplitudes. The width from anti-/kaons is the next largest contribution after the pion. The effect of vector mesons is smaller but significant, especially for the  $K^*$ . The total  $D$  width in a hot meson gas reaches  $\sim 80$  MeV around  $T \simeq 180$  MeV, which should be a lower limit since several channels have still been neglected, e.g., higher partial waves (except for pions) and inelastic channels (e.g.  $DK^* \leftrightarrow D_s\pi$ ).

## 2.3. Hot nuclear matter

To evaluate  $D$  scattering off baryons we follow the same strategy as for mesons, employing vacuum scattering amplitudes. More elaborate many-body calculations for  $D$ -mesons in nuclear matter are available [18,28], but our procedure keeps consistency with the mesonic sector and allows for an estimate of the systematic error due to in-medium effects (e.g., self-consistency of self-energy and in-medium scattering amplitudes).



**Fig. 2.** (a) Elastic cross sections for  $S$ -wave  $D$ - $N$  scattering in the  $I = 1$  (solid line) and  $I = 0$  (dotted line) channel, and  $D$ - $\Delta$  scattering (dashed line,  $I = 1$ ; vertical line:  $D\Delta$  threshold) based on the invariant amplitudes as discussed in the text; (b)  $D$ -meson widths from  $D$ - $N$  scattering (solid line, including both  $I = 1$  and  $I = 0$  channels) and from  $D$ - $\Delta$  scattering (dashed line) at vanishing chemical potential.

We start from Ref. [18] where the  $T$ -matrix results of an effective  $DN$  interaction with coupled channels have been parameterized in analogy to Eq. (4) as

$$T(\sqrt{s}) = \frac{|g|^2}{\sqrt{s} - m_R + i\Gamma_R/2}, \quad (5)$$

where  $m_R$  is the resonance mass and  $\Gamma_R$  the width. The  $T$ -matrix, Eq. (5), is related to the invariant scattering amplitude through  $\mathcal{M}(s) = N(s)T(s)$  with  $N(s) = (s + m_N^2 - m_D^2 + 2m_N\sqrt{s})/(2\sqrt{s})$ . The key states are the dynamically generated  $I = 0$   $\Lambda_c(2595)$  and  $I = 1$   $\Sigma_c(2620)$   $S$ -wave bound states. The former is experimentally well established, while the latter is ca. 180 MeV too deep compared to the empirical  $\Sigma_c(2800)$  state. However, the  $I = 1$   $DN$  cross section above threshold (shown in the upper panel of Fig. 2) is quite comparable to the results of a recent meson-exchange model calculation [29] in which the  $\Sigma_c$  is generated at its experimental mass (in fact, at threshold the  $I = 1$  scattering length in Ref. [29] is significantly larger than for the  $\Sigma_c(2620)$  state of Ref. [18]). In the  $I = 0$  channel, the scattering lengths of Refs. [18] and [29] agree well. The corresponding cross section is also shown in the upper panel of Fig. 2.

The  $D\bar{N}$  scattering amplitude can be inferred from  $\bar{D}N$  due to  $C$ -symmetry of strong interactions. We have found no evidence in the literature for (multi-quark) resonances in this system and adopt the  $D^-N$  elastic  $S$ -wave amplitude calculated in Ref. [18]. The pertinent scattering and relaxation rates are about a factor of  $\sim 3$  smaller than from  $DN$  scattering.

The only available calculation of  $D\Delta$  we are aware of has been conducted in Ref. [30], within the same framework as our  $DN$  amplitudes are based on. In the only available  $I = 1$   $S$ -wave channel

the parametrization, Eq. (5), reflects a rather deep bound state ( $m_R = 2613$  MeV,  $\Gamma_R \simeq 0$ ,  $g = 8.6$ ). The cross section is shown in the upper panel of Fig. 2. Unlike the  $D\bar{N}$  case, the  $D\bar{\Delta}$  system is predicted to support a shallow  $l = 1$  bound state ( $m_R = 2867$  MeV,  $\Gamma_R \simeq 0$ ,  $g = 5.8$ ). As a result, the contribution of  $D\bar{\Delta}$  scattering to the  $D$ -meson width and relaxation rate is about half of that from  $D\Delta$  scattering.

The lower panel of Fig. 2 shows that the width of a  $D$ -meson at rest from scattering off thermally excited nucleons and  $\Delta$ 's at vanishing chemical potential ( $\mu_B = 0$ ) is comparable to that of light vector mesons (cf. lower panel of Fig. 2). When adding  $\bar{N}$  and  $\bar{\Delta}$  contributions, the baryon-induced  $D$ -meson width computed here amounts to ca. 15 MeV at  $T \simeq 180$  MeV. Note again that we have neglected higher partial waves as well as higher excited resonances including strange anti-/baryons.

It is instructive to compare our nucleon-induced width to a self-consistent many-body calculation [17]. From Fig. 6 in Ref. [17] we read off  $\Gamma_D \simeq 100(80)$  MeV at  $T = 100(150)$  MeV and  $\varrho_N = \varrho_0$ , vs.  $\Gamma_D \simeq 75(65)$  MeV in our approach. This indicates that neglecting in-medium effects does not lead to an overestimate in our calculation.

### 3. Thermal relaxation in hadronic matter

The standard expression for the thermal relaxation rate of a particle ( $D$ ) in a heat bath in terms of its scattering amplitude on medium particles ( $h$ ) reads [3]

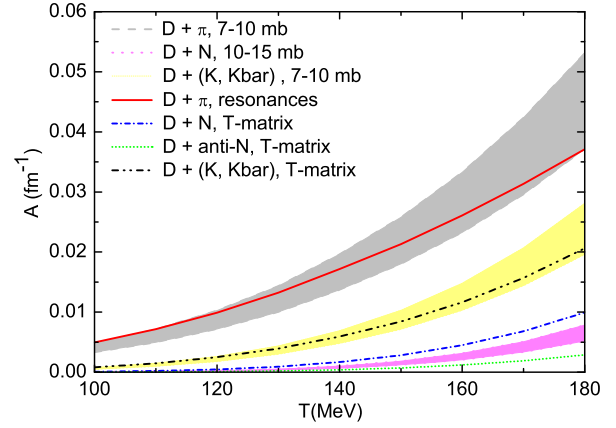
$$A(p, T) = \frac{\gamma_h}{2E_D} \int \frac{1}{(2\pi)^9} \frac{d^3q}{2E_h} \frac{d^3p'}{2E'_D} \frac{d^3q'}{2E'_h} f^h(E_h; T) \times \overline{|\mathcal{M}_{Dh}|^2} (2\pi)^4 \delta^{(4)}(p + q - p' - q') \left(1 - \frac{\vec{p} \cdot \vec{p}'}{\vec{p}^2}\right) \equiv \left\langle \left\langle 1 - \frac{\vec{p} \cdot \vec{p}'}{\vec{p}^2} \right\rangle \right\rangle \quad (6)$$

with  $\vec{p}$  ( $\vec{q}$ ) and  $\vec{p}'$  ( $\vec{q}'$ ) being the  $D$ -meson ( $h$ ) momentum before and after the interaction, respectively. The form of this expression is very similar to the one for the total width, Eq. (3). The latter can be expressed in terms of the average defined above as  $\Gamma = \langle \langle 1 \rangle \rangle$ . In Section 3.1 we evaluate  $A(p, T)$  for  $D$ -mesons in a pion gas ( $h = \pi$ ) and in Section 3.2 for all other hadrons whose amplitudes have been constructed in the previous section. In Section 3.3 we evaluate the relaxation rate at finite baryon and meson chemical potentials as applicable to the hadronic phase below the chemical freeze-out temperature in heavy-ion collisions at RHIC.

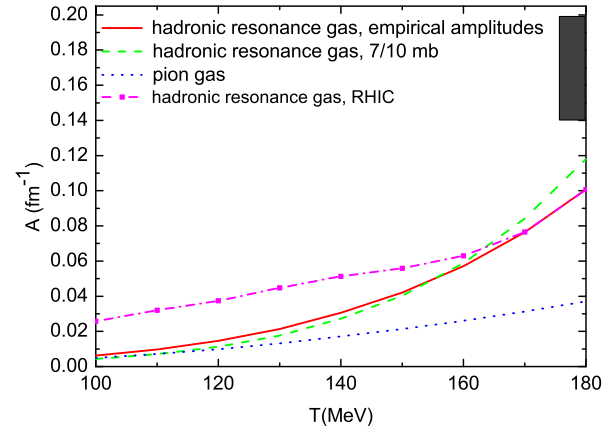
#### 3.1. Pion gas

The thermal relaxation rates for a  $D$ -meson at rest due to scattering off pions in a thermal gas in chemical equilibrium ( $\mu_\pi = 0$ ), using the scattering amplitude of Eq. (1), is displayed in Fig. 3 as a function of temperature. From  $T = 100$  MeV to 180 MeV the rate increases by about a factor of 7, basically following the increase in pion density from 0.2 to 1.4 $\varrho_0$ . Its magnitude at  $T = 180$  MeV,  $A \simeq 1/25$  fm, is small but not negligible. When replacing the  $D\pi$  amplitude in Eq. (3) by one yielding a constant  $S$ -wave cross section of  $\sigma_{D\pi}^S = 7$ –10 mb, the pertinent band for the relaxation rate essentially covers the result of our microscopic calculations. The latter are closer to the upper end of the band at lower  $T$  but to the lower end at higher  $T \gtrsim 150$  MeV. This reflects the increased thermal motion of pions probing higher  $\sqrt{s}$  in the amplitudes where the latter decrease.

Compared to the recent work of Ref. [16], where  $D$ -meson diffusion in a lukewarm pion gas has been evaluated in heavy-



**Fig. 3.** Thermal relaxation rate of  $D$ -mesons at momentum  $p_D = 0.1$  GeV in a gas of pions (solid line), anti-/kaons (dash-double-dotted line), nucleons (dash-dotted line) and antinucleons (dotted line), as a function of temperature at vanishing chemical potentials using the elastic scattering amplitudes constructed in Section 2. The bands represent results obtained with constant  $S$ -wave  $D\pi$ ,  $DK$  and  $DN$  cross sections of 7–10 mb and 10–15 mb, respectively.



**Fig. 4.** Thermal relaxation rate of  $p_D = 0.1$  GeV  $D$ -mesons using empirical amplitudes in a hadron gas at vanishing (solid line) and finite (dash-dotted line with squares) chemical potentials, as well as in a pion gas (dotted line). The dashed line corresponds to isotropic  $D$ -meson cross sections with mesons (7 mb) and baryons (10 mb). The filled box at the upper right indicates charm-quark relaxation rates in a QGP at  $1.2 T_c$  from in-medium  $T$ -matrix calculations [14].

meson chiral perturbation theory, our value for the friction coefficient is much smaller; e.g., at  $T = 50(100)$  MeV, Ref. [16] finds  $A = \kappa/2m_D T \simeq 0.00055(0.05)$ /fm, about a factor of  $\sim 4(10)$  larger than our pion-gas results,  $A \simeq 0.00015(0.005)$ /fm.

#### 3.2. Hadron resonance gas

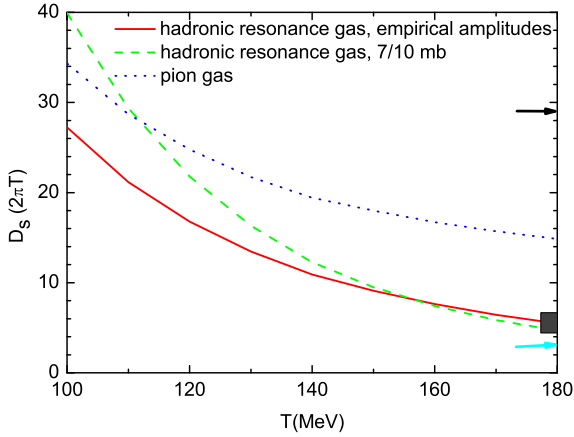
A hot hadron gas in equilibrium is characterized by an increasing abundance of resonances with rising temperature. For example, at  $T = 180$  MeV, the density of baryons plus antibaryons is above  $\varrho_0$  and that of mesons with masses below 2 GeV is above  $3\varrho_0$ . To improve the estimate of  $D$ -meson diffusion in a pion gas for a more realistic hadron-resonance gas, we include rescattering on all particles which at  $T = 180$  MeV and  $\mu_h = 0$  have a density at least  $0.1\varrho_0$ , i.e.,  $\pi$ ,  $K$ ,  $\eta$ ,  $\rho$ ,  $\omega$  and  $K^*(892)$  in the meson sector and anti-/nucleons, and  $\Delta(1232)$ ,  $\bar{\Delta}(1232)$  in the anti-/baryon sector, using all scattering amplitudes of Section 2. The resulting  $D$ -meson friction coefficient in hadronic matter at vanishing chemical potentials increases substantially over the pion-gas result, by about a factor of  $\sim 2(3)$  at  $T = 150(180)$  MeV, see Fig. 4. The individual contributions of  $K + \bar{K}$  and  $N, \bar{N}$  are compared to constant-cross-section calculations in Fig. 3, indicating that a “constituent” light-



**Table 1**

Contributions to the thermal  $D$ -meson thermal relaxation rate at  $T = 180$  MeV indicating the quantum numbers of the included scattering channels with  $L$ : partial wave,  $I$ : isospin and  $J$ : total angular momentum.

Hadrons	$L_{I,J}$	$A$ [ $\text{fm}^{-1}$ ]
$\pi$	$S_{1/2,0}, P_{1/2,2}, D_{1/2,4}, S_{3/2,0}$	0.0371
$K + \eta$	$S_{0,0}, S_{1,0}$	0.0236
$\rho + \omega + K^*$	$S_{1/2,2}, S_{0,2}, S_{1,2}$	0.0129
$N + \bar{N}$	$S_{0,1}, S_{1,1}$	0.0128
$\Delta + \bar{\Delta}$	$S_{1,3}$	0.0144



**Fig. 5.** Spatial  $D$ -meson diffusion coefficient in a pion and hadron-resonance gas based on empirical interactions and constant cross sections, in units of the medium's thermal wavelength,  $1/(2\pi T)$ . To the right, estimates for  $c$ -quark diffusion in the QGP are indicated (filled box: in-medium  $T$ -matrix approach at  $1.2T_c$  [14], lower arrow: AdS/CFT correspondence [9], upper arrow: pQCD with coupling  $\alpha_s = 0.4$ , see also Ref. [2]).

quark cross section of 3–4 mb is compatible with our lower-limit estimates. A quantitative decomposition of the individual hadron contributions to kinetic  $D$ -meson relaxation at  $T = 180$  MeV is given in Table 1. Anti/kaons provide the next-to-leading contributions after the pions, while vector mesons, nucleons and Deltas play a smaller but non-negligible role.

As an estimate of medium effects in our vacuum  $Dh$  amplitudes we have performed a calculation for  $A$  where we have introduced an in-medium broadening of  $\Gamma_R^{\text{med}} = 200$  MeV into the Breit-Wigner parameterizations. The final result for  $A$  changes by less than 5%.

Quantitatively, a value of  $A \simeq 0.1/\text{fm}$  translates into a thermal relaxation time of  $\tau_D \simeq 10$  fm. This time scale is quite comparable to lifetimes of the hadronic phase in Au–Au collisions at RHIC. It is also similar to non-perturbative calculations of charm-quark relaxation in the QGP using in-medium  $T$ -matrix interactions [11, 14]. This is encouraging both from a conceptual (in the sense of a quark–hadron continuity through  $T_c$ ) and a practical point of view (relaxation rates of this magnitude currently provide a fair phenomenology of current heavy-flavor data at RHIC [8,11]).

In Fig. 5 we display the spatial  $D$ -meson diffusion coefficient,  $D_s = T/(m_D A(p=0, T))$ , in hadronic matter. When normalized to the thermal wavelength,  $1/(2\pi T)$ , this quantity decreases with  $T$ , reaching a value of  $\sim 5$  at  $T = 180$  MeV. Again, this is surprisingly close to  $T$ -matrix results for charm quarks in the QGP [14], and, together with those results, suggests a minimum across the hadron-to-quark transition.

Our diffusion coefficient  $D_s$  may be related to the ratio of shear viscosity to entropy density of hadronic matter. With a conversion factor estimated from either kinetic transport or strong-coupling limits (see Refs. [2,31]), we find  $\eta/s \sim (2-5)/4\pi$  at  $T = 180$  MeV. This value is comparable to a recent kinetic theory estimate us-

ing Hagedorn states (i.e., an exponentially rising hadronic mass spectrum) with schematic (linearly rising) collision widths [32]. Estimates based on empirical hadronic scattering rates yield somewhat larger values for  $\eta/s$  [33–35].

### 3.3. RHIC conditions

In relativistic HICs the chemical freeze-out of hadron ratios [36] at a temperature of  $T_{\text{chem}} \simeq 170$  MeV is significantly earlier than thermal freeze-out of the light hadrons at  $T_{\text{fo}} \simeq 100$  MeV. Therefore, to conserve the observed particle ratios in the hadronic evolution, effective chemical potentials are required, reaching appreciable values even at RHIC energies [37], e.g.,  $\mu_\pi(T = 100 \text{ MeV}) \simeq 80$  MeV. We implement the chemical potentials into the thermal hadron distribution functions and recalculate the  $D$ -meson equilibration rate, Eq. (6). As a result, the latter is enhanced at temperatures below  $T_{\text{chem}}$ , staying above  $1/(25 \text{ fm})$  for  $T \geq 130$  MeV (cf. Fig. 4), implying noticeable modifications of  $D$ -meson spectra in the hadronic phase of nuclear collisions at RHIC. For example, if the hadronic evolution lasts for  $\Delta\tau_{\text{had}} \simeq 5$  fm, the expected modification amounts to ca.  $(1 - \exp[-A \Delta\tau_{\text{had}}]) \simeq 20\%$ .

## 4. Summary and conclusion

We have evaluated kinetic transport of  $D$ -mesons in hot hadronic matter by elastic scattering off the 10 most abundant hadron species. The interaction strength with mesons and baryons has been estimated from existing microscopic models for  $D$ -hadron scattering, constrained by chiral symmetry and vacuum spectroscopy. In a pion gas at  $T = 100$  MeV,  $D\pi$  resonance scattering leads to a relaxation rate which is substantially smaller than what has been found in a recent calculation using heavy-meson chiral perturbation theory. Yet, when extrapolating our full results to temperatures in the vicinity of  $T_c$ , the relaxation rate reaches ca.  $0.1/\text{fm}$ , translating into a spatial diffusion coefficient of  $D_s \simeq 5/(2\pi T)$ . This is comparable to non-perturbative  $T$ -matrix calculations of charm-quark relaxation in the QGP. On the one hand, this suggests a rather smooth evolution of charm transport through  $T_c$ , i.e., a kind of “duality” of hadronic and quark-based calculations, reminiscent of what has been found for dilepton emission rates. On the other hand, it implies that quantitative calculations of  $D$ -meson spectra in heavy-ion collisions have to account for hadronic diffusion. This insight is reinforced once chemical freeze-out is implemented into the evolution of the hadronic phase (via effective chemical potentials), with an estimated modification of  $D$ -meson observables of at least 20%. The apparent agreement of hadron- and quark-based approaches, when extrapolated to around  $T_c$ , is encouraging, especially since the magnitude of the transport coefficient is compatible with the phenomenology of current heavy-flavor observables at RHIC. Our findings thus pave the way for an improved theoretical accuracy which will be needed to take advantage of upcoming precision measurements at RHIC and LHC.

### 5. Note added

Two subsequently submitted papers have also addressed hadronic  $D$ -meson diffusion. In Ref. [38] the use of unitarized chiral effective  $D\pi$  interactions leads to relaxation rates in a pion gas in close agreement with our results. In Ref. [39]  $D$ -hadron interactions have been evaluated using Born amplitudes, leading to relaxation rates significantly larger than our results.

## Acknowledgements

We thank L. Tolos for informative discussions. This work has been supported by U.S. National Science Foundation (NSF)

CAREER Award PHY-0847538, by NSF grant PHY-0969394, by the A.-v.-Humboldt Foundation (Germany), by the RIKEN/BNL Research Center and DOE grant DE-AC02-98CH10886, and the JET Collaboration and DOE grant DE-FG02-10ER41682.

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